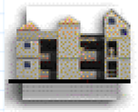


CHAPTER 4: Reinforced Concrete Columns

4.2 Effective Lengths and Critical Loads

Description

An ordinary or first order frame analysis does not include either the effects of the lateral sideways deflections of the column ends, or the effects of the deflections of members along their lengths on the axial loads and moments in a frame. The effects of the deflections of the column ends can be evaluated directly by performing a second order analysis or by using approximate methods. In frames where lateral stability is dependent upon the bending stiffness of rigidly connected beams and columns, the effective length of compression members must be determined (unless a second order analysis meeting the requirements of ACI 318, Section 10.10 is performed). In addition, for frames that are braced laterally, effective length factors less than 1 may be used to permit reduction in the amplified moment, which may be required by Section 10.11.5 of ACI 318. The effective length method uses k factors to equate the strength of a framed compression element of length L to an equivalent pin-ended member of length kL subject to axial load only.

This application calculates the effective length factors and critical loads for rectangular columns in braced or unbraced frames. A single story, all stories, or selected stories may be entered. The summations of the critical loads for each story computed by this application are used in **Section 4.3** to compute the magnified moments to be used for design of the columns.

Mathcad solve blocks are used to solve the equations for effective length factors. This eliminates the need to use either the alignment charts or the simplified formulas shown in the ACI 318 Commentary.

The intermediate variables calculated by this application include the moments of inertia of the gross concrete sections, the relative stiffness of rectangular columns and rectangular or flanged beams, and the ratios of the sum of the column stiffness to the sum of the beam stiffness at each specified joint.

This document is divided into four steps:

1. The user enters section dimensions, member lengths and moments of inertia, and relative member stiffnesses are calculated
2. The user identifies the beams and columns at each joint, and stiffness ratios (y values) at the joints are calculated.
3. The user identifies the stiffness ratios at each end of the columns and the effective length factors are calculated.

4. The user enters the "creep" factor βd and critical loads are calculated.

Note: For entry of more than two floors at a time, the user should have a basic understanding of matrix notation and Mathcad's **vectorize** operator to use this application. The number of joints that may be entered at one time is limited to 100, the maximum matrix size Mathcad (version 3.1) provides for manual entry of elements. For example, if there are five joints per floor with a unique combination of beam and column stiffnesses, a 20 story building could be entered.

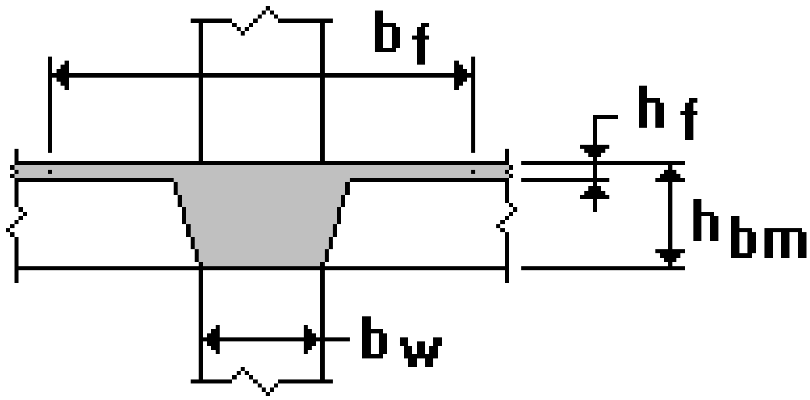
A summary of input and computed variables is shown on pages 17 and 19.

Reference:

ACI 318-89 "Building Code Requirements for Reinforced Concrete." (Revised 1992)

Input

Notation

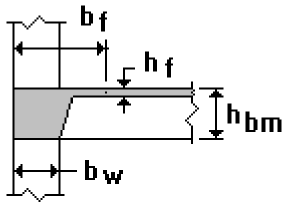


SECTION B

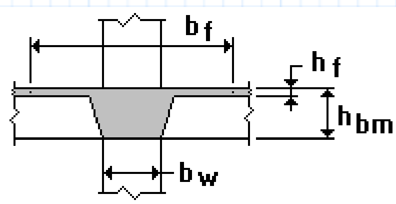
FLOOR PLAN

1 Circled numbers designate joints and columns.

1 Numbers within squares designate beams.



SECTION A



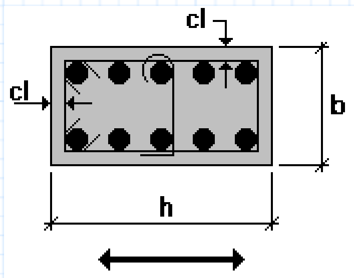
SECTION B

Step 1

Input Variables

The user must enter the section dimensions and clear span or clear height for any member which has a unique combination of section dimensions and clear span or height. In this example frames in the shorter direction are under consideration.

The column thickness h must be the dimension parallel to the frame under consideration.



Direction Under Consideration

The columns above and below each story for which effective lengths and critical loads are being calculated must be entered to calculate the sum of the relative stiffness of all columns at the joint. The number of column levels to be entered will therefore be one or two more than the number of stories being calculated unless you are entering all stories of the building. Figure 1 below shows the case used in this document where the effective lengths and critical loads for levels 1 and 2 are being calculated, Figure 2 shows a case where the effective lengths and critical loads for level 2 are to be calculated, and Figure 3 shows the case where all levels of a three story building are to be calculated. The dashed lines indicate that the member properties are not required.

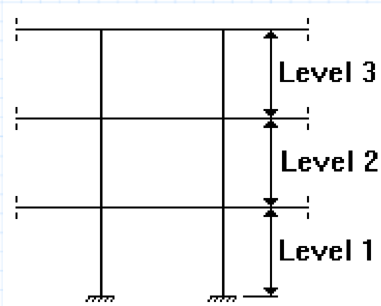


Figure 1

(K values & ΣPcr, Levels 1 & 2)

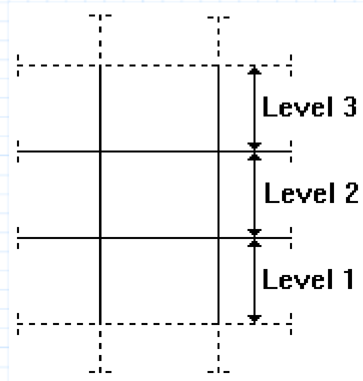


Figure 2

(K values & ΣPcr, Level 2)

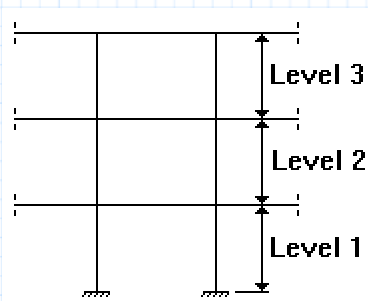


Figure 3

(K values & ΣPcr, Levels 1, 2, & 3)

Enter the size and length of each column with a different combination of size and/or length.

Width of columns: $b_{col} := \begin{bmatrix} 18 & 24 & 12 & 24 \\ 18 & 24 & 12 & 24 \\ 18 & 24 & 12 & 24 \end{bmatrix}^{T} \ in$

← Level 1
← Level 2
← Level 3

Thickness of columns: $h_{col} := \begin{bmatrix} 18 & 12 & 24 & 12 \\ 18 & 12 & 24 & 12 \\ 18 & 12 & 24 & 12 \end{bmatrix}^{T} \ in$

← Level 1
← Level 2
← Level 3

Clear height of columns: $L_{col} := \begin{bmatrix} 14 & 14 & 14 & 14 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \end{bmatrix}^{T} \; ft$ ← Level 1 ← Level 2 ← Level 3

Number of columns at each story with a different combination of size and length (for use in Step 4): $NoCols := \begin{bmatrix} 6 & 6 & 4 & 4 \\ 6 & 6 & 4 & 4 \end{bmatrix}^{T}$ ← Level 1 ← Level 2

Effective width of flanged or rectangular beams: $b_f := \begin{bmatrix} 60 & 60 & 21 & 21 \\ 60 & 60 & 21 & 21 \end{bmatrix}^{T} \; in$ ← Level 1 ← Level 2

Thickness of beams: $h_{bm} := 20 \; in$

Flange thickness: $h_f := 3 \; in$

Beam web width of flanged beams, or width of rectangular beams: $b_w := \begin{bmatrix} 18 & 18 & 12 & 12 \\ 18 & 18 & 12 & 12 \end{bmatrix}^{T} \; in$ ← Level 1 ← Level 2

Clear span of beams: $L_{bm} := \begin{bmatrix} 18.5 & 18.75 & 18 & 18.5 \\ 18.5 & 18.75 & 18 & 18.5 \end{bmatrix}^{T} \; ft$ ← Level 1 ← Level 2

All variables are entered either as single values, or as transposed matrices of the same size, with each column representing a joint and each row a framed level. This requires duplicate entries but provides maximum flexibility for changing dimensions, and avoids additional manipulation required when calculating with arrays of different sizes.

Computed Variables for Steps 1, 2, and 3

I_{col}	moments of inertia of gross concrete section of columns
I_{bm}	moments of inertia of gross concrete section of beams
K_{col}	relative column stiffness, strong or X axis
K_{bm}	relative stiffness of beams
k_s	slenderness factors for unbraced frames
k_b	slenderness factors for braced frames
P_c	critical load computed using ACI 318, Eq. (10-9)

Material Properties and Constants

Enter compressive strength and unit weight of concrete for columns and beams.

Specified compressive strength of concrete for columns: $f'_{c\_col} := 4\ ksi$

Specified compressive strength of concrete for beams: $f'_{c\_bm} := 4\ ksi$

Unit weight of column concrete: $w_{col} := 145\ pcf$

Unit weight of column concrete: $w_{bm} := 145\ pcf$

Modulus of elasticity of reinforcement (ACI 318, 8.5.2): $E_s := 29000\ ksi$

Modulus of elasticity of column concrete for values of w_c between 90 pcf and 155 pcf, (ACI 318, 8.5.1):

$$E_{col} := \left(\frac{w_{col}}{pcf}\right)^{1.5} \cdot 33 \cdot \sqrt{\frac{f'_{c\_col}}{psi}} \cdot psi \qquad E_{col} = 3644\ ksi$$

Modulus of elasticity of beam concrete for values of w_c between 90 pcf and 155 pcf, (ACI 318, 8.5.1):

$$E_{bm} := \left(\frac{w_{col}}{pcf}\right)^{1.5} \cdot 33 \cdot \sqrt{\frac{f'_{c\_bm}}{psi}} \cdot psi \qquad E_{bm} = 3644\ ksi$$

Mathcad variable ORIGIN defined equal to 1 so that index numbers of matrices will correspond to floor levels.

$\mathbf{ORIGIN} := 1$

Calculations for Step 1

Moments of inertia of gross concrete section of columns:

$$I_{col} := \overrightarrow{\frac{1}{12} \cdot b_{col} \cdot h_{col}^{3}}$$

$$I_{col}^{T} = \begin{bmatrix} 8748 & 3456 & 13824 & 3456 \\ 8748 & 3456 & 13824 & 3456 \\ 8748 & 3456 & 13824 & 3456 \end{bmatrix} \mathbf{in}^{4}$$

Calculations for Moments of Inertia of Beam Sections

Distance from the neutral axis of the gross section to the top of the section:

$$y_t := \overrightarrow{\frac{\frac{1}{2} \cdot \left(b_w \cdot h_{bm}^{2} + \left(b_f - b_w\right) \cdot h_f^{2}\right)}{b_w \cdot h_{bm} + \left(b_f - b_w\right) \cdot h_f}}$$

$$y_t^{T} = \begin{bmatrix} 7.796 & 7.796 & 9.14 & 9.14 \\ 7.796 & 7.796 & 9.14 & 9.14 \end{bmatrix} \mathbf{in}$$

Distance from the neutral axis of the gross section to the bottom of the section:

$$y_b := h_{bm} - y_t$$

$$y_b^{T} = \begin{bmatrix} 12.204 & 12.204 & 10.86 & 10.86 \\ 12.204 & 12.204 & 10.86 & 10.86 \end{bmatrix} \mathbf{in}$$

Moment of inertia of gross concrete section beams:

$$I_{bm} := \overrightarrow{\frac{1}{12} \cdot \left(b_w \cdot h_{bm}^{3} + \left(b_f - b_w\right) \cdot h_f^{3}\right) + b_w \cdot h_{bm} \cdot \left(\frac{h_{bm}}{2} - y_t\right)^{2} + \left(b_f - b_w\right) \cdot h_f \cdot \left(\frac{h_f}{2} - y_t\right)^{2}}$$

$$I_{bm}^{T} = \begin{bmatrix} 18837.83 & 18837.83 & 9773.73 & 9773.73 \\ 18837.83 & 18837.83 & 9773.73 & 9773.73 \end{bmatrix} \mathbf{in}^{4}$$

Calculations for Relative Stiffness, EI/L

Relative stiffness of columns:

$$K_{col} := E_{col} \cdot \overrightarrow{\frac{I_{col}}{L_{col}}}$$

$$K_{col}^{T} = \begin{bmatrix} 189756 & 74965 & 299861 & 74965 \\ 265658 & 104951 & 419806 & 104951 \\ 265658 & 104951 & 419806 & 104951 \end{bmatrix} kip \cdot in$$

Relative stiffness of beams:

$$K_{bm} := E_{bm} \cdot 0.5 \cdot \overrightarrow{\frac{I_{bm}}{L_{bm}}}$$

$$K_{bm}^{T} = \begin{bmatrix} 154612 & 152551 & 82447 & 80218 \\ 154612 & 152551 & 82447 & 80218 \end{bmatrix} kip \cdot in$$

Beam stiffness is multiplied by 0.5 to allow for reduced stiffness due to cracking. See the Commentary of ACI 318, Section R10.11.2.

Step 2

In Step 1 the relative stiffness of columns and beams with unique combinations of cross section dimensions and member length were determined. In this section the user must assign the relative column and beam stiffnesses at each joint for calculation of the ratios of the sum of the column stiffness to the sum of the beam stiffnesses. This is a simple calculation. The essential point is to ensure that the correct beam and column stiffnesses for each joint are used.

In this example there are 12 columns and 8 beams. The stiffness of a beam at any particular joint is specified by its matrix row number, which matches the beam number shown on the sketch above, and its matrix column number which corresponds to the level it supports. A column is specified by its matrix row number, which corresponds to the joint number, and its matrix column number, which corresponds to the level it supports.

Example: $K_{col_{1,2}}$ This is the relative stiffness of the column at joint number 1, level 2.

Input Variables for Step 2

ψ = the ratio of ΣKcol of compression members to ΣKbm of flexural members in a plane at one end of a compression member.

Joint 1, Level 1: $\psi_{1,1} := \dfrac{K_{col_{1,1}} + K_{col_{1,2}}}{K_{bm_{1,1}} + K_{bm_{2,1}}}$ $\psi_{1,1} = 1.483$

Joint 1, Level 2: $\psi_{1,2} := \dfrac{K_{col_{1,2}} + K_{col_{1,3}}}{K_{bm_{1,2}} + K_{bm_{2,2}}}$ $\psi_{1,2} = 1.73$

Joint 2, Level 1: $\psi_{2,1} := \dfrac{K_{col_{2,1}} + K_{col_{2,2}}}{K_{bm_{2,1}}}$ $\psi_{2,1} = 1.179$

Joint 2, Level 2: $\psi_{2,2} := \dfrac{K_{col_{2,1}} + K_{col_{2,3}}}{K_{bm_{2,2}}}$ $\psi_{2,2} = 1.179$

Joint 3, Level 1: $\psi_{3,1} := \dfrac{K_{col_{3,1}} + K_{col_{3,2}}}{K_{bm_{3,1}} + K_{bm_{4,1}}}$ $\psi_{3,1} = 4.424$

Joint 3, Level 2: $\psi_{3,2} := \dfrac{K_{col_{3,2}} + K_{col_{3,3}}}{K_{bm_{3,2}} + K_{bm_{4,2}}}$ $\psi_{3,2} = 5.162$

Joint 4, Level 1: $\psi_{4,1} := \dfrac{K_{col_{4,1}} + K_{col_{4,2}}}{K_{bm_{4,1}}}$ $\psi_{4,1} = 2.243$

Joint 4, Level 2: $\psi_{4,2} := \dfrac{K_{col_{4,2}} + K_{col_{4,3}}}{K_{bm_{4,2}}}$ $\psi_{4,2} = 2.617$

Step 3

Now the effective length factors for both unbraced and braced frames are calculated.

Using the joint stiffness ratios calculated in Step 2 the user must assign the stiffness ratio at each end of each column for which effective length factors are to be determined. The essential point is to assign the correct joint stiffness to the ends of each column.

In this example two levels will be calculated. The first level with the lower ends of the columns fixed against rotation, and the second level with a level of columns above. The letters A and B represent the two ends of the column, top and bottom. The theoretical ψ value for a fixed end is 0, however for this example the ψ value at the lower end of the 1st story is assumed equal to 0.2 to allow for small end rotations which may occur unless the foundation is extremely rigid.

Input joint stiffness values:

$$\psi_A^{\langle 1\rangle} := \psi^{\langle 1\rangle} \qquad \left(\psi_A^{\langle 1\rangle}\right)^T = [1.483 \;\; 1.179 \;\; 4.424 \;\; 2.243]$$

$$\psi_B^{\langle 1\rangle} := [0.2 \;\; 0.2 \;\; 0.2 \;\; 0.2]^T$$

$$\psi_A^{\langle 2\rangle} := \psi^{\langle 2\rangle} \qquad \left(\psi_A^{\langle 2\rangle}\right)^T = [1.73 \;\; 1.179 \;\; 5.162 \;\; 2.617]$$

$$\psi_B^{\langle 2\rangle} := \psi^{\langle 1\rangle} \qquad \left(\psi_B^{\langle 2\rangle}\right)^T = [1.483 \;\; 1.179 \;\; 4.424 \;\; 2.243]$$

Calculations for Step 3

Effective length factor for columns in unbraced frames:
(The equation shown within the Mathcad solve block is the equation solved by the Jackson and Moreland Alignment Chart, Fig. 10.11.2 (b) of ACI 318.)

Guess value of k_s: $k_s := 1$ ψ_A and ψ_B range from 0 to ∞
k ranges from 1 to ∞

Constraints

Guess Values

$$\frac{\psi_A \cdot \psi_B \cdot \left(\frac{\pi}{k_s}\right)^2 - 36}{6 \cdot (\psi_A + \psi_B)} = \frac{\frac{\pi}{k_s}}{tan\left(\frac{\pi}{k_s}\right)} \qquad k_s \ge 1$$

Solver

$$f(\psi_A, \psi_B) := \mathbf{Find}(k_s)$$

Effective length factors in unbraced frames:

$$k_s := \overrightarrow{f(\psi_A, \psi_B)}$$

$$k_s^{T} = \begin{bmatrix} 1.255 & 1.215 & 1.511 & 1.34 \\ 1.487 & 1.369 & 2.187 & 1.694 \end{bmatrix}$$

Effective length factors for columns in braced frames:
(The equation shown within the Mathcad solve block is the equation solved by the Jackson and Moreland Alignment Chart, Fig. 10.11.2 (a) of ACI 318.)

Guess value of kb: $k_b := 0.75$

Guess Values

Constraints

$$k_b > 0.5 \qquad k_b < 1$$

$$\frac{\psi_A \cdot \psi_B}{4} \cdot \left(\frac{\pi}{k_b}\right)^2 + \left(\frac{\psi_A + \psi_B}{2}\right) \cdot \left(1 - \frac{\frac{\pi}{k_b}}{tan\left(\frac{\pi}{k_b}\right)}\right) + \frac{2 \cdot tan\left(\frac{\pi}{2 \cdot k_b}\right)}{\frac{\pi}{k_b}} = 1$$

Solver

$$f2(\psi_A, \psi_B) := \mathbf{Find}(k_b)$$

Effective length factors for columns in braced frames:

$$k_b := \overrightarrow{f2\left(\psi_A, \psi_B\right)}$$

$$k_b^{\mathrm{T}} = \begin{bmatrix} 0.697 & 0.686 & 0.735 & 0.715 \\ 0.831 & 0.795 & 0.927 & 0.874 \end{bmatrix}$$

Step 4

Enter the ratio of maximum factored axial dead load to maximum total factored axial load, where the load is due to gravity effects only in the calculation of P_c in Eq. (10-7) of ACI 318:

$$\beta_{d\_b} := 0.735$$

The β_{d_b} factor for gravity loads which produce no appreciable sidesway is the ratio of the factored dead load to the total factored load on the column under consideration. The β_{d_b} factor may calculated for each column and entered as a matrix, with a value for each column to be evaluated. If the loads are essentially uniform, as assumed in this example, a single value may be entered. This factor reduces the column stiffness to allow for the effect of "creep", and consequentially decreases the critical load.

Enter the ratio of the maximum factored sustained lateral load to the maximum total factored lateral load in that story in the calculation of P_c in Eq. (10-8) of ACI 318:

$$\beta_{d\_s} := 0$$

The β_{d_s} factor is 0 for wind or seismic loads. Appreciable sustained lateral load may occur due to an unsymmetrical frame or unsymmetrical dead loads. When there is appreciable sustained load it may be necessary to calculate a value of β_{d_s} for each column.

Calculations for Step 4

Separation of the values of Icol and Lcol for levels 1 and 2 from the matrices containing the values of Icol and Lcol for levels 1, 2 and 3:

$$I'_{col} := \text{augment}\left(I_{col}^{\langle 1\rangle}, I_{col}^{\langle 2\rangle}\right)$$

$$I'^{\,\mathrm{T}}_{col} = \begin{bmatrix} 8748 & 3456 & 13824 & 3456 \\ 8748 & 3456 & 13824 & 3456 \end{bmatrix} in^4$$

$$L'_{col} := \text{augment}\left(L_{col}^{\langle 1\rangle}, L_{col}^{\langle 2\rangle}\right)$$

$$L'^{\,\mathrm{T}}_{col} = \begin{bmatrix} 14 & 14 & 14 & 14 \\ 10 & 10 & 10 & 10 \end{bmatrix} ft$$

Flexural stiffness for braced frames computed by Eq. (10-11) of ACI 318:

$$EI_b := \overrightarrow{\frac{E_{col} \cdot I'_{col}}{2.5} \cdot \frac{1}{1+\beta_{d\_b}}}$$

$$EI_b^{\mathrm{T}} = \begin{bmatrix} 7.35 \cdot 10^6 & 2.904 \cdot 10^6 & 1.161 \cdot 10^7 & 2.904 \cdot 10^6 \\ 7.35 \cdot 10^6 & 2.904 \cdot 10^6 & 1.161 \cdot 10^7 & 2.904 \cdot 10^6 \end{bmatrix} kip \cdot in^2$$

Flexural stiffness for frames subject to loads producing appreciable sidesway, computed by Eq. (10-11) of ACI 318:

$$EI_s := \overrightarrow{\frac{E_{col} \cdot I'_{col}}{2.5} \cdot \frac{1}{1+\beta_{d\_s}}}$$

$$EI_s^{\mathrm{T}} = \begin{bmatrix} 1.275 \cdot 10^7 & 5.038 \cdot 10^6 & 2.015 \cdot 10^7 & 5.038 \cdot 10^6 \\ 1.275 \cdot 10^7 & 5.038 \cdot 10^6 & 2.015 \cdot 10^7 & 5.038 \cdot 10^6 \end{bmatrix} kip \cdot in^2$$

Critical column loads for unbraced frames (ACI 318, Eq. (10-9)):

$$P_{c\_s} := \overrightarrow{\frac{\pi^2 \cdot EI_s}{\left(k_s \cdot L'_{col}\right)^2}}$$

$$P_{c\_s}{}^{\mathrm{T}} = \begin{bmatrix} 2833 & 1193 & 3087 & 981 \\ 3954 & 1841 & 2889 & 1204 \end{bmatrix} kip$$

Critical column loads with sidesway inhibited (ACI 318, Eq. (10-9)):

$$P_{c\_b} := \overrightarrow{\frac{\pi^2 \cdot EI_b}{\left(k_b \cdot L'_{col}\right)^2}}$$

$$P_{c\_b}{}^{\mathrm{T}} = \begin{bmatrix} 5284 & 2156 & 7521 & 1988 \\ 7295 & 3150 & 9258 & 2605 \end{bmatrix} kip$$

Sum of critical loads for all columns with a unique combination of size and length, at each level, with sidesway permitted:

$$P'_{c\_s} := \left(\overrightarrow{(NoCols) \cdot P_{c\_s}}\right)$$

$$P'_{c\_s}{}^{\mathrm{T}} = \begin{bmatrix} 16999.2 & 7158.6 & 12349.3 & 3923 \\ 23726.3 & 11048.6 & 11554.9 & 4815.6 \end{bmatrix} kip$$

Summation of critical loads at each level, sidesway permitted:

$$\sum P'_{c\_s}{}^{\langle 1 \rangle} = \left(2 \cdot 10^8\right) \frac{kg \cdot m}{s^2} \qquad \leftarrow \text{Level 1}$$

$$\sum P'_{c\_s}{}^{\langle 2 \rangle} = \left(2 \cdot 10^8\right) \frac{kg \cdot m}{s^2} \qquad \leftarrow \text{Level 2}$$

Summary

Input Variables

Width of columns:

$$b_{col}^{T} = \begin{bmatrix} 18 & 24 & 12 & 24 \\ 18 & 24 & 12 & 24 \\ 18 & 24 & 12 & 24 \end{bmatrix} \, in$$

Thickness of columns:

$$h_{col}^{T} = \begin{bmatrix} 18 & 12 & 24 & 12 \\ 18 & 12 & 24 & 12 \\ 18 & 12 & 24 & 12 \end{bmatrix} \, in$$

Clear height of columns:

$$L_{col}^{T} = \begin{bmatrix} 14 & 14 & 14 & 14 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \end{bmatrix} \, ft$$

Effective width of flanged or rectangular beams:

$$b_{f}^{T} = \begin{bmatrix} 5 & 5 & 1.75 & 1.75 \\ 5 & 5 & 1.75 & 1.75 \end{bmatrix} \, ft$$

Thickness of beams:

$$h_{bm} = 20 \, in$$

Flange thickness:

$$h_{f} = 3 \, in$$

Beam web width of flanged beams, or width of rectangular beams:

$$b_w^{\mathrm{T}} = \begin{bmatrix} 18 & 18 & 12 & 12 \\ 18 & 18 & 12 & 12 \end{bmatrix} \boldsymbol{in}$$

Clear span of beams:

$$L_{bm}^{\mathrm{T}} = \begin{bmatrix} 18.5 & 18.75 & 18 & 18.5 \\ 18.5 & 18.75 & 18 & 18.5 \end{bmatrix} \boldsymbol{ft}$$

β_{d_b} factors, braced frame:

$$\beta_{d\_b} = 0.735$$

β_{d_s} factors, sway frame:

$$\beta_{d\_s} = 0$$

Computed Variables

Effective length factors k_s, sidesway permitted:

$$k_s^{\mathrm{T}} = \begin{bmatrix} 1.255 & 1.215 & 1.511 & 1.34 \\ 1.487 & 1.369 & 2.187 & 1.694 \end{bmatrix} \begin{matrix} \leftarrow \text{Level 1} \\ \leftarrow \text{Level 2} \end{matrix}$$

Effective length factors k_b, sidesway inhibited:

$$k_b^{\mathrm{T}} = \begin{bmatrix} 0.697 & 0.686 & 0.735 & 0.715 \\ 0.831 & 0.795 & 0.927 & 0.874 \end{bmatrix}$$

Critical column loads, unbraced frame:

$$P_{c\_s}^{\mathrm{T}} = \begin{bmatrix} 2833.2 & 1193.1 & 3087.3 & 980.7 \\ 3954.4 & 1841.4 & 2888.7 & 1203.9 \end{bmatrix} kip$$

Critical column loads, braced frame:

$$P_{c\_b}^{\mathrm{T}} = \begin{bmatrix} 5284.3 & 2156.3 & 7521 & 1987.7 \\ 7295.3 & 3149.6 & 9258 & 2604.6 \end{bmatrix} kip$$

Sum of the critical loads for each level, unbraced frame:

$$\sum P'_{c\_s}{}^{\langle 1 \rangle} = 40430\ kip$$ ← Level 1

$$\sum P'_{c\_s}{}^{\langle 2 \rangle} = 51145\ kip$$ ← Level 2

Moments of inertia are calculated using the gross concrete section neglecting reinforcement. If any input is entered as a vector, the vectors must be "transposed" for horizontal display.