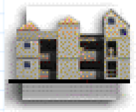


CHAPTER 5: Structural Steel Columns - ASD Design

5.3 Steel Pipe Columns

Description

The selection of steel columns subject to axial load and bending is a tedious trial and error process. The first step is to guess a suitable trial section or to use fairly elaborate approximate formulas to select a reasonable first trial selection. The second step is to check the first trial section to determine if it satisfies the AISC Specification interaction equations. If the first trial selection is not satisfactory the process must be repeated with a second trial section. If the lowest weight section is required, additional cycles of calculations may be required .

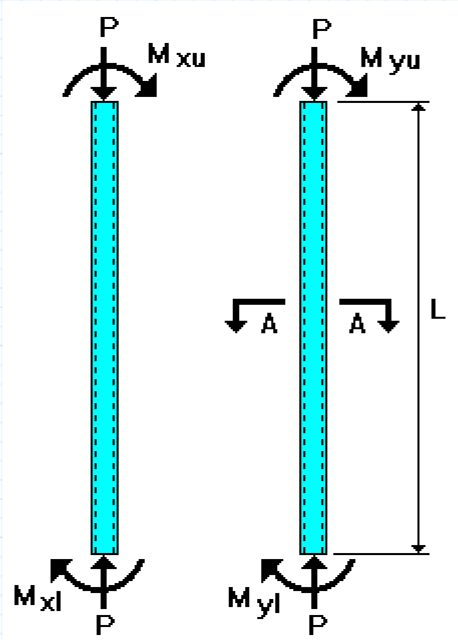
This application computes section properties, slenderness ratios, actual and allowable axial and bending stresses, amplification factors and moment modifiers, and the beam-column interaction equations of the AISC Specification, for hollow circular tubes or pipe column sections with axial load and end moments about one or both axes. The required input includes axial load and end moments about one or both axes, column length, effective length factors, yield strength, indication as to whether the column is free to sway about one or both axis directions, and diameter and thickness of section.

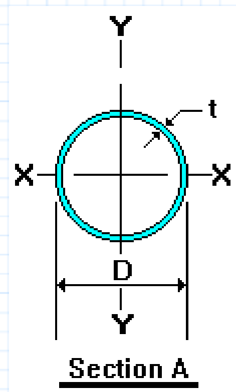
A summary of input, the solution of the interaction equations, and a listing of all computed values is shown on pages 10-12.

Reference: AISC "Specification for Structural Steel Buildings -- Allowable Stress Design and Plastic Design with Commentary." June 1, 1989

Input

Notation





Input Variables

Yield strength of steel:	$F_y := 36 \cdot ksi$
Column axial load:	$P := 250 \cdot kip$
Unbraced column length:	$L := 19.5 \cdot ft$
End moment about the X axis, at the upper end of the column:	$M_{xu} := 12 \cdot kip \cdot ft$
End moment about the X axis, at the lower end of the column:	$M_{xl} := -12 \cdot kip \cdot ft$
End moment about the Y axis, at the upper end of the column:	$M_{yu} := 4 \cdot kip \cdot ft$
End moment about the Y axis, at the lower end of the column:	$M_{yl} := -4 \cdot kip \cdot ft$

Note ⇒ Positive values of M indicate clockwise moments and negative values indicate counterclockwise moments.

Effective length factor relative to the X axis: $K_x := 1.0$

Effective length factor relative to the Y axis: $K_y := 1.0$

Variable "X_Sway" defined as 0 if sidesway is prevented, and 1 if sidesway is permitted: $X\_Sway := 0$

Variable "Y_Sway" defined as 0 if sidesway is prevented, and 1 if sidesway is permitted: $Y\_Sway := 0$

Enter estimated values for the outside diameter and wall thickness.

Outside diameter of pipe: $D := 12.75 \cdot in$

Wall thickness of pipe: $t := 0.500 \cdot in$

Computed Section Properties

A cross section area of member

W weight of section per unit length

I moment of inertia

r radius of gyration

S section modulus

Computed Variables

f_a	axial compressive stress
f_{b_xu}	bending stress about the X axis at the upper end of the column
f_{b_xl}	bending stress about the X axis at the lower end of the column
f_{b_yu}	bending stress about the Y axis at the upper end of the column
f_{b_yl}	bending stress about the Y axis at the lower end of the column
F_a	allowable compressive axial stress (AISC Specification, Eqs. (E2-1) and (E2-2))
F_b	Allowable bending stress for compact or non-compact sections (AISC Specification, Eq. (F3-1))
F'_{ex}	Euler's stress about the X axis divided by a factor of safety (AISC Specification, Sect. H-1)
F'_{ey}	Euler's stress about the Y axis divided by a factor of safety (AISC Specification, Sect. H-1)
C_b	moment modifier factor (AISC Specification, Sect. F1-3)
CF	a variable defined within this application to reflect compact section criteria of AISC S pecification, Table B5.1. Section is compact if CF = 1, and a slender compression element (not covered by this application) if CF = 0.

Calculations

This application is limited to steels with a yield strength less than or equal to 65 ksi:

$$F_y := \text{if}\left(F_y > 65 \cdot ksi, 65 \cdot ksi, F_y\right) = 36\ ksi$$

Section Properties

Cross section area:

$$A := \pi \cdot \left(D \cdot t - t^2\right) = 19.242\ in^2$$

Section weight per unit length:

$$W := A \cdot 3.4 \cdot \frac{lb}{ft \cdot in^2} = 65.424\ \frac{lb}{ft}$$

Moment of inertia:

$$I := \frac{\pi \cdot \left(D^4 - (D - 2 \cdot t)^4\right)}{64} = 361.544\ in^4$$

Section modulus:

$$S := \frac{2 \cdot I}{D} = 56.713\ in^3$$

Radius of gyration:

$$r := \sqrt{\frac{I}{A}} = 4.335\ in$$

Axial stress:

$$f_a := \frac{P}{A} = 12.992\ ksi$$

Bending stress due to moment about the X axis at the upper end of the column:

$$f_{b\_xu} := \frac{M_{xu}}{S} \qquad f_{b\_xu} = 2.5\ ksi$$

Bending stress due to moment about the X axis at the lower end of the column:

$$f_{b\_xl} := \frac{M_{xl}}{S} = -2.539 \ ksi$$

Bending stress due to moment about the Y axis at the upper end of the column:

$$f_{b\_yu} := \frac{M_{yu}}{S} = 0.846 \ ksi$$

Bending stress due to moment about the Y axis at the lower end of the column:

$$f_{b\_yl} := \frac{M_{yl}}{S} = -0.846 \ ksi$$

Slenderness ratios about the X and Y axes:

$$\frac{K_x \cdot L}{r} = 54 \qquad \frac{K_y \cdot L}{r} = 54$$

Larger slenderness ratio:

$$SR := \mathbf{if}\left(\frac{K_x \cdot L}{r} > \frac{K_y \cdot L}{r}, \frac{K_x \cdot L}{r}, \frac{K_y \cdot L}{r}\right) = 53.984$$

If SR is >200 a larger section or bracing is required.

Modulus of elasticity of steel:

$$E := 29000 \cdot ksi$$

Column slenderness ratio separating elastic and inelastic buckling
(see AISC Specification, Eq. (E2-1)):

$$C_c := \sqrt{\frac{2 \cdot \pi^2 \cdot E}{F_y}} = 126.099$$

Section is compact if D/t is less than or equal to 3300/F_y (AISC Specification, Sect. B5 and Table B5.1). Hollow circular sections that are not compact are not covered by this application. Section is compact if CF = 1, and a slender compression element (not covered by this application) if CF = 0.

$$CF := \text{if}\left(\frac{D}{t} \le \frac{3300 \cdot ksi}{F_y}, 1, 0\right) \qquad \frac{D}{t} = 25.5 \qquad CF = 1$$

Allowable axial stress (AISC Specification, Eqs. (E2-1) and (E2-2)) with SR substituted for Kl/r:

(If SR is greater than 200 or if the section is a "slender compression element" this equation will return F_a equal to 0.)

$$F_a := \begin{array}{l} \text{if } (SR \ge C_c) \cdot (SR \le 200) \\ \quad \dfrac{12 \cdot \pi^2 \cdot E}{23 \cdot SR^2} \\ \text{else if } (SR > 200) \lor (CF = 0) \\ \quad 0 \cdot ksi \\ \text{else} \\ \quad \dfrac{\left(1 - \dfrac{SR^2}{2 \cdot {C_c}^2}\right) \cdot F_y}{\dfrac{5}{3} + \dfrac{3 \cdot SR}{8 \cdot C_c} - \dfrac{SR^3}{8 \cdot {C_c}^3}} \end{array} = 17.993\ ksi$$

Ratio of the smaller end moment M_{xa} to the larger end moment M_{xb} about the X axis:

$$M_{xa} := \text{if}\left(|M_{xu}| < |M_{xl}|, M_{xu}, M_{xl}\right)$$

$$M_{xb} := \text{if}\left(M_{xa} = M_{xu}, M_{xl}, M_{xu}\right)$$

$$\frac{M_{xa}}{M_{xb}} = -1$$

Ratio of the smaller end moment M_{ya} to the larger end moment M_{yb} about the Y axis:

$$M_{ya} := \mathbf{if}\left(\left|M_{yu}\right| < \left|M_{yl}\right|, M_{yu}, M_{yl}\right)$$

$$M_{yb} := \mathbf{if}\left(M_{ya} = M_{yu}, M_{yl}, M_{yu}\right)$$

$$\frac{M_{ya}}{M_{yb}} = -1$$

Moment modifier factor Cb (AISC Specification, Sect. F1-3):

$$C_b := 1.75 + 1.05 \cdot \frac{M_{xa}}{M_{xb}} + 0.3 \cdot \left(\frac{M_{xa}}{M_{xb}}\right)^2$$

$$C_b := \mathbf{if}\left(\left(X\_Sway = 0\right) \cdot \left(f_a > 0 \cdot ksi\right), 1.0, \mathbf{if}\left(C_b < 2.3, C_b, 2.3\right)\right) = 1$$

If the ends of the member are braced to prevent sidesway in the X direction and the axial stress is greater than 0 ksi, Cb equals 1.0.

Allowable bending stress for compact or non-compact sections (AISC Specification, Eq. (F3-1)):

$$F_b := \mathbf{if}\left(CF = 1, 0.66 \cdot F_y, 0 \cdot ksi\right) = 23.76\ ksi$$

Euler's stress for the X and Y directions, divided by a factor or safety and using the actual unbraced length and corresponding radius of gyration in the plain of bending (AISC Specification, Sect. H-1):

$$F'_{ex} := \frac{12 \cdot \pi^2 \cdot E}{23 \cdot \left(\frac{K_x \cdot L}{r}\right)^2} = 51.242\ ksi \qquad F'_{ey} := \frac{12 \cdot \pi^2 \cdot E}{23 \cdot \left(\frac{K_y \cdot L}{r}\right)^2} = 51.242\ ksi$$

Moment coefficients for X and Y directions (AISC Specification, Sect. H-1):

$$C_{mx} := \mathbf{if}\left(X\_Sway = 0, 0.6 - 0.4 \cdot \left(\frac{M_{xa}}{M_{xb}}\right), 0.85\right) \qquad C_{mx} = 1$$

$$C_{my} := \mathbf{if}\left(Y\_Sway = 0, 0.6 - 0.4 \cdot \left(\frac{M_{ya}}{M_{yb}}\right), 0.85\right) \qquad C_{my} = 1$$

AISC Specification Eqs. (H1-1) and (H1-2) for combined axial compression and bending, expressed as functions of bending stresses about the X and Y axes:

$$H1\_1\left(f_{bx}, f_{by}\right) := \frac{f_a}{F_a} + \frac{C_{mx} \cdot f_{bx}}{\left(1 - \frac{f_a}{F'_{ex}}\right) \cdot F_b} + \frac{C_{my} \cdot f_{by}}{\left(1 - \frac{f_a}{F'_{ey}}\right) \cdot F_b}$$

$$H1\_2\left(f_{bx}, f_{by}\right) := \frac{f_a}{0.60 \cdot F_y} + \frac{f_{bx}}{F_b} + \frac{f_{by}}{F_b}$$

At upper end of column:

$$H1\_1\left(\left|f_{b\_xu}\right|, \left|f_{b\_yu}\right|\right) = 0.913$$

$$H1\_2\left(\left|f_{b\_xu}\right|, \left|f_{b\_yu}\right|\right) = 0.744$$

At lower end of column:

$$H1\_1\left(\left|f_{b\_xl}\right|, \left|f_{b\_yl}\right|\right) = 0.913$$

$$H1\_2\left(\left|f_{b\_xl}\right|, \left|f_{b\_yl}\right|\right) = 0.744$$

$$HEQ := \begin{bmatrix} H1\_1\left(f_{b\_xu}, f_{b\_yu}\right) & H1\_1\left(f_{b\_xl}, f_{b\_yl}\right) \\ H1\_2\left(f_{b\_xu}, f_{b\_yu}\right) & H1\_2\left(f_{b\_xl}, f_{b\_yl}\right) \end{bmatrix}$$

$$\max\left(HEQ\right) = 0.913$$

If the maximum value of interaction equations H1_1 or H1_2 is less than or equal 1.0 the section is adequate. If the section is too small or too large, a new trial section may be entered.

Summary

Pipe Size

Outside diameter: $D = 12.75\ in$

Wall thickness: $t = 0.5\ in$

Input

Yield strength of steel: $F_y = 36\ ksi$

Column axial load: $P = 250\ kip$

Unbraced column length: $L = 19.5\ ft$

End moment about the X axis, at the upper end of the column: $M_{xu} = 12\ kip \cdot ft$

End moment about the Y axis, at the upper end of the column: $M_{yu} = 4\ kip \cdot ft$

End moment about the X axis, at the lower end of the column: $M_{xl} = -12\ kip \cdot ft$

End moment about the Y axis, at the lower end of the column: $M_{yl} = -4\ kip \cdot ft$

Effective length factor relative to the X axis: $K_x = 1$

Effective length factor relative to the Y axis: $K_y = 1$

Variable "X_Sway" defined as 0 if sidesway is prevented, and 1 if sidesway is permitted: $X\_Sway = 0$

Variable "Y_Sway" defined as 0 if sidesway is prevented, and 1 if sidesway is permitted: $Y\_Sway = 0$

Maximum value of H1-1 or H1-2: $\max(HEQ) = 0.913$

Note ⇒ If the maximum value of H1-1 or H1-2 is significantly larger than 1.0 a stronger section must be used.

Estimated trial tube wall thickness for follow-up estimates, assuming no change in b_f or d. Suggested value of t may be too thin or unavailable:

$\max(HEQ) \cdot t = 0.456\ in$

Intermediate Computed Values:
(Refer to document for definitions.)

$A = 19.242\ in^2$

$I = 361.544\ in^4$

$r = 4.335\ in$

$S = 56.713\ in^3$

$f_a = 12.992\ ksi$

$f_{b\_xu} = 2.539\ ksi$

$f_{b\_xl} = -2.539\ ksi$

$f_{b\_yu} = 0.846\ ksi$

$F_a = 17.993\ ksi$

$F_b = 23.76\ ksi$

$F'_{ex} = 51.242\ ksi$

$F'_{ey} = 51.242\ ksi$

$\frac{K_x \cdot L}{r} = 53.984$

$\frac{K_y \cdot L}{r} = 53.984$

$f_{b\_yl} = -0.846\ ksi$

$C_c = 126.099$

$C_b = 1$

$C_{mx} = 1$

$C_{my} = 1$

$CF = 1$