CHAPTER 5: Structural Steel Columns - ASD Design

5.4 Steel Column Base Plates

Description

This application calculates the plan dimensions and thickness for optimum (least weight) base plates for axially loaded wide flange columns. The plan dimensions and thickness of the plates are optimized for the final rounded plate dimensions and thicknesses.

The required input includes column loads, ratio of the concentric concrete support area to the plate area, the yield strength of the base plate steel, rounding factors for plan dimensions and thickness, and the column widths and depths. Four base plates are shown for illustrative purposes, however, up to 100 plates may be designed and displayed in one run of this application.

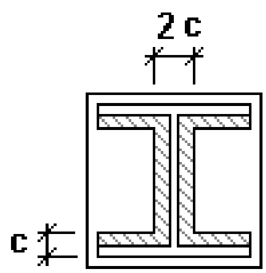
A summary of input and calculated values is shown on pages 11 and 12.

Reference: "Design of Base Plates for Wide Flange Columns - A Concatenation of Methods," Engineering Journal, Volume 27 (4th quarter 1990), by W. A. Thornton

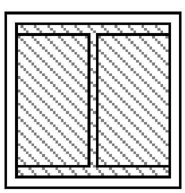
Input

Notation

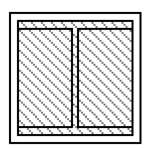
Bearing Area Model
for Small Plates



(Murray-Stockwell Method)
Fig. 2



(Yield Line Method)
Fig. 3



(Yield Line Method)
Fig. 3

Input Variables

Column axial loads:

$$P := \begin{bmatrix} 850 & 425 & 215 & 55 \end{bmatrix}^{T} \cdot kip$$

Column flange widths:

$$b := \begin{bmatrix} 12.515 & 12.08 & 8.00 & 6.50 \end{bmatrix}^{T} \cdot in$$

Column depths:

$$d := [13.88 \;\; 12.38 \;\; 8.12 \;\; 7.75]^{T} \cdot in$$

In this example the column loads and column width and depth are entered in transformed vectors with the number of vector rows corresponding to the number of base plates to be designed.

Computed Variables

f_p	actual bearing pressure on support
B	plate width
N	plate length
t_p	thickness of plate
c	dimension defining the assumed bearing area for the Murray Stockwell Method (See Fig. 2)

Material Properties and Constants

Enter values for f'_c, F_y, SzF, Szf, and R if different from that shown.

Compressive strength of concrete:	$f'_c := 4 \cdot ksi$
Yield strength of base plate steel:	$F_y := 36 \cdot ksi$
Factor for rounding plan dimensions of plates:	$SzF := 1 \cdot in$
Factor for rounding thickness of plates:	$Szf := 0.125 \cdot in$

Ratio of concentric area of concrete supports to the area of the base plates between 1 and 4:

$R := 4$ $\quad$ $R := \textbf{if}(R < 1, 1, \textbf{if}(R > 4, 4, R))$ $\quad$ $R = 4$

Notes ⇒

1) The plan dimensions and plate thicknesses are rounded to multiples of SzF and Szf, respectively.

2) The ratio R may be set at any value from 1 to 4. A value of 1 will set the allowable bearing pressure at 0.35f'c and require the smallest concrete bearing area and the largest base plates. A value of 4 will set the bearing pressure at 0.70f'c and require the smallest base plates and the largest concrete bearing areas.

The following values are computed from the entered material strengths and ratio R.

Ratio of the plan dimensions of the column bearing plate plate to the minimum concentric plan dimensions of the concrete bearing area:

$$\sqrt{R} = 2$$

Allowable bending stress in plates:

$$F_b := 0.75 \cdot F_y \qquad F_b = 27\ ksi$$

Allowable bearing pressure:

$$F_p := \textbf{if}\left((R<4)+(R\geq 1), \sqrt{R}\cdot 0.35\cdot f'_c, \textbf{if}\left(R>4, 0.70\cdot f'_c, 0.35\cdot f'_c\right)\right)$$

$$F_p = 2.8\ ksi$$

Calculations

Range variable i: $i := 0..\text{last}(P)$

Minimum plate area limited to no less than the columns' plan dimensions:

$$A_{pl_i} := \textbf{if}\left(\frac{P_i}{F_p} \leq b_i \cdot d_i, b_i \cdot d_i, \frac{P_i}{F_p}\right) \qquad A_{pl}^{\mathrm{T}} = [303.571\ \ 151.786\ \ 76.786\ \ 50.375]\ in^2$$

All computed values are displayed in transformed vectors or matrices. Values for each column being designed are in the corresponding columns of the transformed vector or matrix. For example, the minimum plate area for the first column entered is in the first column of the transformed vector displayed above.

Minimum plate area before overhangs on both sides may be increased.
(This procedure determines the area and dimensions of the smallest plate where m equals n.):

$$\Delta B_i := \textbf{if}\left(0.1 \cdot b_i \geq 0.025 \cdot d_i, 0 \cdot in, 0.025 \cdot d_i - 0.1 \cdot b_i\right)$$

$$\Delta B^{T} = [0\ \ 0\ \ 0\ \ 0]\ in$$

$$\Delta N_i := \textbf{if}\left(0.025 \cdot d_i \geq 0.1 \cdot b_i, 0 \cdot in, 0.1 \cdot b_i - 0.025 \cdot d_i\right)$$

$$\Delta N^{T} = [0.905\ \ 0.899\ \ 0.597\ \ 0.456]\ in$$

$$A2_i := \left(b_i + \Delta B_i\right) \cdot \left(d_i + \Delta N_i\right)$$

$$A2^{T} = [185.028\ \ 160.404\ \ 69.736\ \ 53.341]\ in^2$$

Optimization Procedure

The optimum or least weight plate is obtained when the plate overhang dimensions m and n are equal. Substituting the variable x for m and n, and equating the area of the base plate in terms of the column dimensions b and d and the plate overhang x, a quadratic equation is obtained which is then solved for the plate overhang x. The plate size with minimum theoretical plan dimensions is designated plate No. 0. Since the plate dimensions obtained are usually fractional dimensions, the next step is to round the theoretical plate width to the nearest multiple of the sizing factor SzF, compute a new theoretical plate length, and round that dimension up to the nearest multiple of SzF. This procedure is then repeated starting with the theoretical minimum plate length. The final step is to select the lighter of the two plates with rounded dimensions. The plates with minimum rounded plan dimensions are designated as plates No. 1 and No. 2. The optimum plate is the plate with the lesser weight, and is designated plate No. 3. The plate numbers correspond to the matrix column index numbers of the computed variables for the plate.

Plate overhang x, (with x = m = n), solved using the quadratic formula:

$$x_i := \frac{-\left(1.6 \cdot b_i + 1.9 \cdot d_i\right) + \sqrt{\left(1.6 \cdot b_i + 1.9 \cdot d_i\right)^2 - 16 \cdot \left(0.76 \cdot b_i \cdot d_i - A_{pl_i}\right)}}{8}$$

$$x^{\mathrm{T}} = [2.948 \;\; 0.826 \;\; 0.865 \;\; 0.449]\ in$$

Plate No. 0, with minimum possible plate dimensions B and N:

$$B_{i,0} := \mathbf{if}\left(A_{pl_i} > A2_i, 0.8 \cdot b_i + 2 \cdot x_i, \mathbf{if}\left(A_{pl_i} = b_i \cdot d_i, b_i, \mathbf{if}\left(\Delta B_i = 0 \cdot \boldsymbol{in}, b_i, \frac{A_{pl_i}}{d_i + \Delta N_i}\right)\right)\right)$$

$$\left(B^{\langle 0 \rangle}\right)^{\mathrm{T}} = [15.908 \;\; 12.08 \;\; 8.13 \;\; 6.5]\ in$$

$$N_{i,0} := \mathbf{if}\left(A_{pl_i} > A2_i, 0.95 \cdot d_i + 2 \cdot x_i, \mathbf{if}\left(A_{pl_i} = b_i \cdot d_i, d_i, \mathbf{if}\left(\Delta N_i = 0 \cdot \boldsymbol{in}, d_i, \frac{A_{pl_i}}{b_i + \Delta B_i}\right)\right)\right)$$

$$\left(N^{\langle 0 \rangle}\right)^{\mathrm{T}} = [19.082 \;\; 12.565 \;\; 9.444 \;\; 7.75]\ in$$

Plate No. 1, with B rounded to nearest multiple of SzF and N rounded up to the nearest multiple of SzF:

$$B_{i,1} := \mathbf{if}\left(SzF \cdot \mathrm{floor}\left(\frac{B_{i,0}}{SzF} + 0.5\right) \geq b_i, SzF \cdot \mathrm{floor}\left(\frac{B_{i,0}}{SzF} + 0.5\right), SzF \cdot \mathrm{ceil}\left(\frac{b_i}{\boldsymbol{in}}\right)\right)$$

$$\left(B^{\langle 1 \rangle}\right)^{\mathrm{T}} = [16 \;\; 13 \;\; 8 \;\; 7]\ in$$

$$N_{i,1} := \mathbf{if}\left(SzF \cdot \mathrm{ceil}\left(\frac{A_{pl_i}}{B_{i,1} \cdot SzF}\right) \geq d_i, SzF \cdot \mathrm{ceil}\left(\frac{A_{pl_i}}{B_{i,1} \cdot SzF}\right), SzF \cdot \mathrm{ceil}\left(\frac{d_i}{\boldsymbol{in}}\right)\right)$$

$$\left(N^{\langle 1 \rangle}\right)^{\mathrm{T}} = [19 \;\; 13 \;\; 10 \;\; 8]\ in$$

Plate No. 2, with N rounded to nearest multiple of SzF and B rounded up to nearest multiple of SzF:

$$N_{i,2} := \text{if}\left(SzF \cdot \text{floor}\left(\frac{N_{i,0}}{SzF} + 0.5\right) \ge d_i, SzF \cdot \text{floor}\left(\frac{N_{i,0}}{SzF} + 0.5\right), SzF \cdot \text{ceil}\left(\frac{d_i}{in}\right)\right)$$

$$\left(N^{\langle 2\rangle}\right)^{\mathrm{T}} = [19\ \ 13\ \ 9\ \ 8]\ in$$

$$B_{i,2} := \text{if}\left(SzF \cdot \text{ceil}\left(\frac{A_{pl_i}}{N_{i,2} \cdot SzF}\right) \ge b_i, SzF \cdot \text{ceil}\left(\frac{A_{pl_i}}{N_{i,2} \cdot SzF}\right), SzF \cdot \text{ceil}\left(\frac{b_i}{in}\right)\right)$$

$$\left(B^{\langle 2\rangle}\right)^{\mathrm{T}} = [16\ \ 13\ \ 9\ \ 7]\ in$$

Plate overhangs m and n:

$$j := 0..2 \qquad m_{i,j} := \frac{N_{i,j} - 0.95 \cdot d_i}{2} \qquad n_{i,j} := \frac{B_{i,j} - 0.80 \cdot b_i}{2}$$

$$m^{\mathrm{T}} = \begin{bmatrix} 2.948 & 0.402 & 0.865 & 0.194 \\ 2.907 & 0.62 & 1.143 & 0.319 \\ 2.907 & 0.62 & 0.643 & 0.319 \end{bmatrix} in$$

$$n^{\mathrm{T}} = \begin{bmatrix} 2.948 & 1.208 & 0.865 & 0.65 \\ 2.994 & 1.668 & 0.8 & 0.9 \\ 2.994 & 1.668 & 1.3 & 0.9 \end{bmatrix} in$$

There are three plates calculated for each of the columns entered. The three rows in the transformed m and n matrices above correspond to the 3 plates calculated for each column. The columns in the matrices correspond to the number of columns entered.

Plate bearing pressures:

$$f_{p_{i,j}} := \frac{P_i}{B_{i,j} \cdot N_{i,j}} \qquad f_p^{\mathrm{T}} = \begin{bmatrix} 2.8 & 2.8 & 2.8 & 1.092 \\ 2.796 & 2.515 & 2.688 & 0.982 \\ 2.796 & 2.515 & 2.654 & 0.982 \end{bmatrix} ksi$$

Yield line method for small base plates, uniform bearing over b x d

Equivalent overhang dimension using the yield line method for small base plates with uniform bearing over the area b x d:

$$n'_i := \frac{1}{4} \cdot \sqrt{d_i \cdot b_i}$$

$$n'^{\mathrm{T}} = \begin{bmatrix} 3.295 & 3.057 & 2.015 & 1.774 \end{bmatrix} \, in$$

When allowable bearing stresses are high this method will often govern over the conventional cantilevered design method.

Murray-Stockwell method for small lightly loaded base plates

Portion of total load acting over H shaped section (Fig. 2):

$$P_{o_{i,j}} := f_{p_{i,j}} \cdot b_i \cdot d_i$$

$$P_o^{\mathrm{T}} = \begin{bmatrix} 486.383 & 418.741 & 181.888 & 55 \\ 485.697 & 376.088 & 174.58 & 49.475 \\ 485.697 & 376.088 & 172.425 & 49.475 \end{bmatrix} kip$$

Intermediate variable for calculation of λ:

$$X_{i,j} := \frac{4 \cdot P_{o_{i,j}}}{\left(b_i + d_i\right)^2 \cdot F_p}$$

$$X^{\mathrm{T}} = \begin{bmatrix} 0.997 & 1 & 1 & 0.387 \\ 0.996 & 0.898 & 0.96 & 0.348 \\ 0.996 & 0.898 & 0.948 & 0.348 \end{bmatrix}$$

Variable for determining which method governs:

$$\lambda_{i,j} := \mathbf{if}\left(\frac{2 \cdot \sqrt{X_{i,j}}}{1+\sqrt{1-X_{i,j}}} \le 1, \frac{2 \cdot \sqrt{X_{i,j}}}{1+\sqrt{1-X_{i,j}}}, 1\right)$$

$$\lambda^{\mathrm{T}} = \begin{bmatrix} 1 & 1 & 1 & 0.698 \\ 1 & 1 & 1 & 0.653 \\ 1 & 1 & 1 & 0.653 \end{bmatrix}$$

When λ is equal to 1 the yield line method governs over the Murray-Stockwell method.

Largest overhang dimension for conventional cantilever design, yield line, or Murray-Stockwell methods:

$$k_{i,j} := \max\left(\begin{bmatrix} m_{i,j} \\ n_{i,j} \\ \lambda_{i,j} \cdot n'_i \end{bmatrix}\right)$$

$$k^{\mathrm{T}} = \begin{bmatrix} 3.295 & 3.057 & 2.015 & 1.238 \\ 3.295 & 3.057 & 2.015 & 1.158 \\ 3.295 & 3.057 & 2.015 & 1.158 \end{bmatrix} \, in$$

Plate thicknesses:

$$t_{p_{i,j}} := 2 \cdot k_{i,j} \cdot \sqrt{\frac{f_{p_{i,j}}}{F_y}}$$

$$t_p^{\mathrm{T}} = \begin{bmatrix} 1.838 & 1.705 & 1.124 & 0.431 \\ 1.837 & 1.616 & 1.101 & 0.383 \\ 1.837 & 1.616 & 1.094 & 0.383 \end{bmatrix} \, in$$

Plate thicknesses rounded up to nearest multiple of Szf:

$$j1 := 1..2$$

$$t_{p_{i,j1}} := Szf \cdot \text{ceil}\left(2 \cdot k_{i,j1} \cdot \sqrt{\frac{f_{p_{i,j1}}}{F_y}} \cdot \frac{1}{Szf}\right)$$

$$t_p^{\mathrm{T}} = \begin{bmatrix} 1.838 & 1.705 & 1.124 & 0.431 \\ 1.875 & 1.625 & 1.125 & 0.5 \\ 1.875 & 1.625 & 1.125 & 0.5 \end{bmatrix} in$$

Weight of plates:

$$Weight_{i,j} := \left(\left(t_{p_{i,j}} \cdot \left(B_{i,j} \cdot N_{i,j}\right)\right) \cdot \frac{3.4}{12} \cdot \frac{lb}{in^3}\right)$$

$$Weight^{\mathrm{T}} = \begin{bmatrix} 158.076 & 73.336 & 24.451 & 6.155 \\ 161.5 & 77.81 & 25.5 & 7.933 \\ 161.5 & 77.81 & 25.819 & 7.933 \end{bmatrix} lb$$

Optimum (least weight) plate:

$$B_{i,3} := \mathbf{if}\left(Weight_{i,1} \le Weight_{i,2}, B_{i,1}, B_{i,2}\right)$$

$$N_{i,3} := \mathbf{if}\left(Weight_{i,1} \le Weight_{i,2}, N_{i,1}, N_{i,2}\right)$$

$$t_{p_{i,3}} := \mathbf{if}\left(Weight_{i,1} \le Weight_{i,2}, t_{p_{i,1}}, t_{p_{i,2}}\right)$$

Weights of the optimum base plates:

$$Weight_{i,3} := \mathbf{if}\left(Weight_{i,1} \le Weight_{i,2}, Weight_{i,1}, Weight_{i,2}\right)$$

$$\left(Weight^{\langle 3 \rangle}\right)^{\mathrm{T}} = \begin{bmatrix} 161.5 & 77.81 & 25.5 & 7.933 \end{bmatrix} lb$$

Actual bearing pressures for the optimum base plates:

$$f_{p_{i,3}} := \frac{P_i}{B_{i,3} \cdot N_{i,3}}$$

$$\left(f_p^{\langle 3 \rangle}\right)^{T} = [2.796 \;\; 2.515 \;\; 2.688 \;\; 0.982]\; ksi$$

Plan dimensions of the minimum required concentric concrete bearing area:

$$W_i := B_{i,3} \cdot \sqrt{R}$$

$$W^{T} = [32 \;\; 26 \;\; 16 \;\; 14]\; in$$

$$L_i := N_{i,3} \cdot \sqrt{R}$$

$$L^{T} = [38 \;\; 26 \;\; 20 \;\; 16]\; in$$

The dimensions W and L are the minimum required concrete pier size for the bearing pressure used. In this example W and L are integers, but they may be fractional dimensions (which should be rounded up) if a different R value and corresponding bearing pressure is used.

Summary

Ratio of concentric area of concrete supports to the area of the base plates:	$R = 4$
Yield strength of base plate steel:	$F_y = 36\; ksi$
Compressive strength of concrete:	$f'_c = 4\; ksi$
Allowable bearing pressure:	$F_p = 2.8\; ksi$

Allowable bending stress in the plates: $F_b = 27\ ksi$

Factor for rounding plan dimensions of plates: $SzF = 1\ in$

Factor for rounding thickness of plates: $Szf = 0.125\ in$

Column loads: $P^{T} = [850\ \ 425\ \ 215\ \ 55]\ kip$

Column flange widths: $b^{T} = [12.515\ \ 12.08\ \ 8\ \ 6.5]\ in$

Column depths: $d^{T} = [13.88\ \ 12.38\ \ 8.12\ \ 7.75]\ in$

Optimum base plate width, length, thickness weight, and bearing pressures:

$$B_{i,3} = \begin{bmatrix} 16 \\ 13 \\ 8 \\ 7 \end{bmatrix} in \qquad N_{i,3} = \begin{bmatrix} 19 \\ 13 \\ 10 \\ 8 \end{bmatrix} in \qquad t_{p_{i,3}} = \begin{bmatrix} 1.875 \\ 1.625 \\ 1.125 \\ 0.5 \end{bmatrix} in$$

$$Weight_{i,3} = \begin{bmatrix} 161.5 \\ 77.81 \\ 25.5 \\ 7.933 \end{bmatrix} lb \qquad f_{p_{i,3}} = \begin{bmatrix} 2.796 \\ 2.515 \\ 2.688 \\ 0.982 \end{bmatrix} ksi$$

Plan dimensions of the minimum required concentric concrete bearing area:

$$B_{i,3} \cdot \sqrt{R} = \begin{bmatrix} 32 \\ 26 \\ 16 \\ 14 \end{bmatrix} in \qquad N_{i,3} \cdot \sqrt{R} = \begin{bmatrix} 38 \\ 26 \\ 20 \\ 16 \end{bmatrix} in$$