CHAPTER 5: Structural Steel Columns - ASD Design

5.5 Elastic Effective Length Factors

Description

In frames where lateral stability is dependent upon the bending stiffness of rigidly connected beams and columns the effective length of compression members must be determined. The effective length method uses K factors to equate the strength of a framed compression element of length L to an equivalent pin-ended member of length KL subject to axial load only. After preliminary trial members have been determined, the alignment chart of AISC Specifications Commentary (Section C-C2) may be used to determine K values. The K values determined from the alignment chart are based on the assumption of purely elastic column behavior and are referred to as elastic K factors. **Section 5.6** may be used to compute inelastic effective length factors for unbraced frames.

This application computes elastic effective length factors for braced or unbraced frames using PTC Mathcad solve blocks to solve the equations for effective length factors, eliminating the need to use the alignment chart in the AISC Standards Commentary.

The required input includes the joint stiffness at each end of the column. Any practical number of K factors may be calculated in one run of this application.

Reference: AISC "Manual of Steel Construction — Allowable Stress Design" June 1, 1989

Reference: "Simple Equations for Effective Length Factors," Engineering Journal (3rd Quarter 1992), by Pierre Dumonteil

Input

Input Variables

Enter the values of G_A and G_B, the ratios of the sum of the column stiffnesses to the sum of the girder stiffnesses at each end (joints A and B, respectively) of the column:

$$G_A := \begin{bmatrix} 0.10 & 0.25 & 0.10 & 0.25 & 0.50 & 0.10 & 0.25 & 0.50 \end{bmatrix}^{T}$$

$$G_B := \begin{bmatrix} 0.40 & 0.25 & 0.90 & 0.75 & 0.50 & 1.90 & 1.75 & 1.50 \end{bmatrix}^{T}$$

Computed Variables

K_b elastic effective length factor for braced frames

K_s elastic effective length factor for unbraced frames

Calculations

Effective length factor for unbraced frames as a function of G_A and G_B:

(The equation shown within the Mathcad solve block is the equation solved by the alignment chart shown in AISC Manual of Steel Construction, Section 3, page 5.)

Guess Values

$$K := 1$$

Constraints

$$K \geq 1$$

$$\frac{G_A \cdot G_B \cdot \left(\frac{\pi}{K}\right)^2 - 36}{6 \cdot \left(G_A + G_B\right)} = \frac{\frac{\pi}{K}}{\tan\left(\frac{\pi}{K}\right)}$$

Solver

$$f_s\left(G_A, G_B\right) := \mathbf{Find}(K)$$

G_A and G_B range from 0 to ∞
K ranges from 1 to ∞

$$K_s := \overrightarrow{f_s\left(G_A, G_B\right)}$$

$$K_s^{T} = \begin{bmatrix} 1.083 & 1.083 & 1.159 & 1.162 & 1.164 & 1.286 & 1.295 & 1.307 \end{bmatrix}$$

Note ⇒ If Ks is not required, inactivate this equation.

Effective length factors for columns in braced frames as a function of GA and GB:

(The equation shown within the Mathcad solve block is the equation solved by the alignment chart shown in the AISC Manual of Steel Construction, Section 3, page 5.)

Guess Values

$$K := 0.75$$

Constraints

$$K > 0.5 \qquad K < 1$$

$$\frac{G_A \cdot G_B}{4} \cdot \left(\frac{\pi}{K}\right)^2 + \left(\frac{G_A + G_B}{2}\right) \cdot \left(1 - \frac{\frac{\pi}{K}}{\tan\left(\frac{\pi}{K}\right)}\right) + \frac{2 \cdot \tan\left(\frac{\pi}{2 \cdot K}\right)}{\frac{\pi}{K}} = 1$$

Solver

$$f_b\left(G_A, G_B\right) := \textbf{Find}(K)$$

$$K_b := \overrightarrow{f_b\left(G_A, G_B\right)}$$

$$K_b^{T} = \begin{bmatrix} 0.603 & 0.611 & 0.648 & 0.672 & 0.686 & 0.683 & 0.716 & 0.751 \end{bmatrix}$$

Note ⇒ If Kb is not required, inactivate this equation.