CHAPTER 5: Structural Steel Columns - ASD Design

5.6 Inelastic Effective Length Factors

Description

In frames where lateral stability is dependent upon the bending stiffness of rigidly connected beams and columns the effective length of compression members must be determined. The effective length method uses K factors to equate the strength of a framed compression element of length L to an equivalent pin-ended member of length KL subject to axial load only. After preliminary trial members have been determined, the alignment chart of AISC Specifications Commentary C-C2 may be used to determine K values. The K values determined from the alignment chart are based on the assumption of purely elastic column behavior and are referred to as elastic K factors. However, most steel columns behave inelastically and the elastic K factors may be reduced using the stiffness reduction factors of the AISC Manual, Table A (Section 3, page 8).

This application computes the stiffness reduction factors for inelastic behavior (Table A, page 3-8 of the AISC Manual), the stiffness ratios at the ends of the column (G values), and the effective length factor k for sidesway permitted. Mathcad's **root** function and solve block are used to compute the stiffness reduction factors and to solve the transcendental equation for effective length factors, eliminating the need to use the alignment chart in the AISC Commentary. If the user does not require the inelastic effective length factors, **Section 5.5** may be used to compute elastic effective length factors.

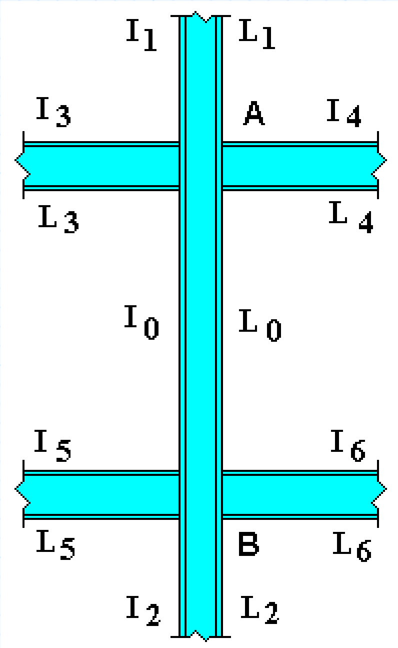
The required input includes the column axial loads on the column being examined and on the columns above and below this column, the cross section area and radius of gyration of the column sections, the moments of inertia and member lengths of all members rigidly connected at each end of the column, and the type of column base when applicable.

A summary of input and computed variables is shown on pages 7 and 8.

Reference: AISC "Manual of Steel Construction — Allowable Stress Design" June 1, 1989

Input

Notation



Input Variables

Enter yield strength of steel: $F_y := 36 \cdot ksi$

Enter axial load, area of the column section and the radius of gyration in the plane of bending for the column under consideration, the column above, and the column below, in sequence starting with subscript 0 for the column under consideration.

Column axial load:

$P_0 := 560 \cdot kip$ $P_1 := 510 \cdot kip$ $P_2 := 610 \cdot kip$

Column section area:

$A_0 := 31.2 \cdot in^2$ $A_1 := 31.2 \cdot in^2$ $A_2 := 39.9 \cdot in^2$

Radius of gyration in the plane of bending:

$r_{b_0} := 5.47 \cdot in$ $\quad r_{b_1} := 5.47 \cdot in$ $\quad r_{b_2} := 5.58 \cdot in$

Enter moments of inertia and lengths of the members framing into the joints at the top and bottom of the column under consideration. Top and bottom joints are designated A and B, respectively. The first I and L entered must be for the column under consideration.

$$I := \begin{bmatrix} 933 \\ 933 \\ 1240 \\ 375 \\ 0 \\ 375 \\ 0 \end{bmatrix} \cdot in^4 \qquad L := \begin{bmatrix} 15 \\ 15 \\ 15 \\ 20 \\ 0 \\ 20 \\ 0 \end{bmatrix} \cdot ft$$

1st row ← column at story
2nd row ← column above
3rd row ← column below
4th row ← girder top
5th row ← girder top
6th row ← girder below
7th row ← girder below

Define the variable "Base" as 0 if lower end is pinned, 1 if fixed and 2 if framed:

$Base := 2$

Computed Variables

f_a	axial compression stress
C_c	column slenderness ratio separating elastic and inelastic buckling (see AISC Specification, Eq. (E2-1))
F_a	allowable axial stress determined by AISC Specification, Eq. (E2-1)
SR	slenderness ratios SR at which axial stress equals the allowable stress determined by AISC Specification, Eq. (E2-1)
F'_e	Euler's stress divided by a factor of safety (AISC Specification, Sect. H1)
SRF	stiffness reduction factor for inelastic behavior, equal to axial stress divided by Euler's stress calculated at the slenderness ratio in the plane of bending at which axial stress equals the allowable axial stress
G_A,G_B	ratios of the sum of the column stiffnesses multiplied by the inelastic stiffness reduction factor, to the sum of the girder stiffnesses at top and bottom (joints A and B, respectively) of the column under consideration
K	effective length factor reduced for inelastic column behavior

Calculations

Axial stress:

$i := 0..2$ $\quad f_{a_i} := \frac{P_i}{A_i}$ $\quad f_a = \begin{bmatrix} 17.949 \\ 16.346 \\ 15.288 \end{bmatrix} ksi$

Modulus of elasticity of steel:

$E := 29000 \cdot ksi$

Column slenderness ratio separating elastic and inelastic buckling
(AISC Specification, Eq. (E2-1)):

$C_c := \sqrt{\frac{2 \cdot \pi^2 \cdot E}{F_y}}$ $\quad C_c = 126.099$

Allowable axial stress determined by AISC Specification, Eq. (E2-1) expressed as a function of the larger slenderness ratio SR:

$$F_a(SR) := \frac{\left(1 - \frac{SR^2}{2 \cdot C_c^{\,2}}\right) \cdot F_y}{\frac{5}{3} + \frac{3 \cdot SR}{8 \cdot C_c} - \frac{SR^3}{8 \cdot C_c^{\,3}}}$$

Function for determining the value of SR at which axial stress equals the allowable stress determined by AISC Specification, Eq. (E2-1):

$f(SR, i) := \left(f_{a_i} - F_a(SR)\right)$

Guess value of slenderness ratio:

$SR := C_c$

Slenderness ratios SR at which axial stress equals the allowable stress determined by AISC Specification, Eq. (E2-1):

$SR_i := \mathbf{root}(f(SR, i), SR)$

$SR^{T} = [54.471\ \ 70.811\ \ 80.601]$

Euler's stress divided by a factor of safety (AISC Specification, Sect. H1):

$$F'_e := \overrightarrow{\frac{12 \cdot \pi^2 \cdot E}{23 \cdot SR^2}} \qquad F'_e{}^{T} = [50.328\ \ 29.782\ \ 22.986]\ ksi$$

Stiffness reduction factor for inelastic behavior, equal to axial stress divided by Euler's stress calculated at the slenderness ratio in the plane of bending at which axial stress equals the allowable axial stress. When the slenderness ratio is greater than Cc, there is no stiffness reduction.

Stiffness reduction factors:

$$i := 0..2 \qquad SRF_i := \text{if}\left(SR_i < C_c, \frac{f_{a_i}}{F'_{e_i}}, 1\right)$$

$SRF^{T} = [0.357\ \ 0.549\ \ 0.665]$

Ratios GA and GB, the sum of the column stiffnesses multiplied by the inelastic stiffness reduction factor, to the sum of the girder stiffnesses at top and bottom (joints A and B, respectively) of the column under consideration:

$$G_A := \frac{SRF_0 \cdot \frac{I_0}{L_0} + SRF_1 \cdot \frac{I_1}{L_1}}{\left(I_3 > 0 \cdot in^4\right) \cdot \frac{I_3}{L_3} + \left(I_4 > 0 \cdot in^4\right) \cdot \frac{I_3}{L_3}} = 3.004$$

$$G_B := \text{if}\left(Base = 0, 10, \text{if}\left(Base = 1, 1, \frac{SRF_0 \cdot \frac{I_0}{L_0} + SRF_2 \cdot \frac{I_2}{L_2}}{\left(I_5 > 0 \cdot in^4\right) \cdot \frac{I_5}{L_5} + \left(I_6 > 0 \cdot in^4\right) \cdot \frac{I_5}{L_5}}\right)\right) = 4.115$$

Effective length factor for Columns with sidesway uninhibited:

(The equation shown within the Mathcad solve block is the equation solved by the alignment chart shown in the AISC Specification Commentary, Section C-C2.)

Guess Values

$$K := 1$$

Constraints

$$K \geq 1$$

$$\frac{G_A \cdot G_B \cdot \left(\frac{\pi}{K}\right)^2 - 36}{6 \cdot \left(G_A + G_B\right)} = \frac{\frac{\pi}{K}}{tan\left(\frac{\pi}{K}\right)}$$

Solver

$$K := \mathbf{Find}(K)$$

G_A & G_B range from 0 to ∞

K ranges from 1 to ∞

K factor, reduced for inelastic column behavior, as applicable:

$$K = 1.938$$

Summary

Yield strength of steel: $F_y = 36\ ksi$

Variable "Base" equal to 0 for pinned base, 1 for fixed base, and 2 for framed lower end: $Base = 2$

Column axial load:

at story → 1st row
story above → 2nd row
story below → 3rd row

$$P = \begin{bmatrix} 560 \\ 510 \\ 610 \end{bmatrix}\ kip$$

Cross section area of column:

$$A = \begin{bmatrix} 31.2 \\ 31.2 \\ 39.9 \end{bmatrix}\ in^2$$

Radius of gyration in the plane of bending of the column section:

at story → 1st row
story above → 2nd row
story below → 3rd row

$$r_b = \begin{bmatrix} 5.47 \\ 5.47 \\ 5.58 \end{bmatrix}\ in$$

Column slenderness ratio separating elastic and inelastic buckling (AISC Specification, Eq. (E2-1)):

$C_c = 126.099$

Stiffness reduction factors: (for inelastic column behavior)

$$SRF = \begin{bmatrix} 0.357 \\ 0.549 \\ 0.665 \end{bmatrix}$$

1st row ← at story
2nd row ← story above
3rd row ← story below

Stiffness factors G_A and G_B reduced by stiffness reduction factor SRF when SR < C_c:

$G_A = 3.004$

$G_B = 4.115$

Column and girder lengths and moments of inertia: (Enter member size designations.)

W12x106

1st row ← W12x106
2nd row ← W12x106
3rd row ← W12x136
4th row ← W16x26
5th row ←
6th row ← W16x26
7th row ←

$$I = \begin{bmatrix} 933 \\ 933 \\ 1.24 \cdot 10^3 \\ 375 \\ 0 \\ 375 \\ 0 \end{bmatrix} in^4 \qquad L = \begin{bmatrix} 15 \\ 15 \\ 15 \\ 20 \\ 0 \\ 20 \\ 0 \end{bmatrix} ft$$

1st row ← column at story
2nd row ← column above
3rd row ← column below
4th row ← girder top
5th row ← girder top
6th row ← girder below
7th row ← girder below

K factor, reduced for inelastic column behavior when applicable:

$K = 1.938$