CHAPTER 7: Reinforced Concrete Column and Wall Footings

7.1 Spread Footings

Description

Spread footings are used under columns and walls to distribute the load to the underlying soil. The plan dimensions of a footing are usually determined by the permissible soil bearing pressure at service (unfactored) loads or by minimum size requirements.

This application calculates the footing thickness required for shear and flexure, and the maximum size and minimum number of reinforcing bars for square or rectangular column or wall footings. The application uses the Strength Design Method of ACI 318-89. Columns may be any square or rectangular size.

The required input includes the strength of the concrete and the reinforcement, the unit weight of concrete, the net allowable soil bearing pressure at service load, the specified reinforcement ratio, sizing factor for rounding footing depth, plan dimensions of the column or wall, and the plan dimensions of the footing.

For illustrative purposes, this application shows five footings. However, any practical number of footings may be entered and displayed at one time.

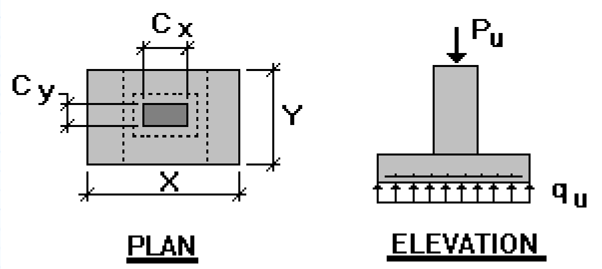
A summary of input and calculated values is shown on pages 15 and 16.

Reference:

ACI 318-89 "Building Code Requirements for Reinforced Concrete." (Revised 1992)

Input

Notation



Input Variables

Allowable net soil bearing pressure at service load: $q_s := 13\ ksf$

Column or wall width: $C_x := [240\ \ 36\ \ 30\ \ 24\ \ 18]^T \cdot in$

Column depth: $C_y := [48\ \ 32\ \ 144\ \ 12\ \ 18]^T \cdot in$

Footing width: $X := [20\ \ 16\ \ 12\ \ 10\ \ 5]^T \cdot ft$

Footing length: $Y := [20\ \ 12\ \ 12\ \ 6\ \ 5]^T \cdot ft$

Computed Variables

The following variables are calculated in this document:

q_u net soil bearing pressure at factored loads

P_s total service load capacity

P_u total factored load capacity

h footing thickness

Numb_X number of reinforcing bars in the X direction

Numb_Y number of reinforcing bars in the Y direction

Size_X size of reinforcing bars in the X direction

Size_Y size of reinforcing bars in the Y direction

β ratio of long side to short side of footing

Material Properties and Constants

Enter values for f'_c, f_y, w_c, w_{rc}, k_v and k_w if different from that shown.

Specified compressive strength of concrete:	$f'_c := 4\ ksi$
Specified yield strength of reinforcement (f_y may not exceed 60 ksi, ACI 318,11.5.2):	$f_y := 60\ ksi$
Unit weight of concrete:	$w_c := 145\ pcf$
Weight of reinforced concrete:	$w_{rc} := 150\ pcf$
Shear strength reduction factor for lightweight concrete k_v = 1 for normal weight, 0.75 for all-lightweight and 0.85 for sand-lightweight concrete (ACI 318, 11.2.1.2.):	$k_v := 1$
Weight factor for increasing development and splice lengths k_w =1 for normal weight and 1.3 for lightweight aggregate concrete (ACI 318, 12.2.4.2):	$k_w := 1$
Modulus of elasticity of reinforcement (ACI 318, 8.5.2):	$E_s := 29000\ ksi$
Strain in concrete at compression failure (ACI 318, 10.3.2):	$\varepsilon_c := 0.003$
Strength reduction factor for flexure (ACI 318, 9.3.2.1):	$\phi_f := 0.9$
Strength reduction factor for shear (ACI 318, 9.3.2.3):	$\phi_v := 0.85$
Sizing factor for rounding footing depths:	$SzF := 2\ in$
Ratio of live load to dead load:	$R := 1$
Combined load factor for dead + live load:	$F := \frac{1.4 + 1.7 \cdot R}{1 + R} = 1.55$

Reinforcing bar number designations, diameters, and areas:

$$No := [0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8\ \ 9\ \ 10\ \ 11\ \ 12\ \ 13\ \ 14\ \ 15\ \ 16\ \ 17\ \ 18]^{T}$$

$$d_b := [0\ \ 0\ \ 0\ \ 0.375\ \ 0.5\ \ 0.625\ \ 0.75\ \ 0.875\ \ 1.00\ \ 1.128\ \ 1.27\ \ 1.41\ \ 0\ \ 0\ \ 1.693\ \ 0\ \ 0\ \ 0\ \ 2.257]^{T} \cdot in$$

$$A_b := [0\ \ 0\ \ 0\ \ 0.11\ \ 0.20\ \ 0.31\ \ 0.44\ \ 0.60\ \ 0.79\ \ 1.00\ \ 1.27\ \ 1.56\ \ 0\ \ 0\ \ 2.25\ \ 0\ \ 0\ \ 0\ \ 4.00]^{T} \cdot in^{2}$$

Bar numbers, diameters and areas are in the vector rows (or columns in the transposed vectors shown) corresponding to the bar numbers. Individual bar numbers, diameters, areas and development lengths and splices of a specific bar can be referred to by using the vector subscripts as shown in the example below.

Example: $No_5 = 5$ $d_{b_5} = 0.625\ in$ $A_{b_5} = 0.31\ in^2$

Limit the value of f'c for computing shear and development lengths to 10 ksi by substituting f'c_max for f'c in formulas for computing shear (ACI 318, 11.1.2, 12.1.2):

$$f'_{c\_max} := \text{if}\left(f'_c > 10 \cdot ksi, 10 \cdot ksi, f'_c\right)$$

The following values are computed from the entered material properties.

Nominal "one way" shear strength per unit area in concrete (ACI 318, 11.3.1.1, Eq. (11-3), 11.5.4.3):

$$v_c := k_v \cdot 2 \cdot \sqrt{\frac{f'_{c\_max}}{psi}} \cdot psi = 126.491\ psi$$

Modulus of elasticity of concrete for values of w_c between 90 pcf and 155 pcf (ACI 318, 8.5.1):

$$E_c := \left(\frac{w_c}{pcf}\right)^{1.5} \cdot 33 \cdot \sqrt{\frac{f'_c}{psi}} \cdot psi = \left(3.644 \cdot 10^{3}\right)\ ksi$$

Strain in reinforcement at yield stress:

$$\varepsilon_y := \frac{f_y}{E_s} = 0.00207$$

Factor used to calculate depth of equivalent rectangular stress block (ACI 318, 10.2.7.3):

$$\beta_1 := \text{if}\left(\left(f'_c \geq 4 \cdot ksi\right) \cdot \left(f'_c \leq 8 \cdot ksi\right), 0.85 - 0.05 \cdot \frac{f'_c - 4 \cdot ksi}{ksi}, \text{if}\left(\left(f'_c \leq 4 \cdot ksi\right), 0.85, 0.65\right)\right) = 0.85$$

Reinforcement ratio producing balanced strain conditions (ACI 318, 10.3.2):

$$\rho_b := \frac{\beta_1 \cdot 0.85 \cdot f'_c}{f_y} \cdot \frac{E_s \cdot \varepsilon_c}{E_s \cdot \varepsilon_c + f_y} = 2.851\%$$

Maximum reinforcement ratio (ACI 318, 10.3.3):

$$\rho_{max} := \frac{3}{4} \cdot \rho_b = 2.138\%$$

Minimum reinforcement ratio for beams (ACI 318, 10.5.1, Eq. (10-3)):

$$\rho_{min} := \frac{200}{f_y} \cdot \frac{lbf}{in^2} = 0.333\%$$

Shrinkage and temperature reinforcement ratio (ACI 318, 7.12.2.1):

$$\rho_{temp} := \begin{array}{l} \text{if } f_y \le 50\ ksi \\ \quad 0.002 \\ \text{else if } f_y \le 60\ ksi \\ \quad 0.002 - \dfrac{f_y}{60\ ksi} \cdot 0.0002 \\ \text{else if } \dfrac{0.0018 \cdot (60\ ksi)}{f_y} \ge 0.0014 \\ \quad \dfrac{0.0018 \cdot (60\ ksi)}{f_y} \\ \text{else} \\ \quad 0.0014 \end{array} = 0.18\%$$

Preferred reinforcement ratio:

$$\rho := \frac{1}{2} \cdot \rho_{max} = 1.069\%$$

Flexural coefficient K, for rectangular beams or slabs, as a function of ρ (ACI 318, 10.2):
(Moment capacity ϕM_n = K(ρ)F, where F = bd^2)

$$K(\rho) := \phi_f \cdot \rho \cdot \left(1 - \frac{\rho \cdot f_y}{2 \cdot 0.85 \cdot f'_c}\right) \cdot f_y$$

Basic tension development length l_{dbt} (ACI 318, 12.2.2 and 12.2.3.6):

No. 3 through No. 11 bars: $n := 3..11$

$$X1_n := 0.04 \cdot A_{b_n} \cdot \frac{f_y}{[illegible]} \qquad X2_n := 0.03 \cdot d_{b_n} \cdot \frac{f_y}{[illegible]}$$

$$l_{dbt_n} := \text{if}\left(X1_n > X2_n, X1_n, X2_n\right)$$

$$l_{dbt}^{\mathrm{T}} = [0\ \ 0\ \ 0\ \ 10.7\ \ 14.2\ \ 17.8\ \ 21.3\ \ 24.9\ \ 30\ \ 37.9\ \ 48.2\ \ 59.2]\ in$$

No. 14 bars:

$$l_{dbt_{14}} := 0.085 \cdot \frac{f_y \cdot \boldsymbol{in}^2}{\sqrt{f'_{c\_max} \cdot \boldsymbol{lbf}}} = 80.638\ \boldsymbol{in}$$

No. 18 bars

$$l_{dbt_{18}} := 0.125 \cdot \frac{f_y \cdot \boldsymbol{in}^2}{\sqrt{f'_{c\_max} \cdot \boldsymbol{lbf}}} = 118.585\ \boldsymbol{in}$$

Tension development length (ACI 318, 12.2.1):

No. 3 through No. 11 bars:

$$l_{dt_n} := \text{if}\left(k_w \cdot l_{dbt_n} \ge 12 \cdot \boldsymbol{in}, k_w \cdot l_{dbt_n}, \text{if}\left(k_w \cdot l_{dbt_n} > 0 \cdot \boldsymbol{in}, 12 \cdot \boldsymbol{in}, 0 \cdot \boldsymbol{in}\right)\right)$$

$$l_{dt}^{\mathrm{T}} = [0\ \ 0\ \ 0\ \ 12\ \ 14.2\ \ 17.8\ \ 21.3\ \ 24.9\ \ 30\ \ 37.9\ \ 48.2\ \ 59.2]\ in$$

No. 14 bars:

$$l_{dt_{14}} := k_w \cdot l_{dbt_{14}} \qquad l_{dt_{14}} = 80.6\ in$$

No. 18 bars

$$l_{dt_{18}} := k_w \cdot l_{dbt_{18}} \qquad l_{dt_{18}} = 118.6\ in$$

Calculations

Net soil bearing pressure at factored load:

$q_u := F \cdot q_s$ $\qquad$ $q_u = 20.15 \; ksf$

The number of footings to be designed N, and range variable i:

$N := \text{length}(X)$ $\qquad$ $N = 5$ $\qquad$ $i := 0 .. N-1$

Total service load capacity:

$P_s := \overrightarrow{q_s \cdot X \cdot Y}$

$P_s^{\mathrm{T}} = \begin{bmatrix} 5.2 \cdot 10^3 & 2.496 \cdot 10^3 & 1.872 \cdot 10^3 & 780 & 325 \end{bmatrix} kip$

Total factored load capacity:

$P_u := \overrightarrow{q_u \cdot X \cdot Y}$

$P_u^{\mathrm{T}} = \begin{bmatrix} 8.06 \cdot 10^3 & 3.869 \cdot 10^3 & 2.902 \cdot 10^3 & 1.209 \cdot 10^3 & 503.75 \end{bmatrix} kip$

Footing projections from face of pier:

$a_{fx} := \dfrac{\overrightarrow{X - C_x}}{2}$

$a_{fx}^{\mathrm{T}} = \begin{bmatrix} 0 & 6.5 & 4.75 & 4 & 1.75 \end{bmatrix} ft$

$a_{fy} := \dfrac{\overrightarrow{Y - C_y}}{2}$

$a_{fy}^{\mathrm{T}} = \begin{bmatrix} 8 & 4.667 & 7.285 \cdot 10^{-16} & 2.5 & 1.75 \end{bmatrix} ft$

Larger footing projection af and corresponding footing width bf:

$a_{f_i} := \textbf{if}\left(a_{fx_i} \ge a_{fy_i}, a_{fx_i}, a_{fy_i}\right)$

$a_f^{\mathrm{T}} = \begin{bmatrix} 8 & 6.5 & 4.75 & 4 & 1.75 \end{bmatrix} ft$

$b_{f_i} := \textbf{if}\left(a_{f_i} = a_{fx_i}, Y_i, X_i\right)$

$b_f^{\mathrm{T}} = [\text{[illegible]}]\; ft$

$$b_f^{\mathrm{T}} = [20\ \ 12\ \ 12\ \ 6\ \ 5]\ ft$$

Effective depth required for beam shear (ACI 318, 11.3.1.1, Eq. (11-3)):

$$d_{bm} := \overrightarrow{\frac{q_u \cdot a_f}{\phi_v \cdot v_c + q_u}}$$

$$d_{bm}^{\mathrm{T}} = [54.288\ \ 44.109\ \ 32.233\ \ 27.144\ \ 11.875]\ in$$

Perimeter of critical section expressed as a function of C_x, C_y and d (ACI 318 11.12.1.2):

$$b_o(C_x, C_y, d) := 2 \cdot (C_x + C_y + 2 \cdot d)$$

Ratio of the longer to the shorter column dimension expressed as a function of C_x and C_y
(ACI 318, 11.12.2.1):

$$\beta_c(C_x, C_y) := \max\left(\begin{bmatrix} \frac{C_x}{C_y} \\ \frac{C_y}{C_x} \end{bmatrix}\right)$$

Nominal "two way" concrete shear strength per unit area in slabs and footings, expressed as a function of C_x, C_y and d, with the constant α_s equal to 40 for interior columns.
(ACI 318, 11.12.2.1, Eqs. (11-36), (11-37) and (11-38)):

$$\alpha_s := 40$$

$$v_{cp}(C_x, C_y, d) := min\left(\begin{bmatrix} 2 + \frac{4}{\beta_c(C_x, C_y)} \\ \frac{\alpha_s \cdot d}{b_o(C_x, C_y, d)} + 2 \\ 4 \end{bmatrix}\right) \cdot \left(k_v \cdot \sqrt{\frac{f'_{c\_max}}{psi}} \cdot psi\right)$$

Effective depth required for peripheral shear d_p (ACI 318 11.12.2.1):

Guess Values

Constraints

$$\frac{q_u \cdot (X \cdot Y - (C_x + d) \cdot (C_y + d))}{b_o(C_x, C_y, d) \cdot d} = \phi_v \cdot v_{cp}(C_x, C_y, d)$$

Solver

$$d_p(X, Y, C_x, C_y, d) := \mathbf{Find}(d)$$

$$d_{p_i} := \text{if}\left(\left(\frac{d_{bm_i}}{2} \ge a_{fx_i}\right) + \left(\frac{d_{bm_i}}{2} \ge a_{fy_i}\right), 0 \cdot in, d_p\left(X_i, Y_i, C_{x_i}, C_{y_i}, d_{bm_i}\right)\right)$$

$$d_p^{\mathrm{T}} = [0 \;\; 44.519 \;\; 0 \;\; 25.432 \;\; 13.482] \; in$$

Maximum bending moments, and minimum effective depths required for flexure with specified reinforcement ratio (ACI 318, 10.2):

$$M_u := \overrightarrow{\frac{1}{2} \cdot q_u \cdot b_f \cdot a_f^{\,2}}$$

$$M_u^{\mathrm{T}} = \left[1.748 \cdot 10^7 \;\; 6.926 \cdot 10^6 \;\; 3.698 \cdot 10^6 \;\; 1.311 \cdot 10^6 \;\; 2.092 \cdot 10^5\right] \frac{kg \cdot m^2}{s^2}$$

$$d_f := \overrightarrow{\sqrt{\frac{M_u}{b_f \cdot K(\rho)}}}$$

$$d_f^{\mathrm{T}} = [35.119 \;\; 28.534 \;\; 20.852 \;\; 17.559 \;\; 7.682] \; in$$

Minimum effective depth required for shear and flexure:

$$d_i := \text{if}\left(\left(d_{f_i} > d_{p_i}\right) \cdot \left(d_{f_i} > d_{bm_i}\right), d_{f_i}, \text{if}\left(d_{p_i} > d_{bm_i}, d_{p_i}, d_{bm_i}\right)\right)$$

$$d^{\mathrm{T}} = [54.288 \;\; 44.519 \;\; 32.233 \;\; 27.144 \;\; 13.482] \; in$$

Index numbers of maximum bar sizes determined by available development lengths:

$$index_{0,i} := 0 \qquad index_{0 \cdot n, i} := \text{if}\left(l_{dt_n} \le \left(a_{fx_i} - 3 \cdot in\right), n, index_{0,i}\right)$$

$$bx_i := index_{0,i}$$

$$index_{0,i} := 0 \qquad index_{0 \cdot n, i} := \text{if}\left(l_{dt_n} \le \left(a_{fy_i} - 3 \cdot in\right), n, index_{0,i}\right)$$

$$by_i := index_{0,i}$$

Sizes of the largest permissible reinforcing bars for the X and Y directions:

$$Size\_X_i := No_{bx_i}$$

$$Size\_X^{\mathrm{T}} = [0 \;\; 11 \;\; 10 \;\; 9 \;\; 5]$$

$$Size\_Y_i := No_{by_i}$$

$$Size\_Y^{\mathrm{T}} = [11 \;\; 10 \;\; 0 \;\; 7 \;\; 5]$$

Total footing thickness rounded up to the nearest multiple of SzF, but not less than the ACI code required minimum of 8 inches:

$$h_i := \mathbf{if}\left(SzF \cdot \text{ceil}\left(\frac{d_i + \frac{d_{b_{bx_i}} + d_{b_{by_i}}}{2} + 3 \cdot in}{SzF}\right) < 8 \cdot in, 8 \cdot in, SzF \cdot \text{ceil}\left(\frac{d_i + \frac{d_{b_{bx_i}} + d_{b_{by_i}}}{2} + 3 \cdot in}{SzF}\right)\right)$$

$$h^{T} = [58 \;\; 50 \;\; 36 \;\; 32 \;\; 18] \; in$$

Effective depths in the X and Y directions :

$$d_{x_i} := \mathbf{if}\left(a_{fx_i} \geq a_{fy_i}, \left(h_i - \frac{d_{b_{bx_i}}}{2} - 3 \cdot in\right), \left(h_i - \frac{d_{b_{bx_i}}}{2} - d_{b_{by_i}} - 3 \cdot in\right)\right)$$

$$d_x^{T} = [53.59 \;\; 46.295 \;\; 32.365 \;\; 28.436 \;\; 14.688] \; in$$

$$d_{y_i} := \mathbf{if}\left(a_{fy_i} > a_{fx_i}, \left(h_i - \frac{d_{b_{by_i}}}{2} - 3 \cdot in\right), \left(h_i - \frac{d_{b_{by_i}}}{2} - d_{b_{bx_i}} - 3 \cdot in\right)\right)$$

$$d_y^{T} = [54.295 \;\; 44.955 \;\; 31.73 \;\; 27.435 \;\; 14.063] \; in$$

Reinforcement As as a function of Mu, bf and d:

$$A_s\left(M_u, b_f, d\right) := \overrightarrow{b_f \cdot d \cdot \left(\left(1 - \left(\sqrt{1 - \frac{2 \cdot M_u}{\phi_f \cdot b_f \cdot d^2 \cdot 0.85 \cdot f'_c}}\right)\right) \cdot \frac{0.85 \cdot f'_c}{f_y}\right)}$$

Bending moments in the X and Y directions:

$$M_{ux} := \overrightarrow{\frac{1}{2} \cdot q_u \cdot Y \cdot a_{fx}^{\,2}}$$

$$M_{ux}^{T} = \left[0 \;\; 5.108 \cdot 10^{3} \;\; 2.728 \cdot 10^{3} \;\; 967.2 \;\; 154.273\right] \; kip \cdot ft$$

$$M_{uy} := \overrightarrow{\frac{1}{2} \cdot q_u \cdot X \cdot a_{fy}^{\,2}}$$

$$M_{uy}^{T} = \left[1.29 \cdot 10^{4} \;\; 3.511 \cdot 10^{3} \;\; 6.416 \cdot 10^{-29} \;\; 629.688 \;\; 154.273\right] \; kip \cdot ft$$

Total required area of reinforcement in X and Y directions:

$$A'_{sx} := A_s\left(M_{ux}, Y, d_x\right)$$

$$A'_{sx}{}^{T} = [0 \;\; 25.371 \;\; 19.445 \;\; 7.822 \;\; 2.391] \; in^2$$

$$A'_{sy} := A_s\left(M_{uy}, X, d_y\right)$$

$$A'_{sy}{}^{T} = [54.816\ \ 17.673\ \ 0\ \ 5.172\ \ 2.503]\ in^2$$

Ratio of the long side to the short side of footing (ACI 318, 15.4.4.2):

$$\beta_i := \textbf{if}\left(X_i \geq Y_i, \frac{X_i}{Y_i}, \frac{Y_i}{X_i}\right)$$

$$\beta^T = [1\ \ 1.333\ \ 1\ \ 1.667\ \ 1]$$

Factor for increasing reinforcement in the short direction to provide uniform spacing in lieu of concentrating reinforcement in a bandwidth equal to the short side of the footing (ACI 318, 15.4.4.2):

$$I := \left(\overrightarrow{\beta \cdot \left(\frac{2}{\beta + 1}\right)}\right)$$

$$I^T = [1\ \ 1.143\ \ 1\ \ 1.25\ \ 1]$$

Increase reinforcement in the short direction of rectangular footings to provide uniform reinforcing bar spacing.

Total required reinforcement areas in the X and Y directions:

$$A_{sx_i} := \textbf{if}\left(X_i \geq Y_i, A'_{sx_i}, I_i \cdot A'_{sx_i}\right)$$

$$A_{sx}{}^{T} = [0\ \ 25.371\ \ 19.445\ \ 7.822\ \ 2.391]\ in^2$$

$$A_{sy_i} := \textbf{if}\left(Y_i \geq X_i, A'_{sy_i}, I_i \cdot A'_{sy_i}\right)$$

$$A_{sy}{}^{T} = [54.816\ \ 20.198\ \ 0\ \ 6.465\ \ 2.503]\ in^2$$

Minimum required reinforcement area for shrinkage and temperature in the X direction (ACI 318, 7.12):

$$A_{sx\_temp} := \overrightarrow{\rho_{temp} \cdot h \cdot Y}$$

$$A_{sx\_temp}{}^{T} = [25.056\ \ 12.96\ \ 9.331\ \ 4.147\ \ 1.944]\ in^2$$

Larger required reinforcement in the X direction:

$$A_{sx_i} := \textbf{if}\left(A_{sx_i} > A_{sx\_temp_i}, A_{sx_i}, A_{sx\_temp_i}\right)$$

$$A_{sx}{}^{T} = [25.056\ \ 25.371\ \ 19.445\ \ 7.822\ \ 2.391]\ in^2$$

Minimum required reinforcement area for shrinkage and temperature in the Y direction (ACI 318, 7.12):

$$A_{sy\_temp} := \overrightarrow{\rho_{temp} \cdot h \cdot X}$$

$$A_{sy\_temp}^{T} = [25.056 \;\; 17.28 \;\; 9.331 \;\; 6.912 \;\; 1.944] \; in^2$$

Larger required reinforcement in the Y direction:

$$A_{sy_i} := \textbf{if}\left(A_{sy_i} > A_{sy\_temp_i}, A_{sy_i}, A_{sy\_temp_i}\right)$$

$$A_{sy}^{T} = [54.816 \;\; 20.198 \;\; 9.331 \;\; 6.912 \;\; 2.503] \; in^2$$

Minimum number of bars to limit spacing of reinforcement to 18 inches (ACI 318 7.6.5):

$$MinNumb\_X := \overrightarrow{\text{ceil}\left(\frac{Y - 6 \cdot in}{18 \cdot in} + 0.5\right)}$$

$$MinNumb\_X^{T} = [14 \;\; 9 \;\; 9 \;\; 5 \;\; 4]$$

$$MinNumb\_Y := \overrightarrow{\text{ceil}\left(\frac{X - 6 \cdot in}{18 \cdot in} + 0.5\right)}$$

$$MinNumb\_Y^{T} = [14 \;\; 11 \;\; 9 \;\; 7 \;\; 4]$$

Maximum bar areas corresponding to minimum specified spacing:

$$MinA\_x := \overrightarrow{\frac{A_{sx}}{MinNumb\_X}} \qquad MinA\_y := \overrightarrow{\frac{A_{sy}}{MinNumb\_Y}}$$

$$MinA\_x^{T} = [1.79 \;\; 2.819 \;\; 2.161 \;\; 1.564 \;\; 0.598] \; in^2 \qquad MinA\_y^{T} = [3.915 \;\; 1.836 \;\; 1.037 \;\; 0.987 \;\; 0.626] \; in^2$$

Index numbers of bar sizes determined by minimum specified spacing:

$$index_{0,i} := 0 \qquad index_{0 \cdot n, i} := \textbf{if}\left(A_{b_n} \le MinA\_x_i, n, index_{0,i}\right)$$

$$ax_i := index_{0,i} \qquad ax^{T} = [11 \;\; 11 \;\; 11 \;\; 11 \;\; 6]$$

$$index_{0,i} := 0 \qquad index_{0 \cdot n, i} := \textbf{if}\left(A_{b_n} \le MinA\_y_i, n, index_{0,i}\right)$$

$$ay_i := index_{0,i} \qquad ay^{T} = [11 \;\; 11 \;\; 9 \;\; 8 \;\; 7]$$

Actual bar sizes in the X and Y directions.

Bar sizes in the X and Y directions (the smaller bar size determined by the required total reinforcement or the specified minimum spacing):

$$Size\_X_i := \mathbf{if}\left(\left(\left|M_{ux_i}\right| \le 10^{-12} \cdot kip \cdot ft\right) + \left(ax_i \le bx_i\right), ax_i, bx_i\right)$$

$$Size\_X^T = [11\ \ 11\ \ 10\ \ 9\ \ 5]$$

$$Size\_Y_i := \mathbf{if}\left(\left(\left|M_{uy_i}\right| \le 10^{-12} \cdot kip \cdot ft\right) + \left(ay_i \le by_i\right), ay_i, by_i\right)$$

$$Size\_Y^T = [11\ \ 10\ \ 9\ \ 7\ \ 5]$$

Subscript variables cx and cy defined as the bar sizes in the X and Y directions, respectively:

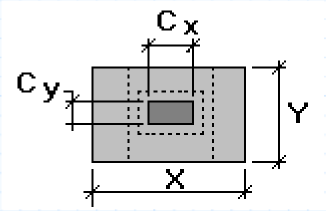
$$cx := Size\_X \qquad cy := Size\_Y$$

Total number of the largest permissible bar size (from No. 3 to No. 11) in the X and Y directions:

X direction: $Numb\_X_i := \text{ceil}\left(\frac{A_{sx_i}}{A_{b_{cx_i}}}\right)$ $\qquad Numb\_X^T = [17\ \ 17\ \ 16\ \ 8\ \ 8]$

Y direction: $Numb\_Y_i := \text{ceil}\left(\frac{A_{sy_i}}{A_{b_{cy_i}}}\right)$ $\qquad Numb\_Y^T = [36\ \ 16\ \ 10\ \ 12\ \ 9]$

Summary



Compressive strength of concrete: $f'_c = 4\ ksi$

Yield strength of reinforcement: $f_y = 60\ ksi$

Unit weight of concrete: $w_c = 145\ pcf$

Service load capacity:

$$P_s^{T} = [5.2 \cdot 10^3 \ \ 2.496 \cdot 10^3 \ \ 1.872 \cdot 10^3 \ \ 780 \ \ 325]\ kip$$

Factored load capacity:

$$P_u^{T} = [8.06 \cdot 10^3 \ \ 3.869 \cdot 10^3 \ \ 2.902 \cdot 10^3 \ \ 1.209 \cdot 10^3 \ \ 503.75]\ kip$$

Soil bearing pressure at service load: $q_s = 13\ ksf$

Soil bearing pressures at factored load: $q_u = 20.15\ ksf$

Column or pier sizes:

$$C_x^{T} = [20 \ \ 3 \ \ 2.5 \ \ 2 \ \ 1.5]\ ft$$

$$C_y^{T} = [4 \ \ 2.667 \ \ 12 \ \ 1 \ \ 1.5]\ ft$$

Footing sizes:

$$X^{T} = [20 \ \ 16 \ \ 12 \ \ 10 \ \ 5]\ ft$$

$$Y^{T} = [20 \ \ 12 \ \ 12 \ \ 6 \ \ 5]\ ft$$

Footing thickness:

$$h^{T} = [58 \ \ 50 \ \ 36 \ \ 32 \ \ 18]\ in$$

Number and size of reinforcing bars in the X direction:

$$Numb\_X^{T} = [17 \ \ 17 \ \ 16 \ \ 8 \ \ 8]$$

$$Size\_X^{T} = [11 \ \ 11 \ \ 10 \ \ 9 \ \ 5]$$

Number and size of reinforcing bars in the Y direction:

$$Numb\_Y^{T} = [36 \ \ 16 \ \ 10 \ \ 12 \ \ 9]$$

$$Size\_Y^{T} = [11 \ \ 10 \ \ 9 \ \ 7 \ \ 5]$$