Article - CS173039

PTC Mathcad Worksheet Library: Finite Element Beginnings

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Applies To

- Mathcad Prime 3.0 to 3.1

Description

- The finite element method is a numerical analysis technique used by engineers, scientists, and mathematicians to obtain solutions to the differential equations that describe, or approximately describe a wide variety of physical (and non-physical) problems. Physical problems range in diversity from solid, fluid and soil mechanics, to electromagnetism or dynamics.
- The underlying premise of the method states that a complicated domain can be sub-divided into a series of smaller regions in which the differential equations are approximately solved. By assembling the set of equations for each region, the behavior over the entire problem domain is determined.
- Each region is referred to as an element and the process of subdividing a domain into a finite number of elements is referred to as discretization. Elements are connected at specific points, called nodes, and the assembly process requires that the solution be continuous along common boundaries of adjacent elements.

Resolution

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