SUBJECT: MOMENT-CURVATURE, ROTATIONS, DEFLECTIONS

CREATED BY: ________________ DATE: ____________

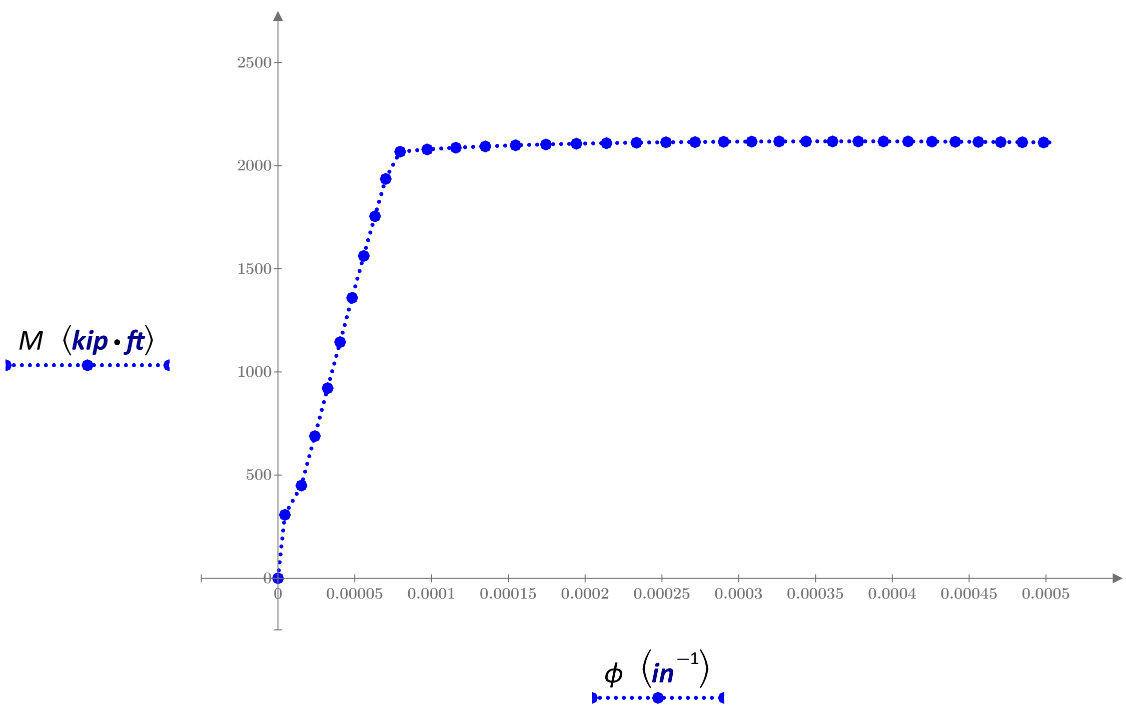
REVISED BY: ________________ DATE: ____________

LOAD MOMENT-CURVATURE ANALYSIS

Read data file: $data := \text{READTEXT}(\text{"Fiber\_Model\_C-M-etop-P-Mc.txt"})$

Extract M-C data: $\phi := data^{\langle 0 \rangle}\ in^{-1}$ $M := data^{\langle 1 \rangle}\ kip \cdot ft$

Plot M-C Interaction:



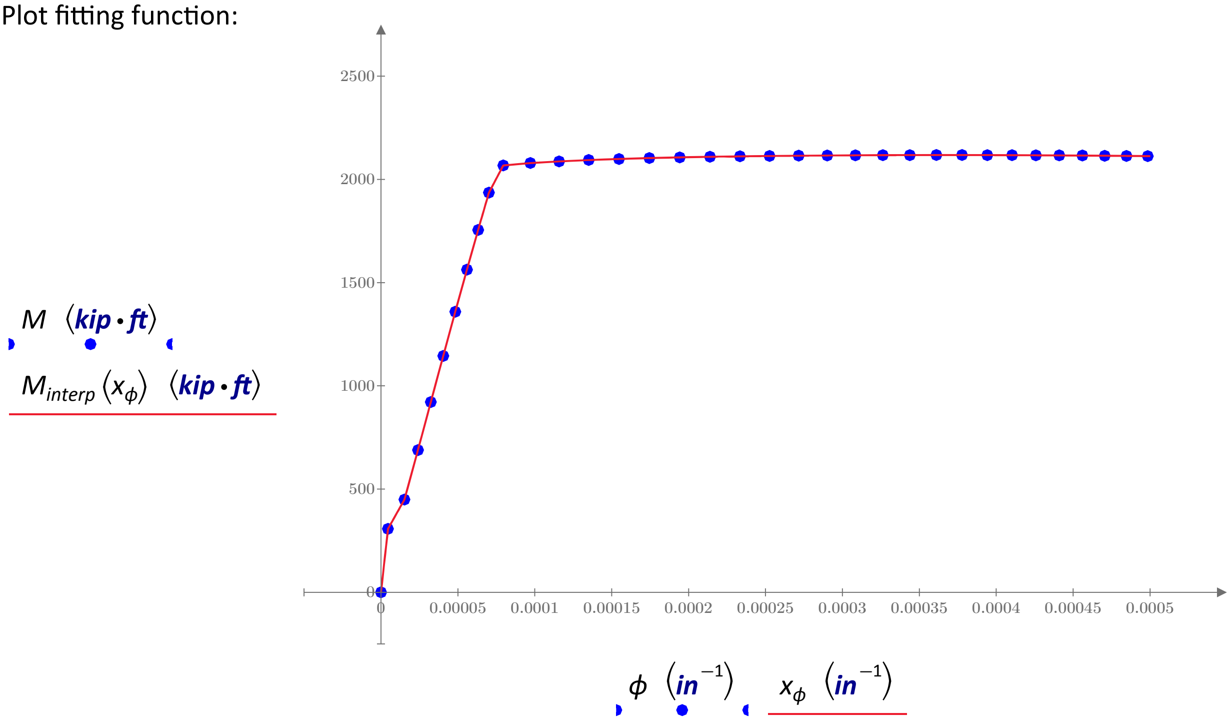
Curvature function of M: $M_{interp}(x) := \text{linterp}(\phi, M, x)$

Plotting range variable: $x_\phi := 0, \frac{\max(\phi)}{1000} .. \max(\phi)$

NOTE ABOUT THIS WORKSHEET

This worksheet uses moment-curvature data (calculated elsewhere in "Fiber Model" analysis worksheet) to define curvature as a function of moment using linearly-interpolation fitting parameters. Curvatures will be mapped to length of beam using moments; curvature as a function of length will be integrated once to calculate rotations, and integrated a second time to calculate beam displacements.

Plot fitting function:



BENDING MOMENT DIAGRAM

Beam length: $L := 24\ ft$

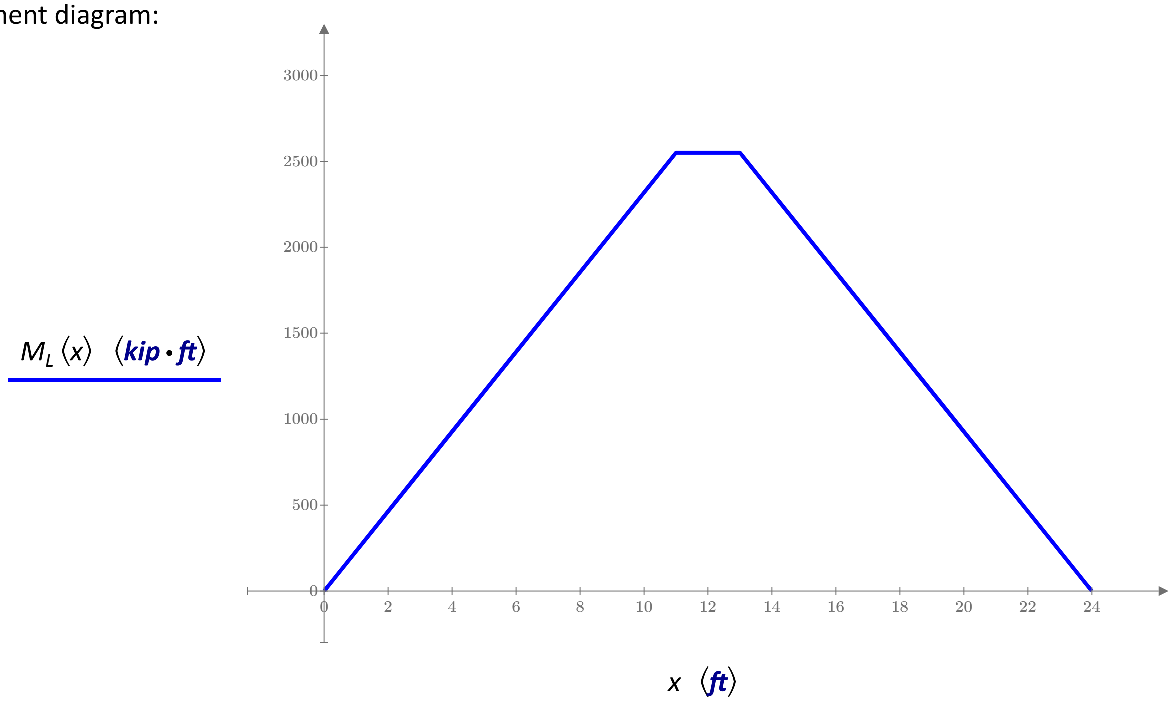
Plotting range variable: $x := 0, 0.1\ ft .. L$

Point load magnitude: $P := 463.6\ kip$

Moment envelope:

$$M_L(x) := \begin{cases} \dfrac{P}{2} \cdot x & \text{if } 0 \le x < 11\ ft \\ \dfrac{P}{2} \cdot 11\ ft & \text{else if } 11\ ft \le x < 13\ ft \\ \dfrac{P}{2} \cdot (L - x) & \text{else if } x \ge 13\ ft \end{cases}$$

Moment diagram:



MATCH CURVATURE ALONG LENGTH OF BEAM

No. of data point to calc.: $N := 50$

Range variable of data pts: $t := 0 \,..\, N$

Vector of beam lengths: $L_{solve_t} := \dfrac{L \cdot t}{N}$

Solve for curvatures:

Guess Values

$\phi_{solve} := \dfrac{in^{-1}}{10000}$

Changing the guess value from 1/1000 to 1/10000 allowed the Solve Block to calculate the correct curvatures.

Constraints

$M_{interp}\left(\phi_{solve}\right) = M_L\langle x\rangle$

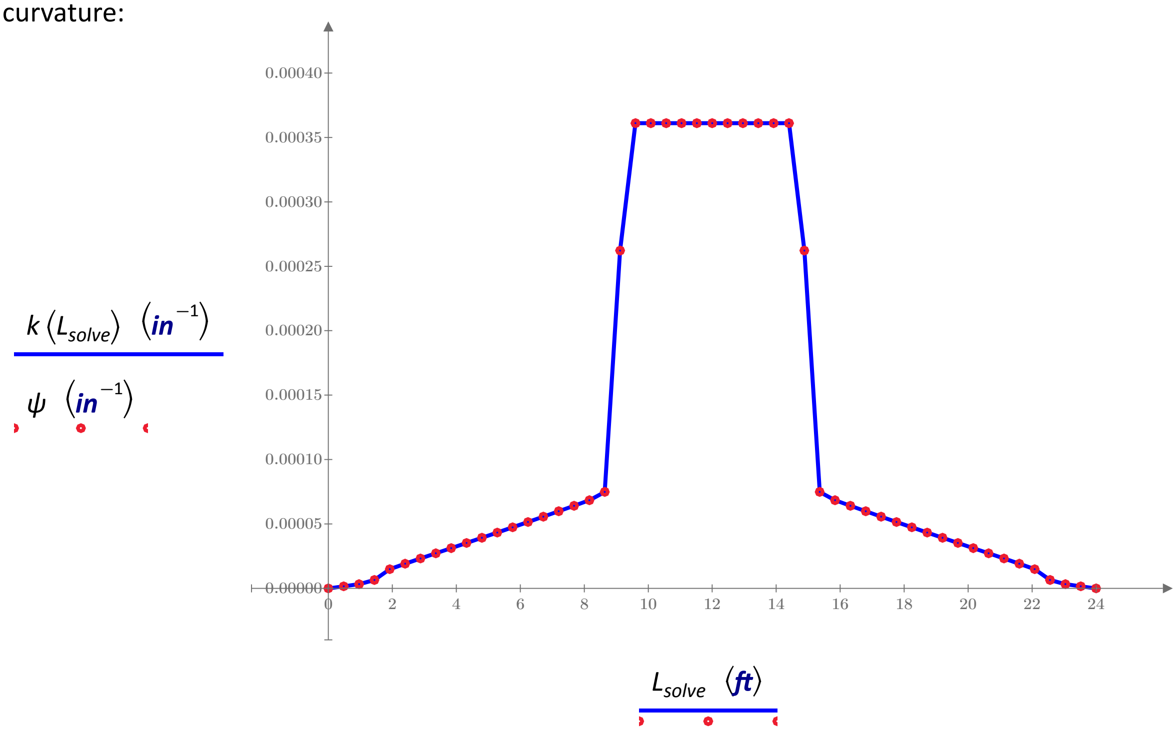
Solver

$\psi_o\langle x\rangle := \mathbf{minerr}\left(\phi_{solve}\right)$

Calculated curvatures vector: $\psi_t := \psi_o\left(L_{solve_t}\right)$

Curvature function of length: $k\langle x\rangle := \text{linterp}\left(L_{solve}, \psi, x\right)$

Plot curvature:



INTEGRATE CURVATURES: ROTATIONS AND DEFLECTIONS

Rotations (i.e. slope): $\vartheta(x) := \int_{0\ in}^{x} k(x)\, dx$

Rotations (i.e. slope): $\Theta(x) := \vartheta(x) - \vartheta\left(\frac{L}{2}\right)$

Deflections: $\Delta(x) := \int_{0\ in}^{x} \Theta(x)\, dx$

SUMMARY OF CALCULATED ROTATIONS, DISPLACEMENTS

Rotations at supports: $\Theta_{end} := \Theta(L)$ .... $\Theta_{end} = 0.017\ rad$

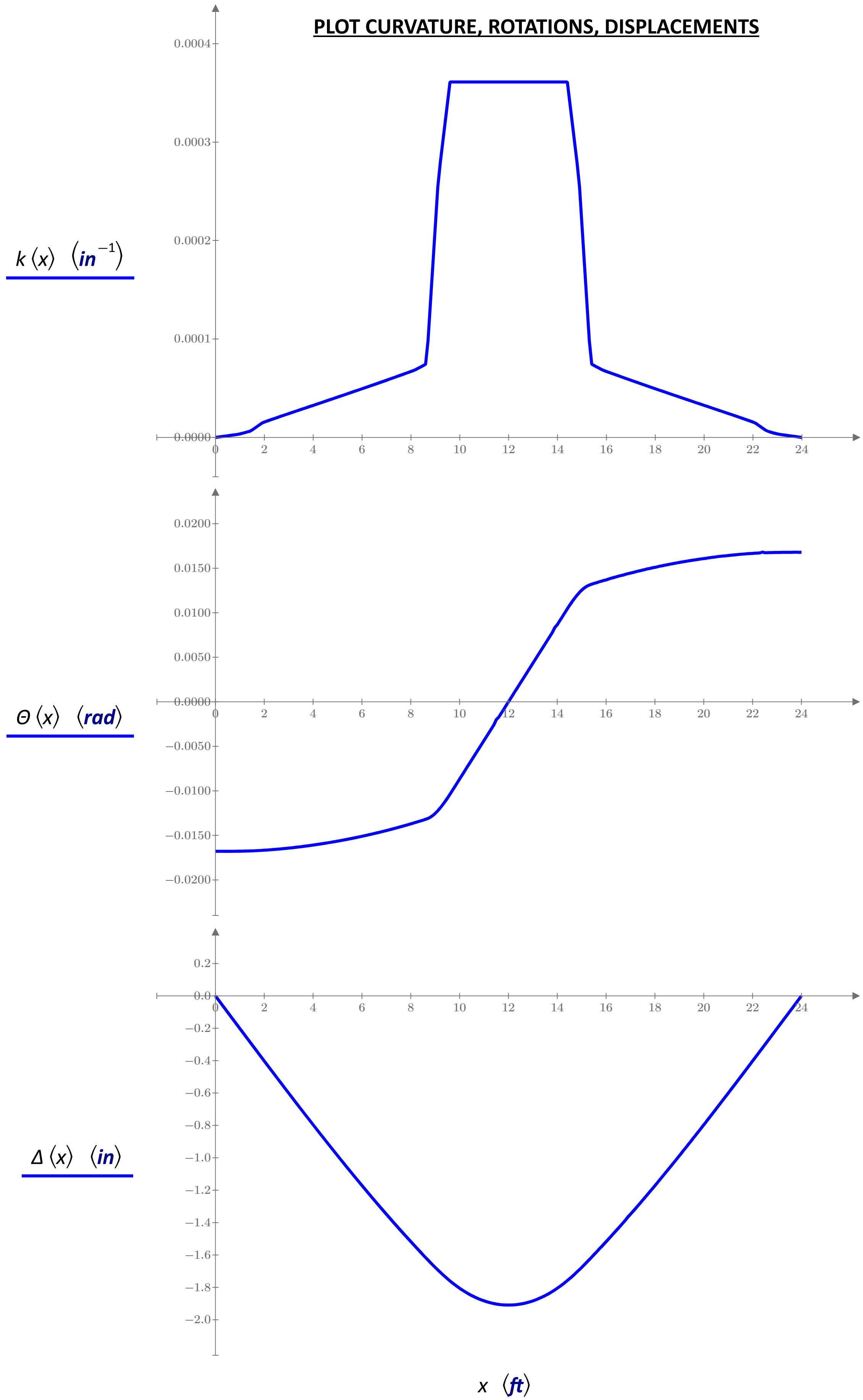
Rotations at mid-span: $\Theta_{mid} := \Theta\left(\frac{L}{2}\right)$ .... $\Theta_{mid} = 0\ rad$

Displacement at supports: $\Delta_{end} := \Delta(L)$ .... $\Delta_{end} = 0\ in$

Displacement at $\frac{L}{4}$: $\Delta_{0.25} := \Delta\left(\frac{L}{4}\right)$ .... $\Delta_{0.25} = -1.17\ in$

Displacement at mid-span: $\Delta_{mid} := \Delta\left(\frac{L}{2}\right)$ .... $\Delta_{mid} = -1.91\ in$

PLOT CURVATURE, ROTATIONS, DISPLACEMENTS

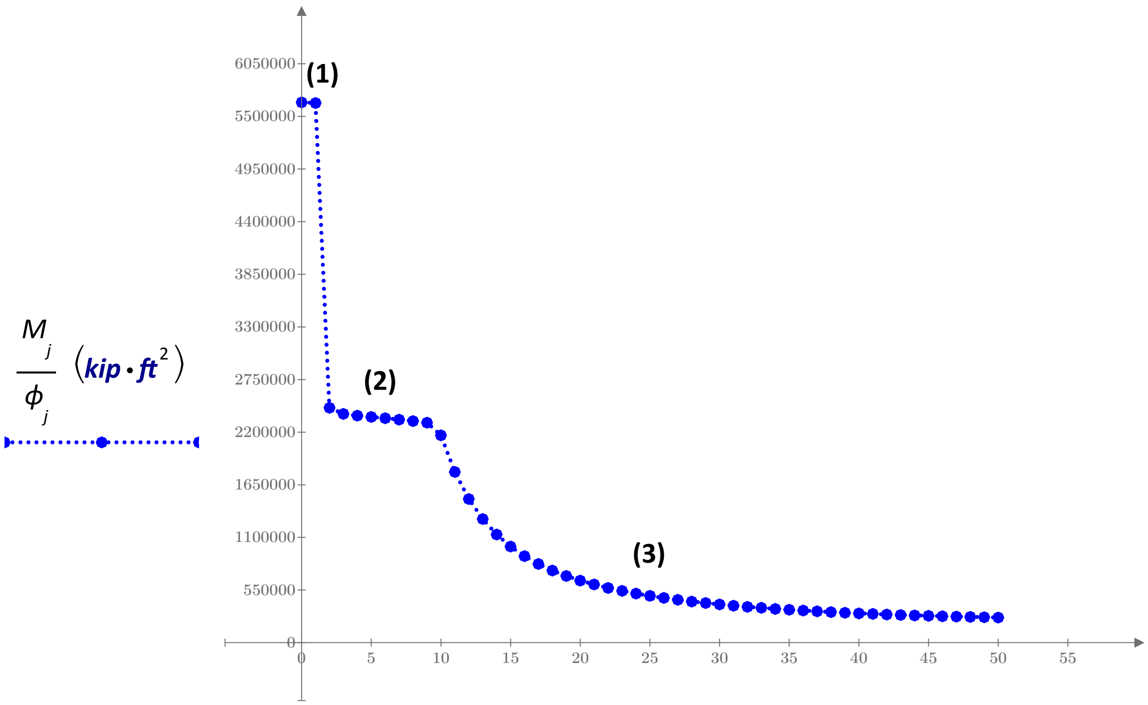


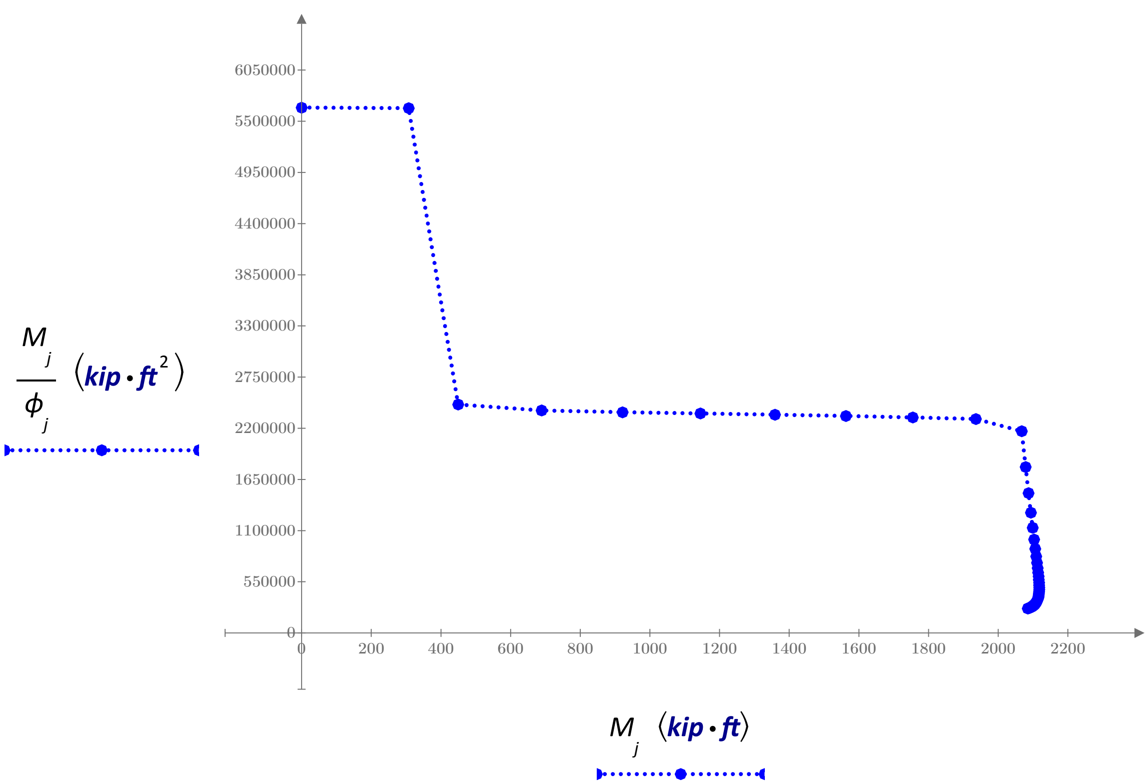
PLOT VARIATION OF FLEXURAL RIGIDITY (EI)

Note: EI is non-linear for concrete; this is especially true once reinforcing steel has yielded. Three plots (below) illustrated the degredation of EI within three identifiable regions,inluding: (1) EI (uncracked-elastic), (2) EI (cracked-elastic), (3) EI post-yield (non-linear).

Plotting range variable: $j := 0 .. \text{rows}\langle\phi\rangle$

From mechanics: $\frac{M}{\phi} = EI$





DEGREDATION OF "EI"

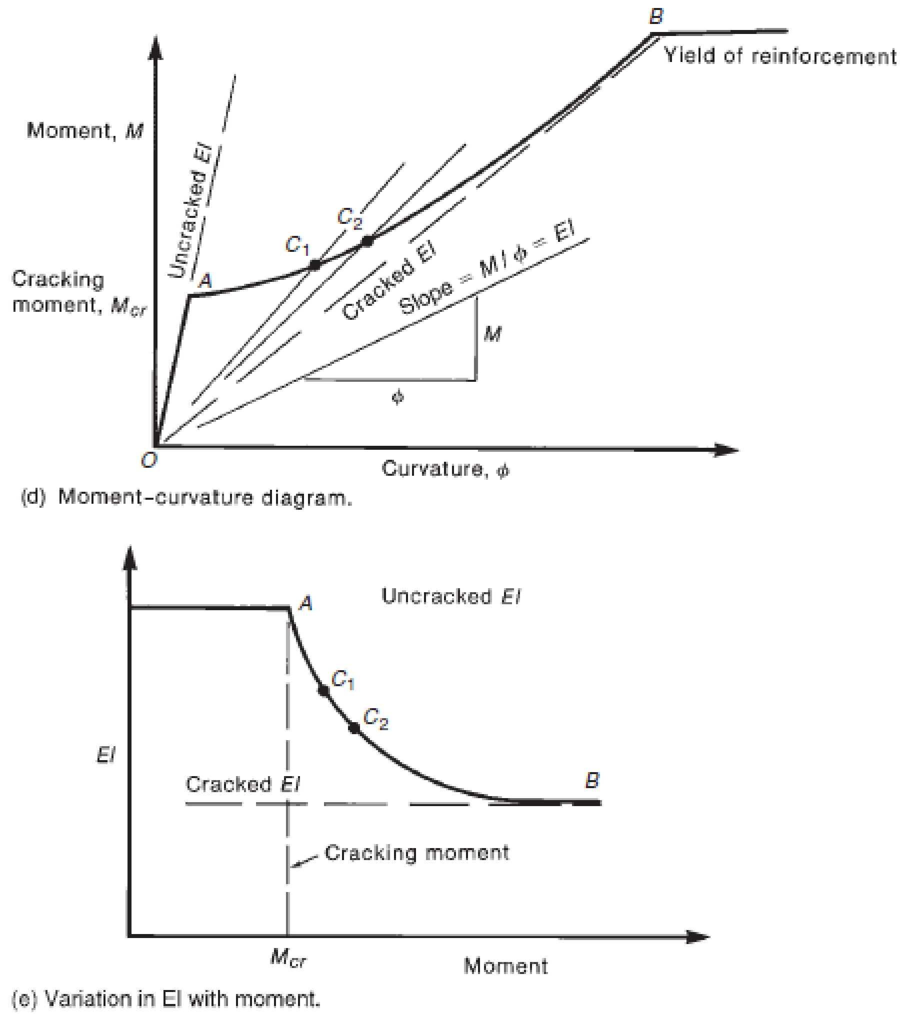


Figure 1. Moment-curvature diagram and variation (degredation) of EI (Wight and MacGregor 2012).