orientation as allowed by system tolerances. "Worst" implies that this orientation will cause the BLF to be the lowest.

Because instability is such a precarious state, you must always ensure that your safety factor is properly selected. Remember that failures due to buckling are usually sudden and disastrous.

Simple Buckling Analyses and Correlation to Theory

Consider standing on an empty soda can that is propped up on level ground. Make the following assumptions.

Fig. 16.1. Simplified soda can with a 200lb person standing squarely on top of it.

- The structure in Fig. 16.1 may be simplified as a cylinder with an outside radius (r_o) of 1.3", a wall thickness (t) of 0.005", a height (L) of 4", and end cap thickness (t_c) of 1/16". The inside radius (r_i) is thus 1.295".
- The structure's material is 6061 aluminum with a modulus (E) of 1e7 psi, a Poisson's ratio (ν) of 0.3, and a yield strength (Sy) of 6e4 psi.
- You weigh (F) 200 lbs. and are able to stand directly above the can, on its upper rim, with your c.g. coinciding with its center axis.
- This scenario may be considered to represent a fixed-free condition. The rotations of the bottom of the can are fixed by its geometry and translation can be assumed fixed by the frictional forces. Hence, the length factor (K) is equal to 2 and the effective length of the can is $L_e = KL = 8''$.

First, you must calculate the can's cross-sectional area, moment of inertia, and radius of gyration.

Eq. 16.1 $$A = \pi(r_o^2 - r_i^2) = 0.0408 in^2$$

Eq. 16.2 $$I = \frac{1}{4}\pi(r_o^4 - r_o^4) = 0.0343 in^4$$

$$Eq.\ 16.3 \quad r = \sqrt{I/A} = 0.917 in$$

With Eq. 2.60, you can determine whether the soda can falls into the Euler ("slender column") category.

$$Eq.\ 16.4 \quad \frac{L_e}{r} = 8.72 < \sqrt{\frac{\pi^2 E}{S_y}} = 40.6$$

Because the first quantity is not greater than the second, the column is nonEuler and its buckling mode will belong to the short to intermediate height regime. As shown below, it is interesting to erroneously continue with a Euler type analysis. According to this type of analysis, the column will buckle under the critical load as seen in Eq. 16.5.

$$Eq.\ 16.5 \quad P_{cr} = \frac{\pi^2 EI}{L_e^2} = 52900 lbs!(BLF = 265)$$

Note that all BLFs reported in this example correspond to the 200lb loading scenario. You probably would not trust the column under anything close to this loading–remember that Euler's hyperbola is being extended into the nonEuler range. At this point, you should calculate the compressive stress caused by the 200lb weight on the can's wall as follows:

$$Eq.\ 16.6 \quad \sigma = F/A = 4910 psi$$

The calculated stress is well below the yield point of the aluminum; thus, were the can to buckle under your weight, it would be an elastic phenomenon. Calculating the force (F_y) necessary for the can to reach yield is useful, so that a nonlinear buckling analysis can be anticipated.

$$Eq.\ 16.7 \quad F_y = S_y A = 2450 lbs(BLF = 12.25)$$

It is hard to believe that the can would remain stable under this much weight. If you have ever seen a soda can buckle under a coaxial load, it is likely you have noticed an "accordion-like" crushing effect of the can wall as it collapses. This elastic, buckling crush mode has been quantified theoretically, and the equation describing it appears below.

$$Eq.\ 16.8 \quad P_{cr} = \frac{\pi E t^2}{\sqrt{3(1-\nu^2)}}\left(2 - \frac{t}{r_o}\right) = 949 lbs(BLF = 4.75)$$

Eq. 16.8 is valid for thin-walled circular tubes under uniform longitudinal compression whose radius to wall ratio is greater than 10 and whose length is several times greater than the quantity $1.72\sqrt{r_o t}$. Note that all conditions are met by the soda can example. In addition, note that the critical load equation is independent of length, which is not entirely intuitive when dealing with buckling phenomena. This crush mode has also been treated empirically, resulting in an approximate equation of the form:

$$\textit{Eq. 16.9} \qquad P_{cr} = 0.3\pi E t^2\left(2-\frac{t}{r_o}\right) = 470lbs(BLF = 2.35)$$

which assumes the same conditions as its theoretical counterpart. Because it is based on testing data, Eq. 16.9 is the most indicative of the load necessary to crush the can.

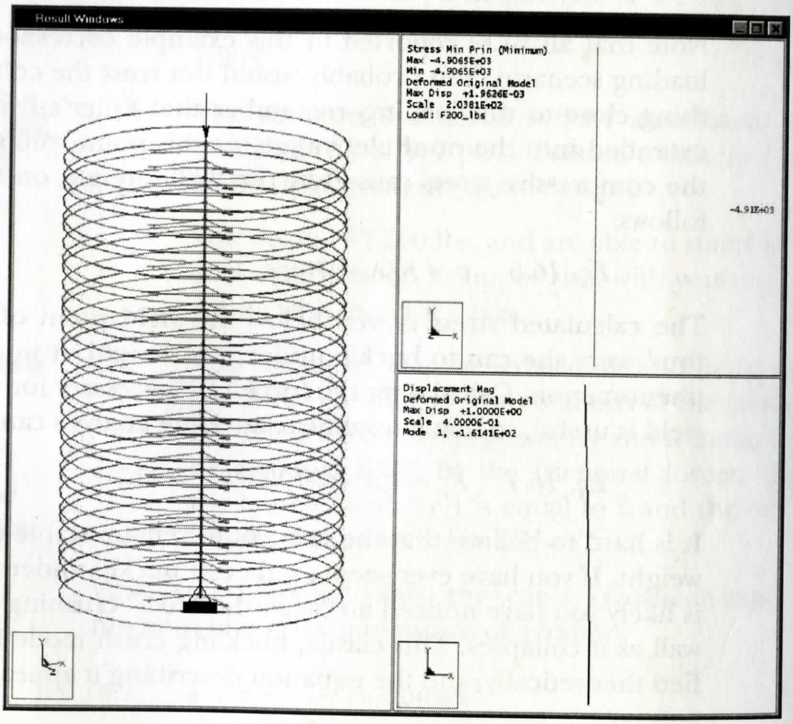

Fig. 16.2. Beam FEA model of the can, and results of its first buckling mode.

Now you are ready to perform FEA on the soda can. The simplest type of FEA you can conduct will make use of beam elements. Keep in mind, however, that because these elements have no real wall, they are not likely to capture the accordion crush mode. Note also that these elements cannot be "capped" (although the constraint at the bottom endpoint will act like a cap). Fig. 16.2 shows a Pro/MECHANICA beam model of the can with corresponding first mode buckling results (BLF = 161). The mode shape is that of a Euler column's first mode, yet the BLF is 60% of the hand calculated value. Note that the BLF is also nearly 70 times greater than that calculated using the empirical crush mode equation. Hence, you should avoid the use of beams for the buckling analysis of nonslender structures. This should not come as a surprise, because beam elements generally should not be used to represent structures that are short relative to respective cross sections.

Next, a Pro/MECHANICA shell model is built. This FEM and corresponding results are shown in Fig. 16.3 (BLF = 5). The mode shape is quite different from that of the beam. It is the accordion shape predicted by the crush mode theory. Note that the BLF is only 7% higher than that predicted by Eq. 16.8, yet over twice the value predicted by Eq. 16.9. As mentioned in the introduction to this chapter, buckling analyses require a robust safety factor and a thorough test correlation plan.

The above numbers mean that the can will not buckle under your perfectly centered weight, yet the comfort zone varies quite a bit depending on the analysis used. In addition, when dealing with such a thin-walled structure, tolerances are a big issue. A single thousandth of an inch difference in wall thickness (t = 0.004") will bring the empirical crush mode BLF down to 1.5–precariously close to unity.

Investigating the change in results caused by increasing the length of the soda can, while keeping all of its other properties and boundary conditions the same is extremely interesting. By reviewing the required conditions stated above, note that as L grows, both the theoretically and empirically derived crush mode BLFs become even more valid. Table 16.1 presents the manually calculated and FEA results of three new length scenarios in addition to the original.

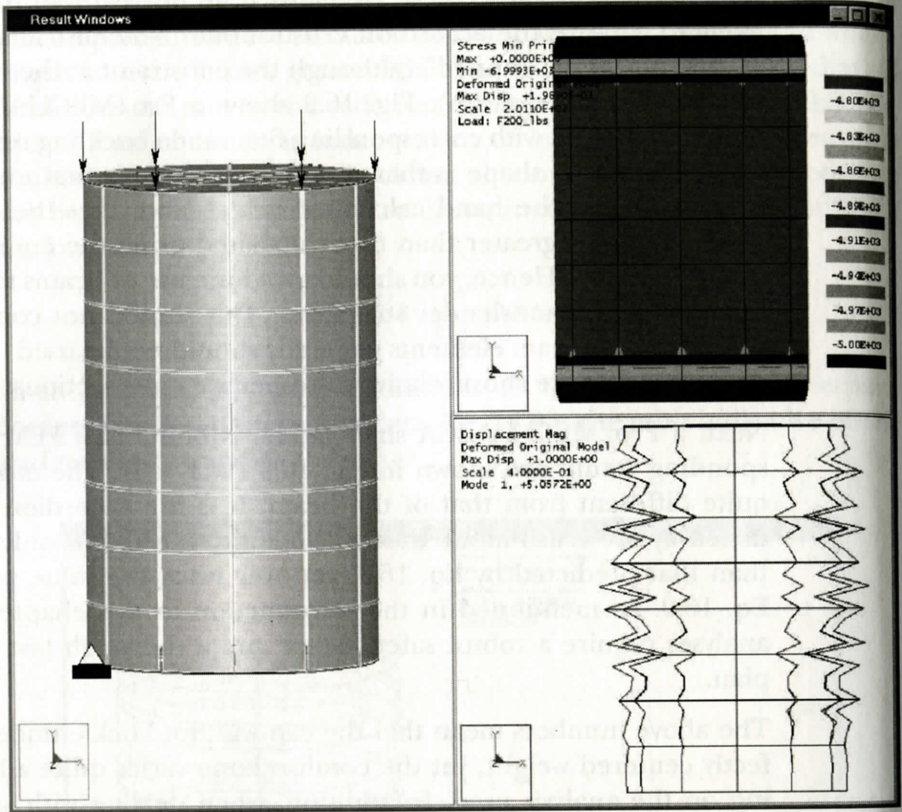

Fig. 16.3. Shell FEA model of the can, and results of its first buckling mode.

Table 16.1. Analytical buckling results of a fixed-free soda can subject to a coaxial 200lb compressive load applied to its free rim: r_0 = 1.3", t = 0.005", t_c = 1/16", K = 2, S_y = 6e4 psi, E = 1e7 psi, ν = 0.3

L	L_e	L_e/r	$\sqrt{\pi^2 E/S_y}$	Euler BLF	Theoretical Crush BLF	Empirical Crush BLF	Beam FEA BLF	Shell FEA BLF
4"	8"	8.72	40.6	265	4.75	2.35	161	5.06
20"	40"	43.6	40.6	10.6	4.75	2.35	10.3	10.3
65"	130"	142	40.6	1.00	4.75	2.35	0.999	1.00
100"	200"	218	40.6	0.425	4.75	2.35	0.425	0.425

Many interesting phenomena are occurring here. First, note once again that the crush mode BLFs are independent of column length. In addition, note that all three longer cans meet the slender, Euler column criterion.

The most difficult analysis, in terms of dealing with its results, turns out to be that of the 20" can, which can be barely considered as a Euler column. (Refer to Fig. 16.4.) Note that both the beam and shell FEA results show an Euler-type buckling mode with identical BLFs, which agree within 3% with the manually calculated Euler BLF. Yet, these values are all higher than their crush mode equivalents. It appears that as the column becomes Euler, FEA shell models no longer predict the accordion mode as the first buckling mode. Alternatively, it may be that to capture this mode, the model requires many more elements (although Pro/MECHANICA reported the results as having converged within 2.5%). Experiment using your own code. Note that all analyses still indicate that the longer can will still handle your perfectly centered weight, but the comfort zone derived from FEA may be dangerously misleading.

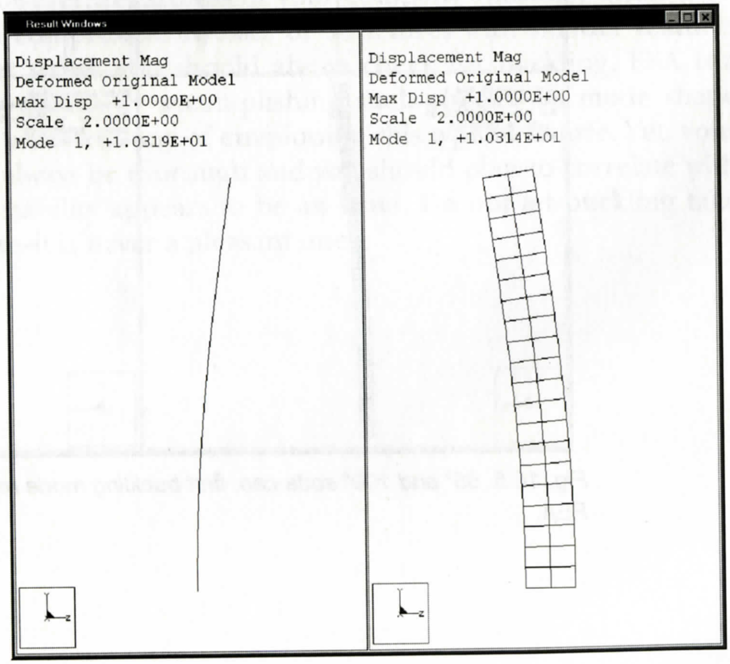

Fig. 16.4. 20" soda can, first buckling mode results of beam FEA and shell FEA.

The last two analysis results are easier to reconcile. Both 65" and 100" columns reside overwhelmingly in the Euler regime. Fig. 16.5 shows Euler type, buckling mode shapes from FEA. The corresponding BLFs are identical to the calculated Euler. Note that at the 65" length, these BLFs have reached unity–for the first time the can is no longer stable under your weight. Note as well that these BLFs are smaller than their crush mode equivalents. Had you taken for granted that the first mode shape would always be the accordion type, you would have underestimated the can at these lengths.

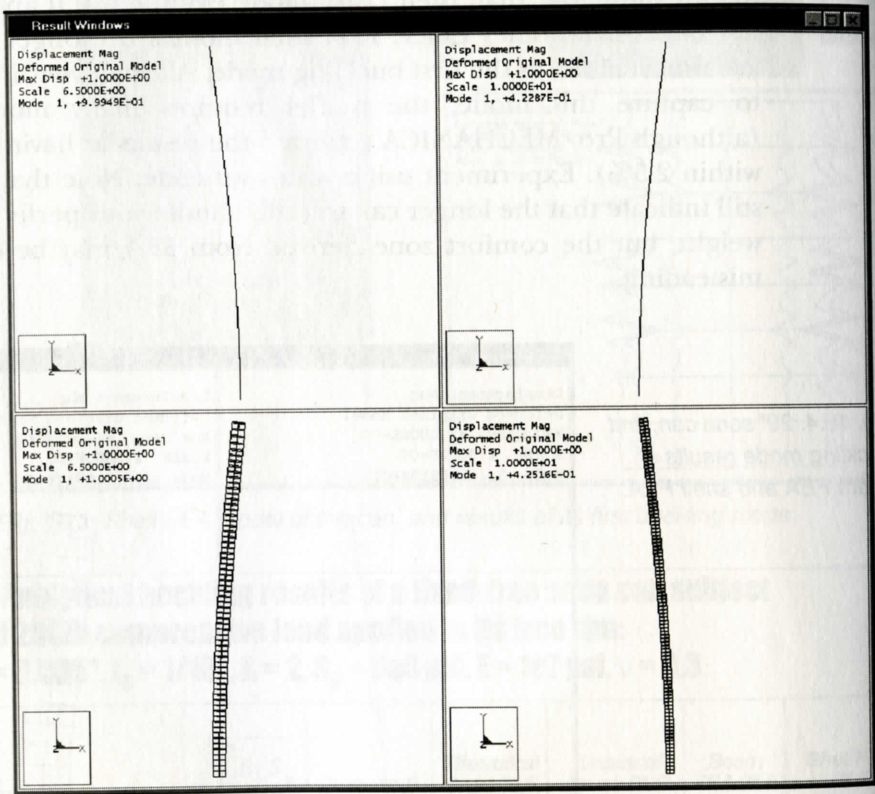

Fig. 16.5. 65" and 100" soda can, first buckling mode results of beam FEA and shell FEA.

Summary

Buckling analysis is a very difficult topic indeed. Any assumptions made regarding surface smoothness, geometric consistency, load vector placement, and orientation variances must be well founded and thoroughly reviewed. Worst case scenarios must always be utilized in the analysis. Manual calculations should be used against the FEA results whenever possible. Always review the buckling mode shape to verify that it makes sense and is not entirely unexpected. Always use large safety factors, especially when dealing with relatively short or nonlinear structures.

Most companies know whether buckling is a commonly encountered phenomenon in their product lines. If it is, it is well known that its results are sudden and spectacularly catastrophic. FEA users in these companies should spend quality time with test models, calculations, and empirical testing before basing a buckling critical design on analysis results. If buckling has not historically been a problem in your company, you should still take to heart the statements made earlier about considering tolerances and all load orientation options.

In general, however, regardless of your company's history, when dealing with high compressive stresses or structures with slender features that are under stress, you should always check for buckling. FEA is a great tool for efficiently accomplishing such checks. Its mode shape results are an excellent way of envisioning this type of failure. Yet, your analysis must always be thorough and you should plan to correlate with testing when stability appears to be an issue. Do not let buckling take you by surprise–it is never a pleasant one.