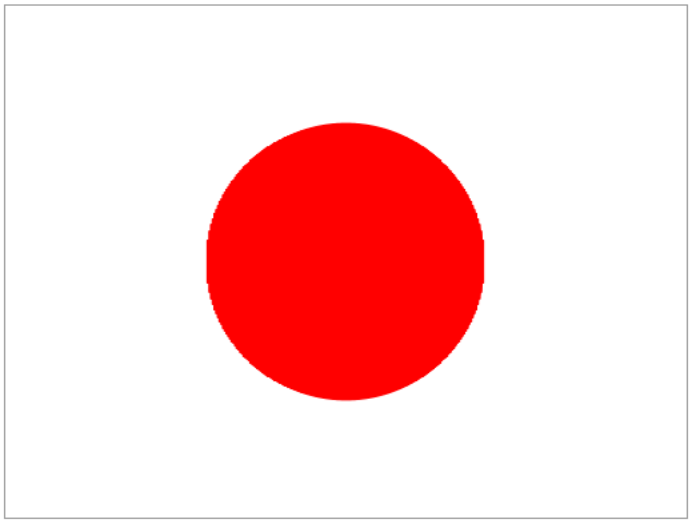
Using Hatch function by Werner_E Level 20 , T.Tokoro.

$$f(x) := 0.2\mathrm{i} \cdot \sqrt{5.0 \cdot x - 6.0} \cdot \sqrt{5.0 \cdot x - 14.0} \qquad (x-2)^2 + y^2 = .64 \xrightarrow{solve,\, y} \begin{bmatrix} -\left(0.2\mathrm{i} \cdot \sqrt{5.0 \cdot x - 6.0} \cdot \sqrt{5.0 \cdot x - 14.0}\right) \\ 0.2\mathrm{i} \cdot \sqrt{5.0 \cdot x - 6.0} \cdot \sqrt{5.0 \cdot x - 14.0} \end{bmatrix}$$

$$g(x) := -f(x)$$

$$Hatch(f1, f2, x1, x2, \Delta x) := \left\| \begin{array}{l} H \leftarrow [NaN \;\; NaN] \\ \text{for } i \in 0 .. \operatorname{trunc}\left(\frac{x2 - x1}{\Delta x}\right) \\ \quad \left\| \begin{array}{l} x \leftarrow x1 + i \cdot \Delta x \\ y1 \leftarrow f1(x) \\ y2 \leftarrow f2(x) \\ H \leftarrow \operatorname{stack}\left(H, \begin{bmatrix} x & y1 \\ x & y2 \\ NaN & NaN \end{bmatrix}\right) \end{array} \right. \\ H \end{array} \right.$$

$$H := Hatch(f, g, 0, 4, 0.01)$$

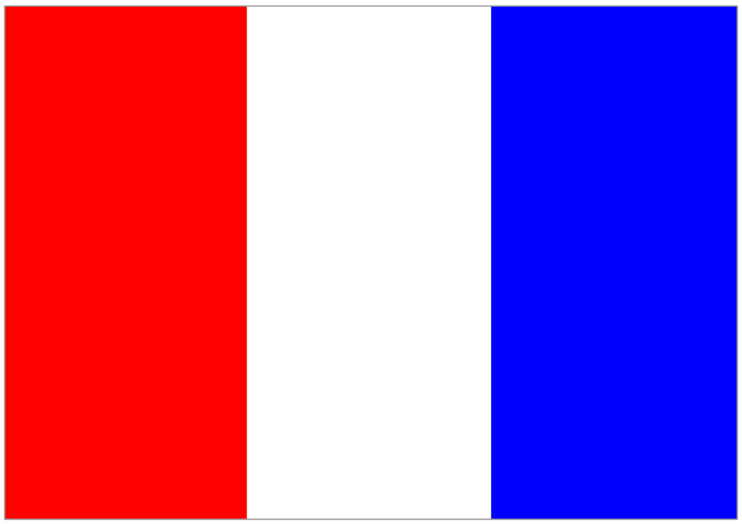


$$f(x) := 1.5 \qquad g(x) := 0.5 \qquad h(x) := -0.5 \qquad i(x) := -1.5$$

$$H1 := Hatch(f, g, 0, 4, 0.01) \qquad H2 := Hatch(g, h, 0, 4, 0.01) \qquad H3 := Hatch(h, i, 0, 4, 0.01)$$

$f(x) := 1.5 \quad g(x) := -1.5$

$H1 := Hatch\left(f, g, 0, \frac{4}{3}, 0.01\right) \qquad H2 := Hatch\left(f, g, \frac{4}{3}, \frac{8}{3}, 0.01\right) \qquad H3 := Hatch\left(f, g, \frac{8}{3}, 4, 0.01\right)$



$f(x) := x^2$

$g(x) := x + 1$

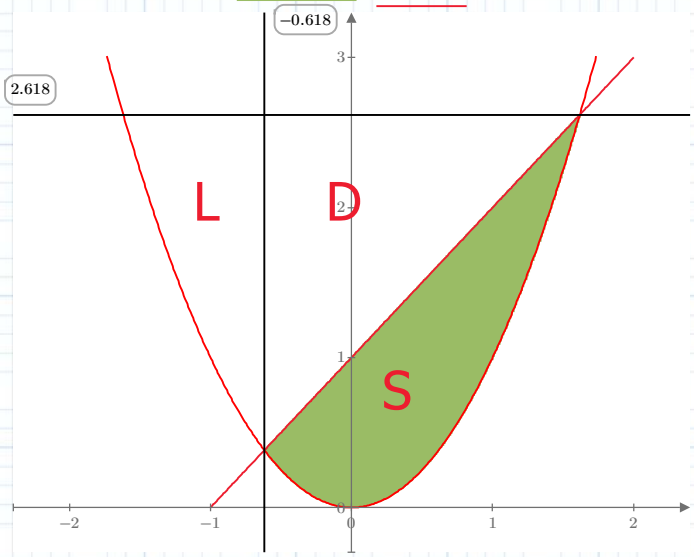
$H := Hatch\left(f, g, \frac{1}{2} - \frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2} + \frac{1}{2}, 0.01\right)$

$x^2 = x + 1 \xrightarrow{solve, x} \begin{bmatrix} \frac{\sqrt{5}}{2} + \frac{1}{2} \\ \frac{1}{2} - \frac{\sqrt{5}}{2} \end{bmatrix}$

$up := \frac{\sqrt{5}}{2} + \frac{1}{2}$

$low := \frac{1}{2} - \frac{\sqrt{5}}{2}$

$H^{\langle 0 \rangle}$ x



$H^{\langle 1 \rangle}$

$f(x)$

$g(x)$

$S := \int_{low}^{up} (g(x) - f(x))\, dx \rightarrow \frac{5 \cdot \sqrt{5}}{6}$

$L := \int_{-up}^{low} ((up + 1) - f(x))\, dx \rightarrow \frac{\sqrt{5}}{2} + \frac{1}{6}$

$D := \int_{low}^{up} ((up + 1) - g(x))\, dx \rightarrow \frac{5}{2}$

$S + L + D \rightarrow \frac{4 \cdot \sqrt{5}}{3} + \frac{8}{3}$

$\int_{-up}^{up} ((up + 1) - f(x))\, dx \rightarrow \frac{4 \cdot \sqrt{5}}{3} + \frac{8}{3}$