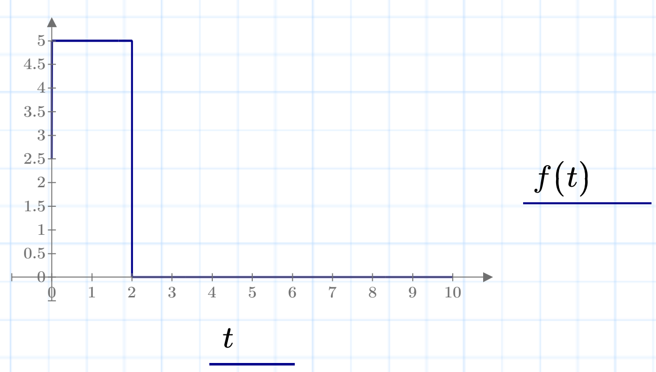
Circuit Analysis using Laplace Transform 2nd order LowPassRC

$u := \Phi$ "<—— For piecewise functions must be used heaviside"
$p := 5$ "<—— Signal period"
$n := 10$ "<—— Number of periods"

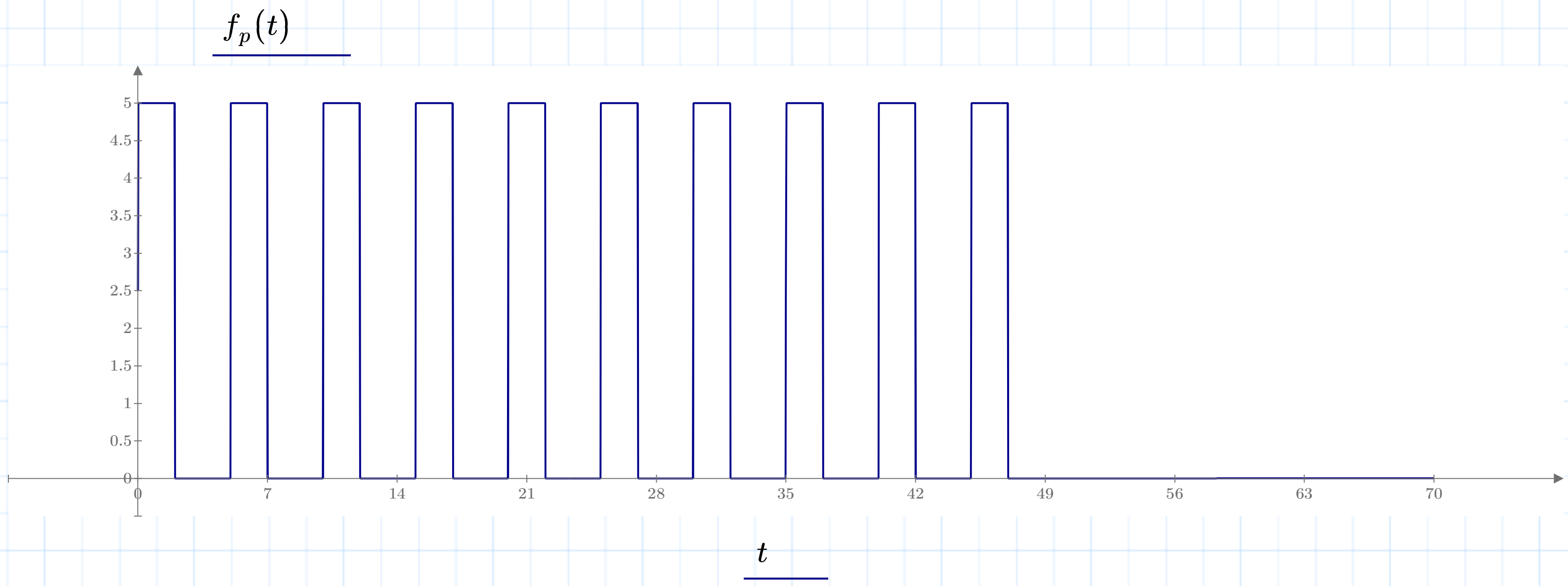
"Input Signal:"

$$f(t) := 5\,(u(t) - u(t-2))$$



"Incremented f(t):"

$$f_p(t) := \begin{Vmatrix} f\_(t) \leftarrow f(t) \\ \text{for } i \in 1..n-1 \\ \quad \Vert\, f\_(t) \leftarrow f\_(t) + f(t - i \cdot p) \\ f\_(t) \end{Vmatrix}$$

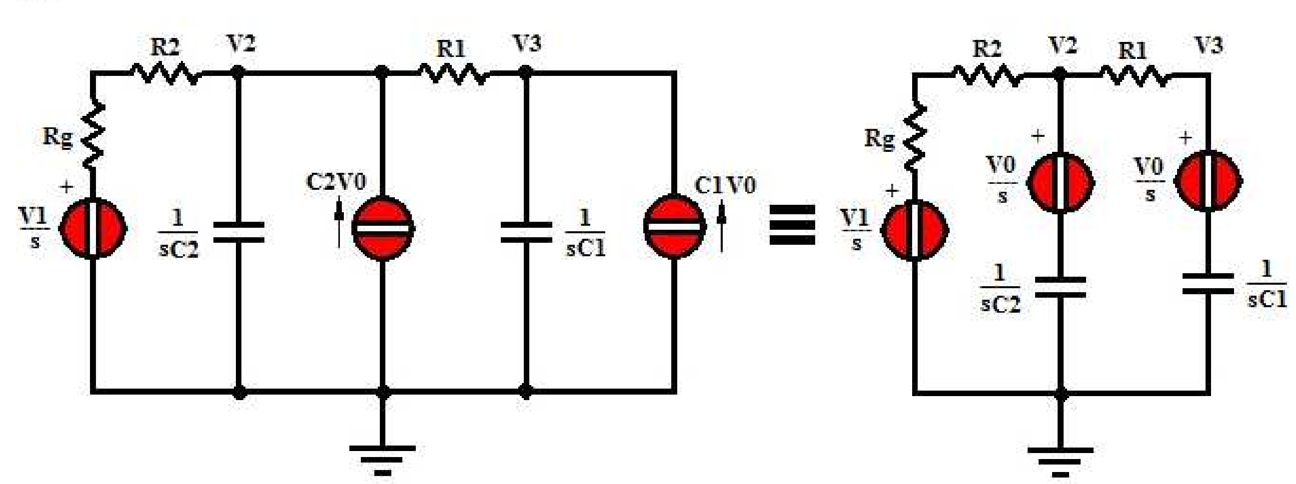


"Apply Laplace transform to f(t):"

$$f(s) := f(t) \xrightarrow{laplace} -\frac{5 \cdot e^{-2 \cdot s} - 5}{s}$$

$$g(s) := f_p(t) \xrightarrow{laplace} \frac{5 \cdot e^{-10 \cdot s} - 5 \cdot e^{-2 \cdot s} - 5 \cdot e^{-12 \cdot s} + 5 \cdot e^{-20 \cdot s} - 5 \cdot e^{-22 \cdot s} + 5 \cdot e^{-30 \cdot s} - 5 \cdot e^{-32 \cdot s} + \cdots}{}$$

"Components values:"



$$V_2(s) = \frac{\dfrac{f(s)}{R_2 + R_g} + \dfrac{\frac{V_0}{s}}{\frac{1}{s \cdot C_2}} + \dfrac{\frac{V_0}{s}}{R_1 + \frac{1}{s \cdot C_1}}}{\dfrac{1}{R_2} + s \cdot C_2 + \dfrac{1}{R_1 + \frac{1}{s \cdot C_1}}}$$

$C_2 := 2 \cdot 10^{-6}$

$C_1 := 2 \cdot 10^{-6}$

$R_1 := 10$

$R_2 := 10$

$V_0 := 5$

$V_1 := 15$

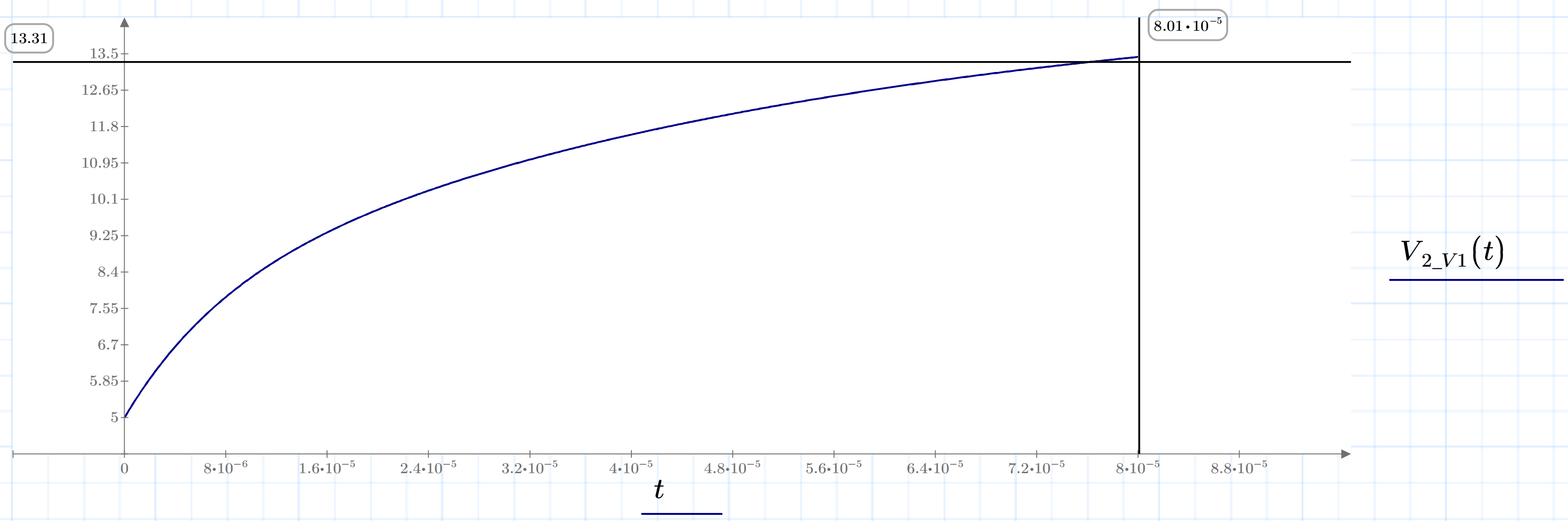
$R_g := 0$

$$\frac{\dfrac{\frac{V_1}{s}}{R_2+R_g}+\dfrac{\frac{V_0}{s}}{\frac{1}{s \cdot C_2}}+\dfrac{\frac{V_0}{s}}{R_1+\frac{1}{s \cdot C_1}}}{\dfrac{1}{R_2}+s \cdot C_2+\dfrac{1}{R_1+\frac{1}{s \cdot C_1}}} \xrightarrow[\text{collect}, C_1]{\substack{\text{simplify} \\ \text{collect}, s \\ \text{collect}, s^2 \\ \text{collect}, V_0 \\ \text{collect}, C_2}} \frac{\dfrac{2500000 \cdot s^2}{s^3+150000 \cdot s^2+2500000000 \cdot s}+\dfrac{500000 \cdot (1250000 \cdot s+37500000000)}{s^3+150000 \cdot s^2+2500000000 \cdot s}}{500000}$$

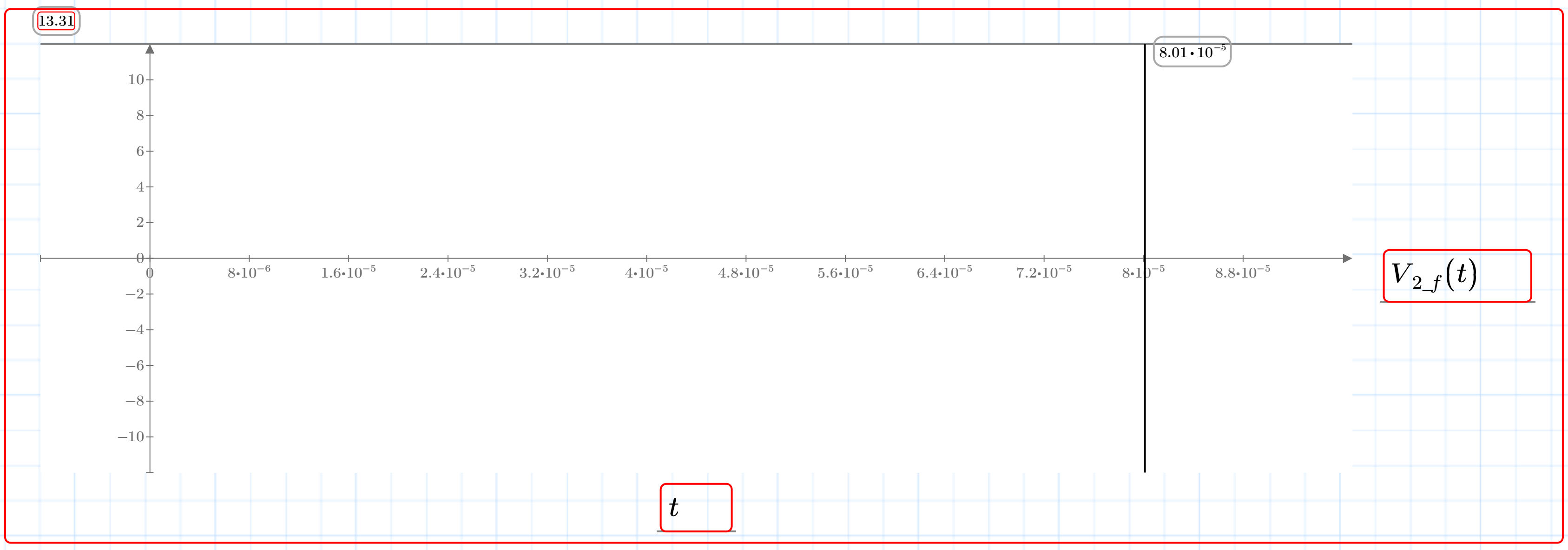
$$\frac{\dfrac{f(s)}{R_2+R_g}+\dfrac{\frac{V_0}{s}}{\frac{1}{s \cdot C_2}}+\dfrac{\frac{V_0}{s}}{R_1+\frac{1}{s \cdot C_1}}}{\dfrac{1}{R_2}+s \cdot C_2+\dfrac{1}{R_1+\frac{1}{s \cdot C_1}}} \xrightarrow[\text{collect}, C_1]{\substack{\text{simplify} \\ \text{collect}, s \\ \text{collect}, s^2 \\ \text{collect}, V_0 \\ \text{collect}, C_2}} \frac{\dfrac{2500000 \cdot s^2}{s^3+150000 \cdot s^2+2500000000 \cdot s}+\dfrac{500000 \cdot (750000 \cdot s-12500000000 \cdot e^{-2 \cdot s}-250000 \cdot s \cdot e^{-2 \cdot s}+12500000000)}{s^3+150000 \cdot s^2+2500000000 \cdot s}}{500000}$$

$$\frac{\dfrac{g(s)}{R_2+R_g}+\dfrac{\frac{V_0}{s}}{\frac{1}{s \cdot C_2}}+\dfrac{\frac{V_0}{s}}{R_1+\frac{1}{s \cdot C_1}}}{\dfrac{1}{R_2}+s \cdot C_2+\dfrac{1}{R_1+\frac{1}{s \cdot C_1}}} \xrightarrow[\text{collect}, C_1]{\substack{\text{simplify} \\ \text{collect}, s \\ \text{collect}, s^2 \\ \text{collect}, V_0 \\ \text{collect}, C_2}} \frac{\dfrac{2500000 \cdot s^2}{s^3+150000 \cdot s^2+2500000000 \cdot s}-\dfrac{500000 \cdot \left(250000 \cdot s \cdot e^{-47 \cdot s}-750000 \cdot s+25000000000 \cdot \sinh\left(\frac{750000 \cdot s}{500000}\right) \cdot e \ldots\right.}{\ldots}}{\ldots}$$

$$V_{2\_V1}(t) := \frac{\dfrac{2500000 \cdot s^2}{s^3+150000 \cdot s^2+2500000000 \cdot s}+\dfrac{500000 \cdot (1250000 \cdot s+37500000000)}{s^3+150000 \cdot s^2+2500000000 \cdot s}}{500000} \xrightarrow{\text{invlaplace}} 15-2 \cdot \sqrt{5} \cdot \sinh\left(25000 \cdot \sqrt{5} \cdot t\right) \cdot e^{-75000 \cdot t}-1\ldots$$



$$V_{2\_f}(t) := \frac{\dfrac{2500000 \cdot s^2}{s^3+150000 \cdot s^2+2500000000 \cdot s}+\dfrac{500000 \cdot (750000 \cdot s-12500000000 \cdot e^{-2 \cdot s}-250000 \cdot s \cdot e^{-2 \cdot s}+12500000000)}{s^3+150000 \cdot s^2+2500000000 \cdot s}}{500000} \xrightarrow{\text{invlaplace}} 5 \cdot e^{150000-75000 \cdot t} \cdot \cosh\ldots$$



$$V_{2\_g}(t) := \frac{\dfrac{2500000\cdot s^2}{s^3+150000\cdot s^2+2500000000\cdot s} - \dfrac{500000\cdot\left(250000\cdot s\cdot e^{-47\cdot s} - 750000\cdot s + 25000000000\cdot \sinh\left(\dfrac{750000\cdot s}{500000}\right)\cdot e^{-\frac{11750000\cdot s}{500000}} - 25000000000\cdot \sinh\left(\dfrac{17\ldots}{5\ldots}\right.\right.}{\ldots}}{\ldots}$$