$$IsPoly2(f,x,y) := \left| \begin{array}{l} aux(f) \leftarrow \left| \begin{array}{l} v(x\_) \leftarrow f(x\_) \text{ coeffs}, x\_ \rightarrow \\ v \end{array} \right. \\ v \leftarrow aux(f) \\ \text{return } 1 \text{ if } v(x) = v(y) \\ 0 \end{array} \right.$$

$$IsDenomPoly(f) := \left| \begin{array}{l} v(x\_) \leftarrow denom(f(x\_)) \text{ coeffs}, x\_ \rightarrow \begin{pmatrix} 7 \\ 0 \\ 5 \\ -3 \end{pmatrix} \\ \text{return } 0 \text{ on error } R \leftarrow IsPoly2(v) \rightarrow \\ 1 \end{array} \right.$$

$$f_1(x) := \frac{3 + \sin\left(\ln(x) + x^4\right) \cdot x^2 + x^4}{3x^2 \cdot \tan(x) + 5} \qquad f_2(y) := \frac{3y^2 \cdot \tan(y) + 5}{5y^2 - 3y^3 + 7}$$

$IsDenomPoly(f_1) \rightarrow 0$ $\qquad$ $IsDenomPoly(f_2) \rightarrow 1$

$$IsPoly2\_(f) := \left| \begin{array}{l} v(x\_) \leftarrow f(x\_) \text{ coeffs}, x\_ \rightarrow \\ \text{return } 1 \text{ if } v(x) = v(y) \\ 0 \end{array} \right.$$

$$IsDenomPoly\_(f) := \left| \begin{array}{l} v(x\_) \leftarrow denom(f(x\_)) \text{ coeffs}, x\_ \rightarrow \begin{pmatrix} 7 \\ 0 \\ 5 \\ -3 \end{pmatrix} \\ \text{return } 0 \text{ on error } R \leftarrow IsPoly2\_(v) \rightarrow \\ 1 \end{array} \right.$$

$IsDenomPoly\_(f_1) \rightarrow 0$ $\qquad$ $IsDenomPoly\_(f_2) \rightarrow 1$

So here it looks that we don't need "aux", etc. in IsPoly2

$f(x,y) := x + y^2$

$g := f(3)$

$g(10) = 103$ $\qquad$ $g(2) = 7$

$g(a) \rightarrow a^2 + 3$

History:

$g_1(x) := 3x^2 \cdot \tan(x) + 5$ $\qquad$ $g_2(y) := 5y^2 - 3y^3 + 7$

First approach was to compare the result of "coeffs" for different values of x
We have a polynom if coeffs returns a vector of constat scalars, otherwise the results should be different depending on the arguments

$$test0(f) := \left| \begin{array}{l} v(x\_) \leftarrow f(x\_) \text{ coeffs}, x\_ \rightarrow \\ v(x) \end{array} \right. \qquad test0(g_1) \rightarrow \begin{pmatrix} 5 \\ 0 \\ 3 \cdot \tan(x) \end{pmatrix} \qquad test0(g_2) \rightarrow \begin{pmatrix} 7 \\ 0 \\ 5 \\ -3 \end{pmatrix}$$

$$IsPoly0(f) := \left| \begin{array}{l} v(x\_) \leftarrow f(x\_) \text{ coeffs}, x\_ \rightarrow \\ \text{return } 1 \text{ if } v(1) = v(2) \\ 0 \end{array} \right.$$

$IsPoly0(g_1) \rightarrow 0$ $\qquad$ $IsPoly0(g_2) \rightarrow 0$

Obviously this approach does not work as the second call should return a 1.

Next try:

$$\text{test1}(f) := \left| \begin{array}{l} v(x\_) \leftarrow f(x\_) \text{ coeffs}, x\_ \rightarrow \\ v \end{array} \right.$$

$v := \text{test1}(g_1)$ $\quad v(1) \rightarrow \begin{pmatrix} 5 \\ 0 \\ 3\cdot\tan(1) \end{pmatrix}$ $\quad v(2) \rightarrow \begin{pmatrix} 5 \\ 0 \\ 3\cdot\tan(2) \end{pmatrix}$

$v := \text{test1}(g_2)$ $\quad v(1) \rightarrow \begin{pmatrix} 7 \\ 0 \\ 5 \\ -3 \end{pmatrix}$ $\quad v(2) \rightarrow \begin{pmatrix} 7 \\ 0 \\ 5 \\ -3 \end{pmatrix}$

I dont understand the error message which says

This value must be a scalar.

but the function seems to work as expected nonetheless

$$\text{IsPoly1}(f) := \left| \begin{array}{l} \text{aux}(f) \leftarrow \left| \begin{array}{l} v(x\_) \leftarrow f(x\_) \text{ coeffs}, x\_ \rightarrow \\ v \end{array} \right. \\ v \leftarrow \text{aux}(f) \\ \text{return } 1 \text{ if } v(1) = v(2) \\ 0 \end{array} \right.$$

$\text{IsPoly1}(g_1) \rightarrow 0$ $\qquad \text{IsPoly1}(g_2) \rightarrow 1$

$$\text{IsPoly1\_}(f) := \left| \begin{array}{l} v(x\_) \leftarrow f(x\_) \text{ coeffs}, x\_ \rightarrow \\ \text{return } 1 \text{ if } v(1) = v(2) \\ 0 \end{array} \right.$$

$\text{IsPoly1\_}(g_1) \rightarrow 0$ $\qquad \text{IsPoly1\_}(g_2) \rightarrow 0$ WRONG!

???? For some reason we need aux here - no idea, why!

So this approach seems to work, but it may happen that we run into functions not defined for arguments 1 and 2 or which by chance return the very same result for both and so we would get a 1 while a 0 would be correct.

So the next idea was to use two different variables instead of the constants 1 and 2. After all we evaluate symbolically and expressions which include those variables should return a zero if compared, but, alas, we get an error:

$$\text{IsPoly2}(f) := \left| \begin{array}{l} \text{aux}(f) \leftarrow \left| \begin{array}{l} v(x\_) \leftarrow f(x\_) \text{ coeffs}, x\_ \rightarrow \\ v \end{array} \right. \\ v \leftarrow \text{aux}(f) \\ \text{return } 1 \text{ if } v(x) = v(y) \\ 0 \end{array} \right.$$

$\text{IsPoly2}(g_1) \rightarrow$ $\qquad \text{IsPoly2}(g_2) \rightarrow 1$

The error message is again This value must be a scalar.

$$\text{IsPoly2\_}(f) := \left| \begin{array}{l} v(x\_) \leftarrow f(x\_) \text{ coeffs}, x\_ \rightarrow \\ \text{return } 1 \text{ if } v(x) = v(y) \\ 0 \end{array} \right.$$

$\text{IsPoly2\_}(g_1) \rightarrow$ $\qquad \text{IsPoly2\_}(g_2) \rightarrow 1$

Here omitting "aux" still yields the same correct result!! Strange!

So this function alone is not capable to give us the result 0 in case of a non-polynomial.
We would need a second function which simply looks if an error occurs when calling IsPoly2 and the yields 0.
We can use either IsPoly2 or IsPoly2_ (without aux) here.

$$\text{IsPoly}_{\text{final}}(f) := \left| \begin{array}{l} \text{return } 0 \text{ on error IsPoly2}(f) \\ 1 \end{array} \right.$$

$\text{IsPoly}_{\text{final}}(g_1) \rightarrow 0$ $\qquad \text{IsPoly}_{\text{final}}(g_2) \rightarrow 1$

The error message is "This variable is undefinded". It can be avoided by using local symbolic evaluation:

$$\text{IsPoly}_{\text{final2}}(f) := \left| \begin{array}{l} \text{return } 0 \text{ on error IsPoly2}(f) \rightarrow \\ 1 \end{array} \right.$$

$\text{IsPoly}_{\text{final2}}(g_1) \rightarrow 0$ $\qquad \text{IsPoly}_{\text{final2}}(g_2) \rightarrow 1$

My attempts to make IsPoly2 a local function to IsPoly.final to make an all-in-one solution unfortunately failed.