Tooth-root stress calculation of internal spur gears

A Kawalec* and **J Wiktor**
Rzeszów University of Technology, Rzeszów, Poland

Abstract: This paper is focused on analysis of tooth-root strength of internal gears. Methods of calculation of the tooth-root stress of internal gears according to the ISO 6336-3:1996 and DIN 3990-3:1987 standards are briefly described. Reasons for potential errors in the strength calculations of internal gears performed in accordance with the ISO 6336-3:1996 and DIN 3990-3:1987 standards are discussed. Determination of the precise values of the parameters of the critical section and introducing them to use with the ISO 6336-3:1996 standard, proposed in the paper and called the modified ISO 6336-3:1996 method, is presented. Finite element analysis of various accurate models of internal gears is performed and location of critical section as well as its parameters are determined for all considered models of gears. On the grounds of results of precise finite element computations made for a large number of models of gears the influences of several main parameters of the machined gear and applied tool on the tooth-root strength of internal gears are investigated. General conclusions concerning the tooth-root strength of internal gears are formulated. They concentrate on the influences of several main parameters of the machined gear and tool on the tooth-root stresses of gear. They are useful for correct determination of the parameters of the critical section of the internal spur gear and for proper selection of the main parameters of the machined gear and applied tool.

Keywords: gear design and manufacturing, finite element analysis, numerical modelling, computer aided engineering (CAE) techniques

1 INTRODUCTION

Internal spur gears are used in machine design more often than before, mainly due to more wide application of planetary gear transmissions. Development of such transmissions follows their general advantages [**1**, **2**]:

(a) compact structure (relatively small dimensions and weight, compared with external gear transmissions with the same gear ratio and the same transmitted power), which is especially important in the case of aviation gear transmissions;
(b) better load distribution, especially in planetary transmissions, in which the load is distributed between several planet gears, giving more possibilities for compensation of manufacturing deviations and assembly errors (e.g. due to flexible bearing structure of a sun gear and due to flexible wheel rims of internal gears);
(c) possibility of summing rotations, usually applied in differential gear transmissions;
(d) relatively silent-running work of gear transmission.

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**Corresponding author: Rzeszów University of Technology, W. Pola 2, 35-959 Rzeszów, Poland.*

The strength of internal gears has not been of great concern of designers, until recently. The thickness of the internal gear at the tooth root is larger than in the case of external gears with the same module. Therefore, it was assumed that the risk of damage of the internal gear is much smaller than in the case of the external gear. Thus, most work concerning the strength of gears has been devoted to external gears. Computational models initially developed for use with external gears were transferred to internal gears. Good examples of such approach are the well-known ISO 6336-3:1996 [**3**–**5**] and DIN 3990-3:1987 [**6**] standards, which focus on the analysis of gear strength. These standards are almost completely devoted to analysis of the strength of external gears. In the case of an internal gear, a strength analysis is performed for a virtual and simplified model of a tooth, which does not consider the real tooth form resulting from gear machining. Also further calculations are conducted using formulae developed for external gears. The other well-known standards, ANSI–AGMA 6002-B93 [**7**] and [**8**], do not cover the analysis of the strength of internal gears at all, as discussed in reference [**9**].

In recent years, because of the constantly increasing load applied in internal gear transmissions the above-named safety margin, coming from a thicker tooth root in internal gears, became significantly smaller. Therefore,

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precise strength analysis of this type of gear transmission, considering real and not virtual parameters comes to be significant and necessary. Experimental investigations [**1**] as well as precise numerical calculations made with the use of the finite element method (FEM) or the boundary element method (BEM) [**1**] have shown significant differences in the strengths of internal and external gears. These differences consider especially the location of the critical section (the place at the tooth root where the maximum stress occurs) and the influence of the thickness of the gear rim on the magnitude of stresses at the tooth root.

This paper also focuses on the problem of the strength of internal gears. The aims of the described investigations are as follows:

(a) development of a geometric model of internal gearing with consideration of the real tooth form, resulting from the applied method of machining and parameters of the tool;
(b) performing calculations of stresses for the above-named model, using the FEM: determination of the distribution of stress at the tooth root, computation of the maximum stresses and location of the critical section;
(c) computation, for internal gears, of the precise values of the parameters of the critical section, s_{Fn}, h_{Fe} and ρ_{F}, considering a real tooth fillet and not the virtual tooth fillet suggested by the ISO 6336-3:1996 and DIN 3990-3:1987 standards;
(d) calculation of stresses at the tooth root of an internal gear according to the formulae given in the ISO 6336-3:1996 and DIN 3990-3:1987 standards with consideration of the precise values of the parameters of the critical section (in this paper this method is called the modified ISO 6336-3:1996 method);
(e) comparison of results of calculations made with the use of three methods: FEM for the precise model of tooth, the numerical procedure given in the ISO 6336-3:1996 and DIN 3990-3:1987 standards (method B) and the modified ISO 6336-3:1996 method;
(f) analysis of the influence of the most important gear and tool geometric parameters on the tooth root strength of internal gears.

2 CALCULATION OF THE TOOTH-ROOT STRESS OF AN INTERNAL GEAR ACCORDING TO THE ISO 6336-3:1996 AND DIN 3990-3:1987 STANDARDS

According to the ISO 6336-3:1996 and DIN 3990-3:1987 standards, the bending strength of spur gears is determined on the basis of the tooth-root stress σ_{F}, which is computed from the formula

$$\sigma_{\mathrm{F}} = \sigma_{\mathrm{F0}} K_{\mathrm{A}} K_{\mathrm{v}} K_{\mathrm{F}\beta} K_{\mathrm{F}\alpha} \qquad (1)$$

where

σ_{F0} = nominal local stress at the tooth root
K_{A} = application factor for bending strength considering variation in load in gear transmission caused by external factors (variation in the real acting load from nominal value)
K_{v} = dynamic factor for bending strength considering the increase in load in gear transmission caused by internal factors (dynamic loads caused by vibrations of the gear and pinion)
$K_{\mathrm{F}\beta}$ = face load factor considering the non-uniform distribution of load along the path of contact used for calculation of the tooth strength for fatigue bending
$K_{\mathrm{F}\alpha}$ = transverse load factor considering the non-uniform distribution of load between pairs of teeth in contact used for calculation of the tooth strength for fatigue bending

In order to compare different computational methods and to determine the influences of selected machined gear and tool geometric parameters on the tooth-root strength of internal gears it is assumed in this work that external conditions of gear transmission operation [and, in consequence, values of load coefficients used in equation (1)] are the same in all considered cases. Therefore, only the nominal stress σ_{F0} is calculated and non-uniform distribution of loads as well as additional dynamic loads are not considered; i.e., for comparison reasons described above, it is assumed that $K_{\mathrm{A}} = K_{\mathrm{v}} = K_{\mathrm{F}\beta} = K_{\mathrm{F}\alpha} = 1.0$.

The nominal local stress at the tooth root, σ_{F0}, is calculated from

$$\sigma_{\mathrm{F0}} = \frac{F_{\mathrm{t}}}{m_{\mathrm{n}} b} Y_{\mathrm{F}} Y_{\mathrm{S}} Y_{\beta} \qquad (2)$$

where

F_{t} = transmitted tangential load at the pitch cylinder
m_{n} = normal module
b = effective face width
Y_{F} = tooth form factor, which considers the influence of the tooth form on the nominal stress at the tooth root
Y_{S} = stress correction factor, which considers both the concentration of stress induced by notch at tooth root (fillet) and the complex stress state at tooth root
Y_{β} = helix angle factor, which considers the influence of the helix angle on the stress at the tooth root; for straight tooth gears, $Y_{\beta} = 1$

The tooth form factor Y_{F} and stress correction factor Y_{S} are calculated from

$$Y_{\mathrm{F}} = \frac{6(h_{\mathrm{Fe}}/m_{\mathrm{n}}) \cos \alpha_{\mathrm{Fen}}}{(s_{\mathrm{Fn}}/m_{\mathrm{n}})^2 \cos \alpha_{\mathrm{n}}} \qquad (3)$$

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and

$$Y_S = (1.2 + 0.13L)q_s^{e_1} \tag{4}$$

where $e_1 = (1.21 + 2.3/L)^{-1}$, $L = s_{Fn}/h_{Fe}$ and $q_s = s_{Fn}/(2\rho_F)$ and where

α_{Fen} = load angle
s_{Fn} = tooth thickness at the critical section
h_{Fe} = bending moment of the arm
ρ_F = radius of curvature of fillet curve (i.e. radius of curvature of fillet in its transverse section)

As follows from equations (3) and (4) the coefficients Y_F and Y_S are calculated on the basis of parameters of critical section: s_{Fn}, h_{Fe} and ρ_F. Values of these parameters are strongly related to the form of fillet at the tooth root mapped by a particular tool during gear machining. This form depends on the applied method of machining and geometric parameters of machined gear and applied tool.

In the ISO 6336-3:1996 and DIN 3990-3:1987 standards, only a simplified model of both teeth is considered for calculation of parameters of the critical section of the internal gear. This model of tooth represents a trapezoid rack tooth with the same standard basic rack tooth profile as the gear (Fig. 1). In the most precise computational method, method B, it is assumed that the load is applied at the highest point of single pair tooth contact (HPSTC) and the load angle equals nominal value of standard normal pressure angle $\alpha_{Fen} = \alpha_n$. It is also assumed that the critical section is determined by the point of tangency of the tooth fillet with the straight line inclined at an angle $\zeta_F = 30^\circ$ to the tooth centre-line, as is presumed in the case of external gears. For this model the following formulae for tooth thickness at the critical section s_{Fn} and bending moment of the arm h_{Fe} in internal gears are developed (Fig. 1):

$$\frac{s_{Fn2}}{m_n} = 2\left[\frac{\pi}{4} + \tan\alpha_n \frac{h_{fP2} - \rho_{fP2}}{m_n} + \frac{\rho_{fP2} - s_{pr}}{m_n \cos\alpha_n} - \frac{\rho_{fP2}}{m_n}\cos\left(\frac{\pi}{6}\right)\right] \tag{5}$$

$$\frac{h_{Fe2}}{m_n} = \frac{d_{en2} - d_{fn2}}{2m_n} - \left[\frac{\pi}{4} + \left(\frac{h_{fP2}}{m_n} - \frac{d_{en2} - d_{fn2}}{2m_n}\right)\tan\alpha_n\right]\tan\alpha_n - \frac{\rho_{fP2}}{m_n}\left[1 - \sin\left(\frac{\pi}{6}\right)\right] \tag{6}$$

In this work, only internal gears without an intentional undercut at the tooth root are analysed. For such gears, according to the ISO 6336-3:1996 standard [**3**], $s_{pr} = 0$.

For a helix gear, related calculations are carried out for a virtual gear with a straight tooth, as is done in the case of external gears. For this purpose, auxiliary virtual cylinders and their diameters d_{n2}, d_{fn2} and d_{en2} are determined. Then, the tooth height h_{fP2} is calculated from

$$h_{fP2} = \frac{d_{n2} - d_{fn2}}{2} \tag{7}$$

where

d_{n2} = diameter of the reference cylinder
d_{fn2} = diameter of tooth root of virtual gear

In the ISO 6336 draft international standard [**10**] it was assumed that the tooth fillet curve can be replaced by a circular arc of radius

$$\rho_{F2} = \frac{c_P}{2(1 - \sin\alpha_n)} \tag{8}$$

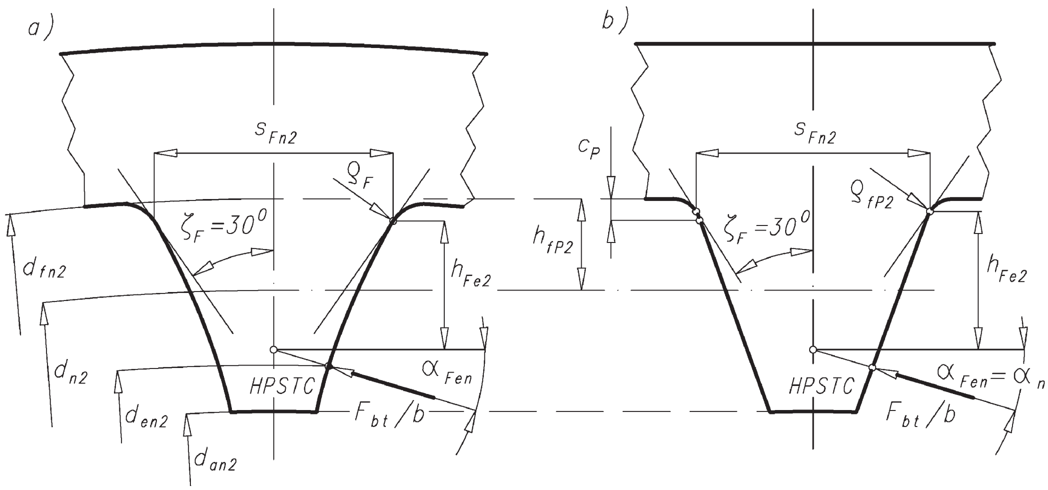

Fig. 1 Form of the internal gear tooth and parameters of the critical section: (a) real tooth form obtained after generation; (b) virtual tooth form of the rack, according to the ISO 6336-3:1996 and DIN 3990-3:1987 standards

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where c_P denotes the tooth tip clearance in internal gear transmission.

Only in the case of complete rounding of the tooth root, according to the ISO 6336 draft international standard, were values twice the radius ρ_{F2} in equation (8) assumed:

$$\rho_{F2} = \frac{c_P}{1 - \sin \alpha_n} \tag{9}$$

In ISO 6336-3:1996, for proper strength calculations of the tooth root, the radius ρ_{F2} is calculated from

$$\rho_{F2} = \rho_{fP2} = \frac{c_P}{1 - \sin \alpha_n} = \frac{d_{Nf2} - d_{f2}}{2(1 - \sin \alpha_n)} \tag{10}$$

where d_{Nf2} denotes the diameter of the circle close to the foot circle, grasping the usable part of the tooth profile.

In the computations and analyses performed by the present authors a real, precisely computed tooth profile with real radius of curvature of the fillet is used. The algorithm of computation of those geometric characteristics of spur gears is based on the analytical and numerical method developed by the present authors, described in detail in reference [**11**].

The radius ρ_{F2} has a significant influence on the maximum stress at the tooth root. Therefore, in the following sections the results of calculations of both geometric parameters of the fillet and the maximum stress at the tooth root obtained with consideration of the different methods of their determination are given. From the related figures it follows, among other things, that the ISO 6336-3:1996 standard [**3**] [equation (10)] gives results much closer to the results obtained in precise geometric computations and in finite element analysis, than the ISO 6336 draft international standard [**10**] [equation (8)].

The assumptions made in the ISO 6336-3:1996 and DIN 3990-3:1987 standards significantly simplify calculations but may cause errors. In the following sections it is shown that stresses at the tooth root calculated from equation (2) with consideration of equations (3) to (10) differ from the values obtained in precise computations based on the FEM, which take into account the real tooth profile of the internal gear [**1**]. The main reasons for such errors are as follows:

1. The real involute tooth of an internal gear is replaced by a straight-line rack tooth.
2. The tooth fillet at the tooth root is substituted by a circular arc of radius ρ_F, calculated from equation (10). The tooth fillet resulting from gear generation is in fact a trochoid with varying radius of curvature. This trochoid is created by the tool tip in its rolling movement. The form of this fillet depends on different factors. The main factors are the geometric parameters of the gear (module, pressure angle, number of teeth and addendum modification), the assumed method of machining (kinematics of machining) and the type and parameters of the applied tool (number of teeth of tool, radius of tooth tip of tool and sharpening of gear-type tool).
3. The same criterion for internal gears concerning the location of the critical section ($\zeta_F = 30°$) is assumed as in the case of external gears. Numerous investigations, both experimental and computational [**1**] show that in the case of internal gears the angle ζ_F of the tangent line, which determines the location of the critical section, is larger than 30° and may reach values within the range $\langle 35°, 80° \rangle$.

The presented formulae, defined for geometric calculations of straight-tooth internal gears [equations (3) to (10)], are also suitable for calculations of helical internal gears. According to the ISO 6336-3:1996 standard, in geometric calculations, instead of real helical gearing a virtual straight-tooth gearing with the virtual number of teeth, z_n, is introduced [**3**] and used for further geometric calculations.

3 DETERMINATION OF THE PRECISE VALUES OF THE PARAMETERS OF THE CRITICAL SECTION: THE MODIFIED ISO 6336-3:1996 METHOD

Spur gears can be machined with tools of rack form (Maag and Sunderland rack tools or hobbing cutters) or with gear-type tools (gear-shaper cutters). In the ISO 6336-3:1996 and DIN 3990-3:1987 standards [**3–5**, **12**] there are formulae for calculation of parameters of critical section only for the case of external gears machined with racks. In the case of external gears machined with gear-type tools, both standards suggest performing calculations for a virtual rack with the same standard basic rack tooth profile as a gear-type tool. In the case of internal gears, additional simplifications are introduced. They consist in replacing the real tooth form of the internal gear with a straight-line rack profile and replacement of the trochoidal tooth fillet with a circular arc.

The parameters of the critical section depend on the shape of the tooth fillet. In gears machined by generation a tooth fillet arises as an envelope of successive rolling locations of tool tooth tip [**4**, **5**, **13**, **14**]. Therefore, the shape of this curve depends both on the parameters of the machined gear and on the type as well as the parameters of the applied tool. The shape of the tooth fillet created by the gear-type tool may significantly differ from the shape of the same gear machined with a rack [**11**]. This may lead to substantial differences in the calculated parameters of the critical section and stresses. Therefore, in order to make a precise strength analysis of the tooth root in gears machined with a gear-type tool, it is necessary to calculate parameters of the critical section with consideration of the tooth fillet created by a real gear-type tool.

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The method of analytical determination of the tooth form and parameters of critical section based on computer simulation of generation of spur gears has been presented previously [**11**, **15**, **16**]. This method considers all types of spur gear (external and internal; straight tooth and helix) machined with racks and gear tools with various types of tool tip (point type, rounded tip and tip with protuberance). This method is used in this paper for precise calculation of parameters of critical section of internal gears. According to the assumptions made in the ISO 6336-3:1996 and DIN 3990-3:1987 standards, calculations are made for the critical section determined by the angle between the tooth centre-line and the tangent to the fillet at the critical point $\zeta_F = 30^\circ$. The parameters computed this way are used for calculation of the coefficients Y_F [equation (3)] and Y_S [equation (4)] and further for calculation of the nominal stress at the tooth root, σ_{F0}. This method of calculation, proposed in this paper and called the modified ISO 6336-3:1996 method, is compatible with the main assumptions of the ISO 6336-3:1996 and DIN 3990-3:1987 standards but considers precise and not simplified parameters of the critical section.

4 DETERMINATION OF STRESSES IN INTERNAL GEARS WITH THE USE OF THE FINITE ELEMENT METHOD

The most accurate methods of strength analysis of spur gears named in the ISO 6336-3:1996 and DIN 3990-3:1987 standards are as follows:

(a) experimental methods (photoelastic methods or methods based on electric resistance strain gauges), which give the most credible information on the magnitude and distribution of stresses at the tooth root;
(b) precise computational methods, especially the FEM.

One of the advantages of the FEM, compared with experimental methods, is relatively low cost and possibility of application of this method also at the initial stage of gear transmission design. Investigations made by Jahn [**1**] showed that the maximum stress at the tooth root of an internal gear determined in experiments using electrical resistance wire strain gauges is very close to the maximum stress at the tooth root determined in finite element calculations.

4.1 Geometric and finite element model of an internal gear

In order to compute stresses at the tooth root using the FEM, precise geometric models of internal gears are generated. They consider an accurate tooth shape, which results from machining of gear with assumed tool, including accurate data describing the tooth fillet. The shape of tooth including the fillet is calculated using the analytical and numerical method described in detail in references [**11**] and [**16**].

For precise representation of the tooth shape, cubic splines are used for modelling its profile. Local coordinate systems are used for calculations of splines in order to omit large values of the first derivatives of the functions describing both the active tooth profile and the fillet. These local coordinate systems are linked by suitable affine transforms with global coordinate system in which the whole finite element models of gears are generated. Considering the principal idea of spur gears, both straight tooth and helical, plain stress finite elements are used for analysis of the tooth-root strength. This approach assumes that, in each considered case, the load is equally distributed along the tooth line and deflections and their distributions of stresses in the transverse sections of the tooth and gear are the same. Axial symmetry of the gear allows the finite element model to be reduced to a representative segment of gear, as far as the distribution of stress in the loaded tooth and in its surrounding is concerned. Therefore, not the whole gears need to be modelled, but certain segments of them, containing for example three, four, five or more teeth. In this case, the load is applied to the central tooth of the gear segment and boundary conditions are applied to external teeth of the segment (Fig. 2).

A spur gear tooth loaded with in-plane forces (concentrated and/or distributed) orthogonal to the gear axis can be analysed as a plane stress model, representing a unit-length slice of gear in its transverse section. In this model, a two-dimensional stress state occurs. Considering the z axis normal to the transverse section of a gear, it can be concluded that all components of the stress vector equal zero, except normal components of the stress vector τ_{xx} and τ_{yy}, and the shear stress τ_{xy}, which are uniformly distributed across the tooth thickness.

The solution of finite element computations is found, assuming linear elastic continuum body with zero initial stress and isotropic model of material of gear, with consideration of the minimum of the fundamental functional $\Pi[\boldsymbol{u}(\boldsymbol{x})]$, representing the total potential energy of the model expressed in terms of field of displacements $\boldsymbol{u}(\boldsymbol{x})$ [**17**, **18**]:

$$\begin{aligned}\Pi[\boldsymbol{u}(\boldsymbol{x})] &= U + W \\ &= \frac{1}{2}\int_V \boldsymbol{\varepsilon}^{\mathrm{T}}(\boldsymbol{x})\boldsymbol{\tau}(\boldsymbol{x})\,\mathrm{d}V - \int_V \boldsymbol{u}^{\mathrm{T}}(\boldsymbol{x})\boldsymbol{f}(\boldsymbol{x})\,\mathrm{d}V \\ &= \frac{1}{2}\int_V [\mathbf{B}\,\boldsymbol{u}(\boldsymbol{x})]^{\mathrm{T}}\mathbf{C}[\mathbf{B}\,\boldsymbol{u}(\boldsymbol{x}) - \boldsymbol{\varepsilon}_0(\boldsymbol{x})]\,\mathrm{d}V \\ &\quad - \int_V \boldsymbol{u}^{\mathrm{T}}(\boldsymbol{x})\boldsymbol{f}(\boldsymbol{x})\,\mathrm{d}V \end{aligned} \tag{11}$$

where U denotes the strain energy of the analysed system, W is the potential energy of the external loads,

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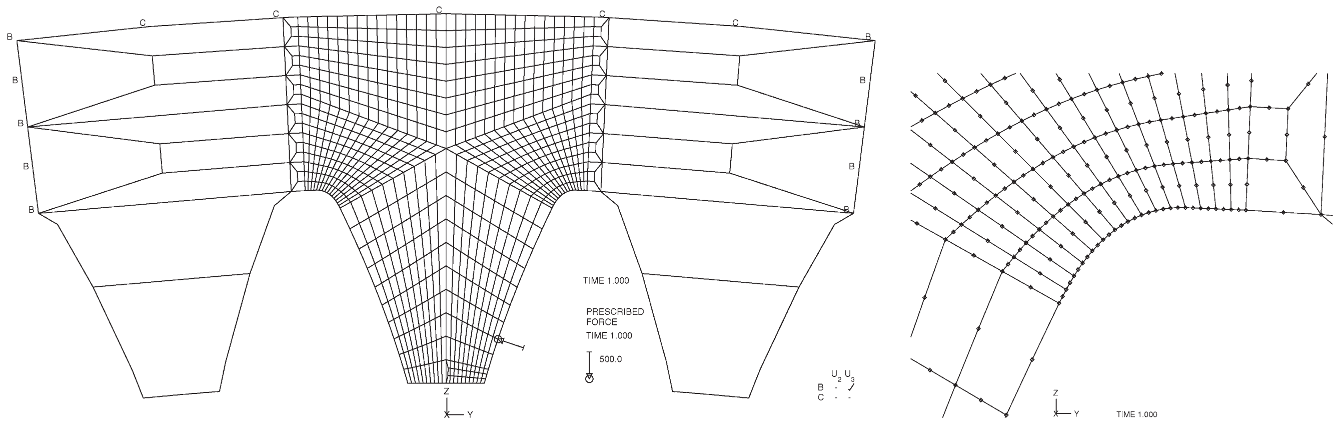

Fig. 2 Finite element model of the segment of an internal gear machined with a gear-type tool. The main tool parameters are $z_0 = 28$, $\rho_{a0} = 0.5$ mm and $s_0 = 4.32$ mm. The main gear parameters are $m_n = 2.75$ mm, $\alpha_n = 20°$, $\beta = 0°$, $s_1 = 4.265$ mm and $z_1 = 73$

$\boldsymbol{u} = [u(\boldsymbol{x}), v(\boldsymbol{x})]^T$ is the field of displacements depending on the vector of coordinates, $\boldsymbol{x} = [x, y]^T$ is of an arbitrary point belonging to the analysed two-dimensional body (region V), $[\,]^T$ denotes transposition of $[\,]$, $\boldsymbol{\varepsilon} = [\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{xy}]^T$ and $\boldsymbol{\tau} = [\tau_{xx}, \tau_{yy}, \tau_{xy}]^T$ are the strain and stress vectors respectively, $\boldsymbol{\varepsilon}_0$ is the initial strain vector, $\boldsymbol{f}(\boldsymbol{x})$ is the vector of loads, components of which correspond to the components of the displacement vector $\boldsymbol{u}(\boldsymbol{x})$, **B** is the matrix of differential operators and **C** is the strain–stress matrix, defined by

$$\mathbf{B} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}, \qquad \mathbf{C} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

In numerical computations the following average values for steel, the material of the analysed models of gears, are considered: Young's modulus $E = 2.1 \times 10^5$ MPa and Poisson's ratio $\nu = 0.3$.

Continuous loads distributed along boundaries of finite elements are replaced by discrete equivalent loads and are computed from the condition of conservation of the same potential energy W as the energy associated with the continuous loads.

The distributions of strains and stresses in the considered models of gears are computed from equations

$$\varepsilon = \mathbf{B}\,\boldsymbol{u}(\boldsymbol{x}), \qquad \tau = \mathbf{C}(\varepsilon - \varepsilon_0) \tag{12}$$

for the field of displacements $\boldsymbol{u}(\boldsymbol{x})$ satisfying the requirement of the minimal total potential energy $\Pi[\boldsymbol{u}(\boldsymbol{x})]$ given in equation (11).

Strength computations of gears considered in this work are performed with the use of the eight-node isoparametric finite elements. The obtained results show that the distribution of maximum principal stress at the tooth root of loaded tooth, σ_{P1}, is similar to the distribution of the effective von Mises stress σ_{eff} computed in the same area (Fig. 3). Therefore, in further analysis the effective von Mises stress σ_{eff} is considered for all models of gears for comparison reasons, as this model well represents the stress effort at the tooth root.

4.2 Load of gear transmission

In this work it is assumed, according to the ISO 6336-3:1996 and DIN 3990-3:1987 standards, that the contact force F_{bt} is normal to the profile of the internal gear and is inclined with an angle α_{Fen} to the X axis (Fig. 1). Calculations are made for two different loads:

(a) for a load $F_{bt} = 500$ N applied at the HPSTC;
(b) for a load $F_{bt} = 250$ N applied at the tooth tip.

The load at the tooth tip, which is half that at the HPSTC, follows from the load distribution between two pairs of teeth in contact.

4.3 Analysis of results of finite element computations

Fields of deflections and stresses are calculated with the use of the ADINA finite element system [**19**]. Some of the results are presented in Figs 3 and 4.

The obtained results show that in the area at tooth root, where undercutting of the active involute profile occurs, a significant increase in the maximum principal stress σ_P takes place. The related von Mises effective stress σ_{eff}, calculated also in the same area, follows the distribution of the maximum principal stress σ_P (Fig. 3). Moreover, with increasing depth, both the average magnitude of stress and the concentration of stress along fillet are significantly reduced.

Analysis of the results also shows that both the form and the strength properties of the tooth with the fillet determined for a virtual rack (a rack with the same standard basic rack tooth profile as the gear-type tool

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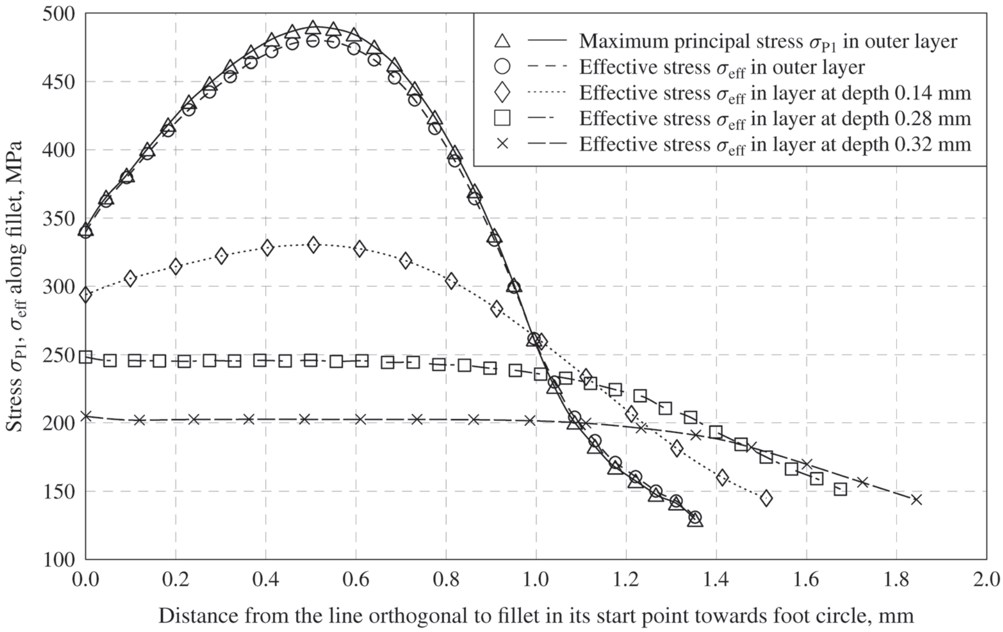

Fig. 3 Distribution of maximum principal stresses σ_P and von Mises effective stresses σ_{eff} along the fillet in the outer layer and distribution of the von Mises effective stresses σ_{eff} in layers at 0.14 mm, 0.28 mm and 0.32 mm depth, calculated for the model of a gear machined with a gear-type tool. The main tool parameters are $z_0 = 28$, $\rho_{a0} = 0.5$ mm and $s_0 = 4.32$ mm. The main gear parameters are $m_n = 2.75$ mm, $\alpha_n = 20°$, $\beta = 0°$, $s_1 = 4.265$ mm and $z_1 = 73$

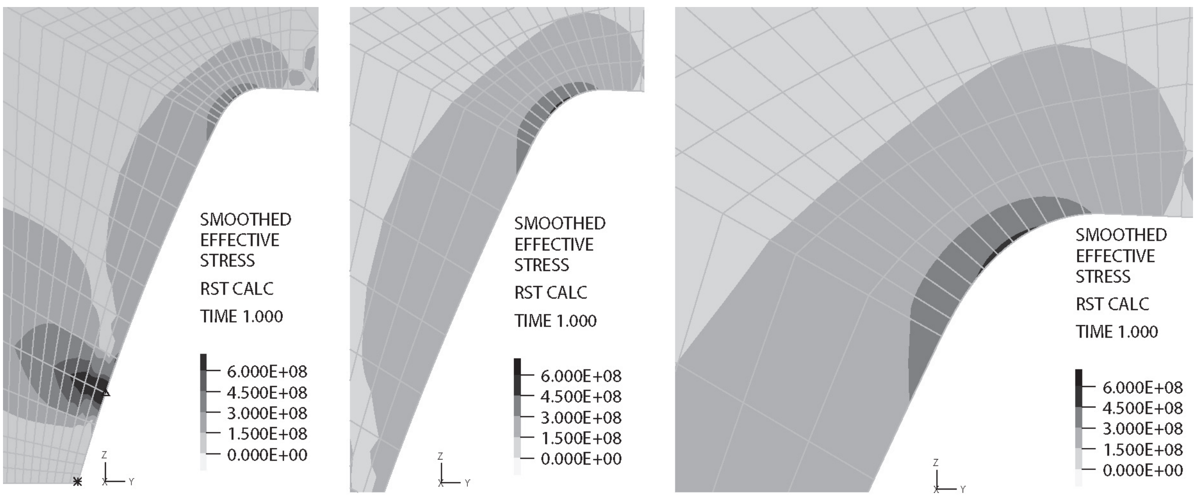

Fig. 4 Distribution of the von Mises effective stresses σ_{eff}, calculated for the model of a gear with 73 teeth machined with a gear-type tool with tip radius $\rho_{a0} = 0.5$ mm. The main tool parameters are $z_0 = 28$, $\rho_{a0} = 0.5$ mm and $s_0 = 4.32$ mm. The main gear parameters are $m_n = 2.75$ mm, $\alpha_n = 20°$, $\beta = 0°$, $s_1 = 4.265$ mm and $z_1 = 73$

used for machining) and for a real gear-type tool significantly differ from each other.

5 INFLUENCE OF THE GEAR AND TOOL PARAMETERS ON THE STRESSES AT THE TOOTH ROOT OF THE INTERNAL GEAR

FEM calculations of stress are performed on models of internal gears (section 4.1) determined for various combinations of gear and tool parameters. Then, the influences of the most significant gear and tool parameters on the tooth-root stress is analysed. Results of the FEM calculations are compared with results obtained from calculations based on simplified models used in the ISO 6336-3:1996 and DIN 3990-3:1987 standards as well as with results obtained from calculations based on the modified ISO 6336-3:1996 method. The influences of the module and pressure angle on the tooth strength are well known. Therefore, the influences of these two

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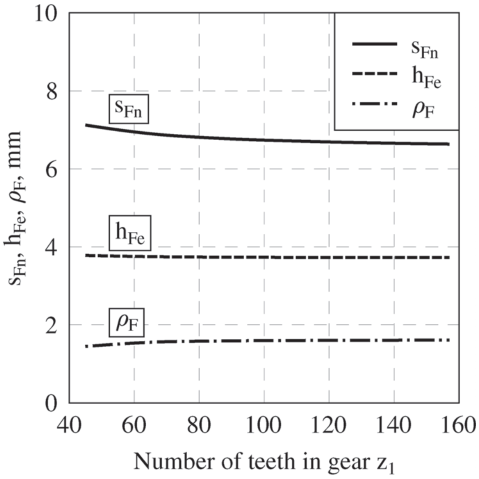

Fig. 5 Influence of the number of teeth in the internal gear, z_1, on the parameters of the critical section; the parameters are calculated according to the method given in reference [**11**]. The main tool parameters are $z_0 = 28$, $\rho_{a0} = 0.5\,\text{mm}$ and $s_0 = 4.32\,\text{mm}$. The main gear parameters are $m_n = 2.75\,\text{mm}$, $\alpha_n = 20°$, $\beta = 0°$, $s_1 = 4.265\,\text{mm}$ and z_1 variable

parameters on the tooth-root stress are not analysed in this paper.

5.1 Number of teeth in the internal gear, z_1

The number of teeth in the gear, z_1, influences both the tooth fillet profile and the involute tooth profile. On varying z_1 the radius of base cylinder from which involute curve is developed also varies. Computations made using the FEM and with the modified ISO 6336-3:1996 method show, however, that the influence of the number of teeth in the gear, z_1, on the parameters of the critical

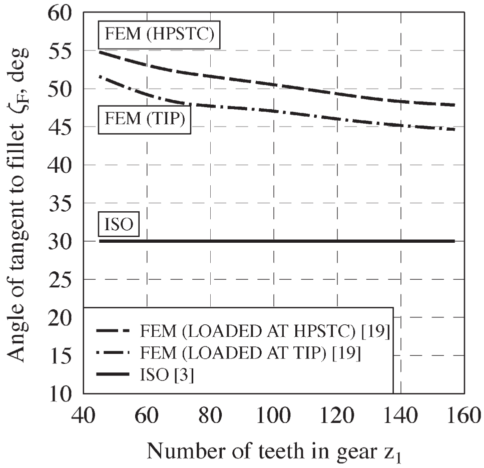

Fig. 7 Influence of the number of teeth in the internal gear, z_1, on the angle ζ_F between the tooth centre-line and the tangent to the fillet at the critical point F. The main tool parameters are $z_0 = 28$, $\rho_{a0} = 0.5\,\text{mm}$ and $s_0 = 4.32\,\text{mm}$. The main gear parameters are $m_n = 2.75\,\text{mm}$, $\alpha_n = 20°$, $\beta = 0°$, $s_1 = 4.265\,\text{mm}$ and z_1 variable

section (Fig. 5) and on the maximum stress at the tooth root is negligible (Fig. 6).

With an increasing number of teeth, z_1, the angle ζ_F between the tooth centre-line and the tangent to the fillet at the critical point F, which determines the location of the critical section (Fig. 7), decreases.

Stresses calculated according to the ISO 6336-3:1996 and DIN 3990-3:1987 standards and the ISO 6336 draft international standard are constant, because in these standards it is assumed that the parameters of the critical section do not depend on the number of teeth in the internal gear, z_1. Stresses calculated in this way

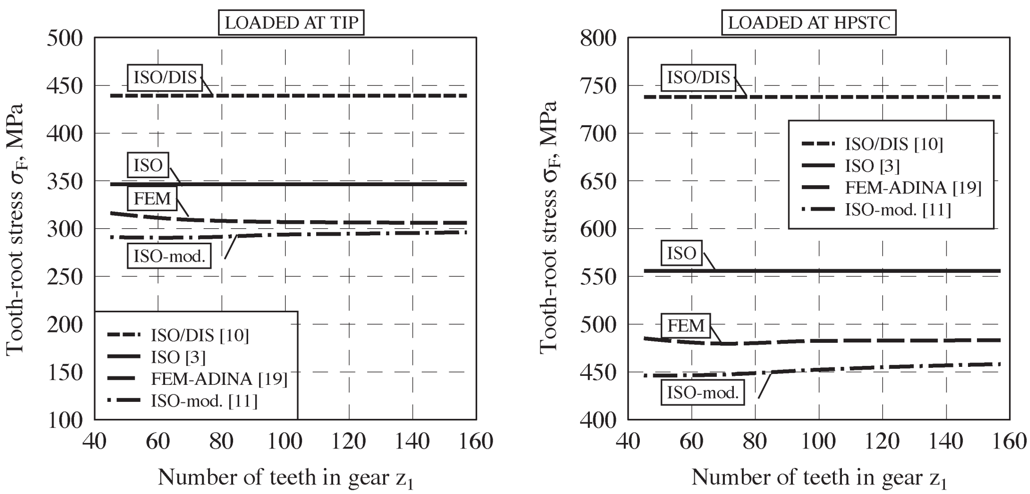

Fig. 6 Influence of the number of teeth in the internal gear, z_1, on the maximum stress at the tooth root for a load applied at the tooth tip (TIP) (left) and a load applied at the HPSTC (right). The main tool parameters are $z_0 = 28$, $\rho_{a0} = 0.5\,\text{mm}$ and $s_0 = 4.32\,\text{mm}$. The main gear parameters are $m_n = 2.75\,\text{mm}$,

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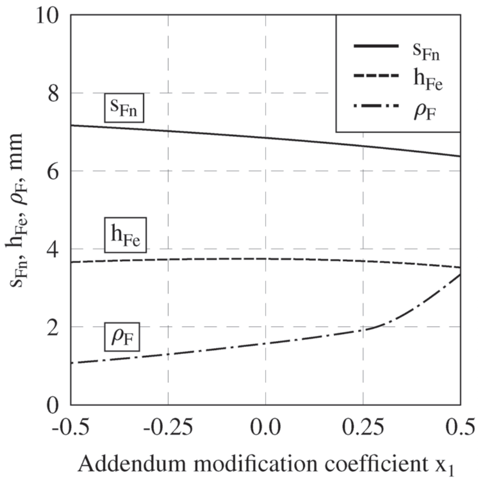

Fig. 8 Influence of the addendum modification coefficient x_1 on the parameters of the critical section; the parameters are calculated according to the method given in reference [**11**]. The main tool parameters are $z_0 = 28$, $\rho_{a0} = 0.5$ mm and $s_0 = 4.32$ mm. The main gear parameters are $m_n = 2.75$ mm, $\alpha_n = 20°$, $\beta = 0°$, $z_1 = 73$ and x_1 variable

are, for both cases of applied load, larger than the stresses obtained using the FEM or with the modified ISO 6336-3:1996 method (Fig. 6). However, the results obtained according to the ISO 6336-3:1996 standard [**3**] are much closer to the results of FEM calculations, compared with the results obtained according to the ISO 6336 draft international standard [**10**].

5.2 Addendum modification of internal gear, x_1

In gear transmissions, addendum modification is frequently used for both design and technological reasons. The magnitude of the tooth addendum is determined by the coefficient x_1. In gears machined by generation this addendum is obtained by increasing or decreasing the distance between the gear and the generating tool. A positive addendum (in both external and internal gears) is associated with a shift in the tooth location in the direction towards the tooth tip cylinder).

The change in the location of the tool associated with addendum modification results in a change in the tooth-root shape. Therefore, it influences the parameters of the critical section (Fig. 8), especially the radius of curvature of the fillet, ρ_F. This radius increases with increasing tooth addendum coefficient x_1, which results in a decrease in the maximum tooth-root stress when the modified ISO 6336-3:1996 method is used.

Finite element analysis of all the considered models of internal gears precisely representing real tooth profiles obtained after machining shows, however, that the influence of the tooth addendum coefficient x_1 on the tooth-root stress (Fig. 9, left) and on the angle between the tooth centre-line and the tangent to the fillet at the critical point, ζ_F (Fig. 9, right) is negligible.

5.3 Tool tip radius ρ_{a0}

The tooth fillet during gear generation is formed as an envelope of consecutive rolling locations of the tool tooth tip. The tool tooth tip radius ρ_{a0} is one of the most important geometric tool parameters influencing both the shape of the tooth fillet and the strength of the tooth. Neither the ISO 6336-3:1996 standard nor the DIN 3990-3:1987 standard consider this important influence. In these standards the parameters of the critical section are calculated for a virtual tooth profile

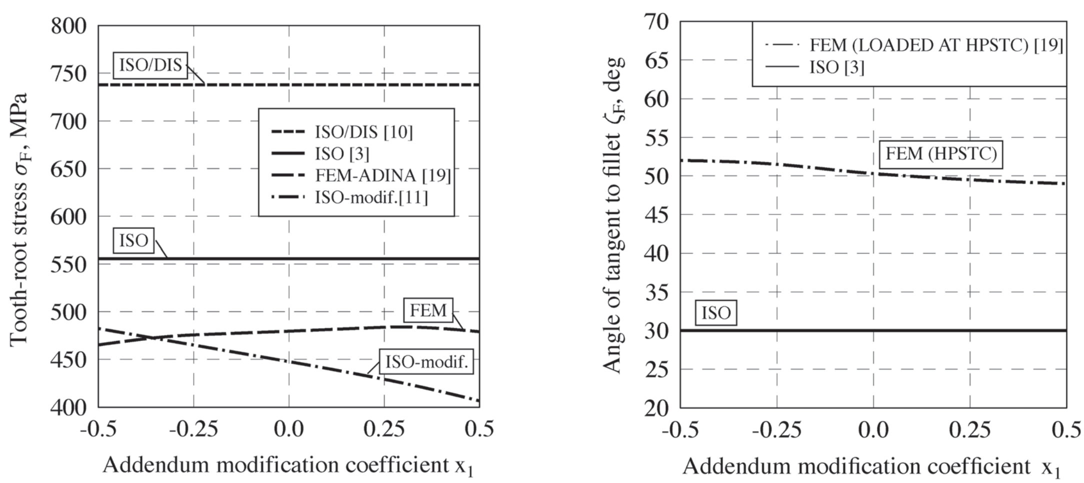

Fig. 9 Influence of the addendum modification coefficient on the maximum stress at the tooth root of the internal gear (left) and influence of the addendum modification coefficient on the angle ζ_F between the tooth centre-line and the tangent to the fillet at the critical point F (right). The main tool parameters are $z_0 = 28$, $\rho_{a0} = 0.5$ mm and $s_0 = 4.32$ mm. The main gear parameters are $m_n = 2.75$ mm, $\alpha_n = 20°$, $\beta = 0°$, $z_1 = 73$ and x_1 variable

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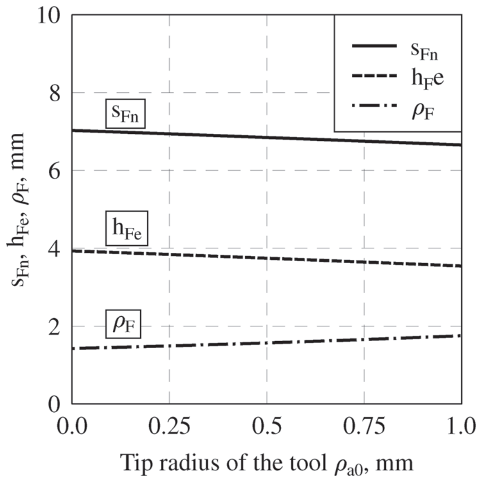

Fig. 10 Influence of the tool tip radius ρ_{a0} on the parameters of the critical section; the parameters are calculated according to the method given in reference [**11**]. The main tool parameters are $z_0 = 28$, $s_0 = 4.32$ mm and ρ_{a0} variable. The main gear parameters are $m_n = 2.75$ mm, $\alpha_n = 20°$, $\beta = 0°$ and $z_1 = 73$

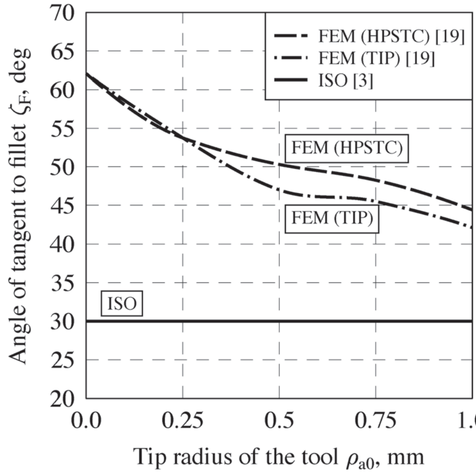

Fig. 12 Influence of the tool tip radius ρ_{a0} on the angle ζ_F between the tooth centre-line and the tangent to the fillet at the critical point F. The main tool parameters are $z_0 = 28$, $s_0 = 4.32$ mm and ρ_{a0} variable. The main gear parameters are $m_n = 2.75$ mm, $\alpha_n = 20°$, $\beta = 0°$ and $z_1 = 73$

without taking into account the real parameters of the generating tool.

The strength analysis of the tooth of an internal gear made according to the modified ISO 6336-3:1996 standard (in which a real tooth profile is considered, but the parameters of the critical section are determined from the angle between the tooth centre-line and the tangent to fillet at the critical point F, i.e. $\zeta_F = 30°$) shows that relations between the tool tip radius ρ_{a0} and the parameters of the critical section (Fig. 10) as well as between the tool tip radius ρ_{a0} and the tooth-root stress (Fig. 11) can be neglected.

Finite element computations of precisely prepared models of gears considering a real tooth profile obtained after machining show, however, that with increasing radius ρ_{a0} the maximum tooth-root stress significantly decreases. This conclusion follows analysis of the results of computations made for the whole range of variation in radius ρ_{a0}, bounded between the lower limit $\rho_{a0} = 0$ (sharp tool tip) and the upper limit $\rho_{a0} = \rho_{a0max}$ (a tool with a fully rounded tooth point). Figure 11 shows that stresses calculated using the FEM for $\rho_{a0} = \rho_{a0max}$ are more than 30 per cent less than the stresses calculated for $\rho_{a0} = 0$. At the same time also the angle between the tooth centre-line and the tangent to the fillet at the critical point decreases from $\zeta_F = 62°$ to $\zeta_F = 45°$ (Fig. 12).

In the developed modified ISO 6336-3:1996 method of calculation of the tooth-root stress, given in detail in

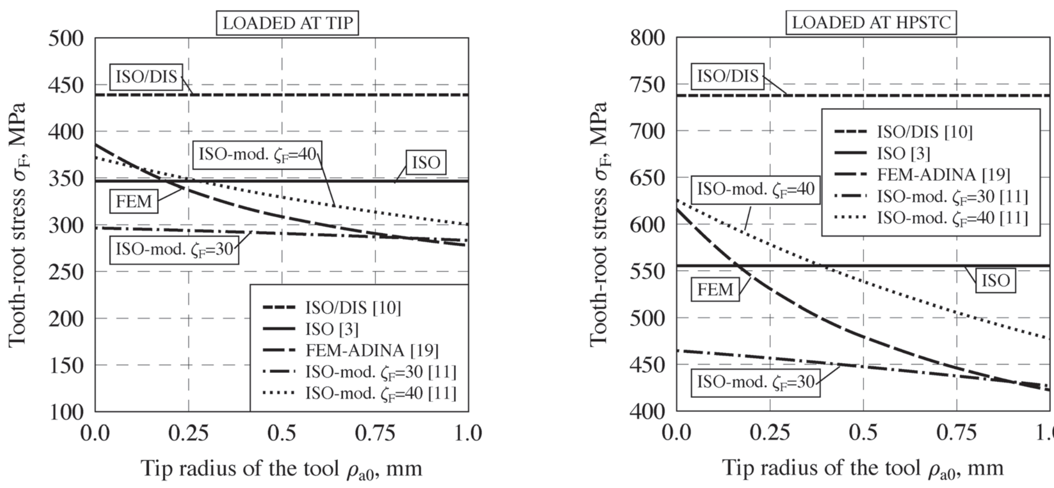

Fig. 11 Influence of the tool tip radius ρ_{a0} on the maximum stress at the tooth root for a load applied at the tooth tip (left) and a load applied at the HPSTC (right). The main tool parameters are $z_0 = 28$, $s_0 = 4.32$ mm and ρ_{a0} variable. The main gear parameters are $m_n = 2.75$ mm, $\alpha_n = 20°$, $\beta = 0°$ and $z_1 = 73$

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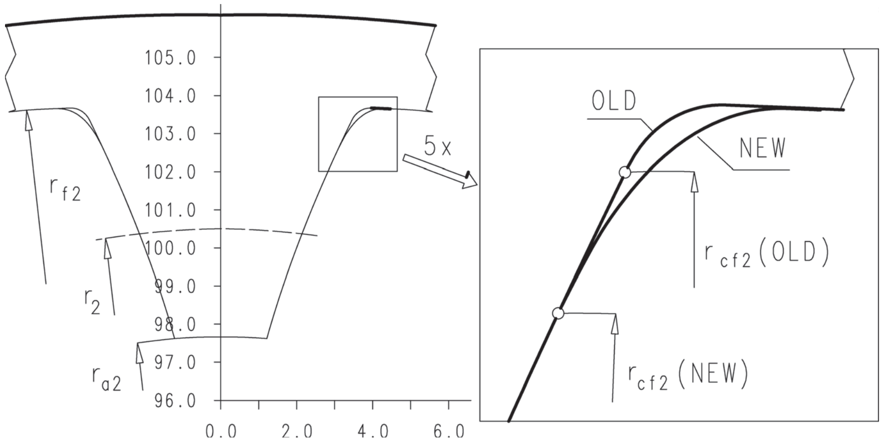

Fig. 13 Influence of the tool regrinding rate of a gear-type tool (parameter u) on the tooth-root form of internal gear

reference [**11**], the parameters of the critical section are computed for the real tooth profile, including the fillet, and not for the virtual tooth profile, which is used in the ISO 6336-3:1996 standard. However, according to the ISO 6336-3:1996 standard, the location of the critical section is determined by the angle between the tooth centre-line and the tangent to the fillet at the critical point $\zeta_F = 30^\circ$. That assumption causes differences in the results of stress calculations between the developed method and the precise finite element analysis of the same models of internal spur gears (Fig. 11).

The method proposed in reference [**11**] allows analyses to be performed for arbitrary values of the angle ζ_F. Following that possibility, numerous models of internal gears were analysed and their maximum tooth-root stresses were calculated considering different values of the angle ζ_F. In the analyses performed, the angle ζ_F varied in the range 30–60°. The best results, compared with the results of the finite element computations of the relevant models, were obtained for the angle $\zeta_F = 40^\circ$ (Fig. 11).

5.4 Tool regrinding rate

Internal gears are machined, in almost all cases, with gear-type tools (gear-shaper cutters and rotary shaving cutters). One characteristic property of such gear-type tools that is different from racks is that sharpening of gear tool causes change in the geometric parameters of such tools.

Sharpening of a gear-shaper cutter causes changes in the following geometric parameters: external diameter d_{a0}, tooth thickness at the pitch circle, s_0, tooth thickness at the tooth tip, s_{a0}, and the diameter of the pitch cylinder of the tool. As a result, also the profile of the tooth fillet (Fig. 13) as well as some parameters of the machined gear, namely r_{cf1}, the radius of the tooth active profile at the tooth root, and r_{f1}, the root radius of the gear, and the parameters of the critical section become different.

Relations between these parameters concerning external gears have been described in detail in references [**11**], [**16**] and [**20**]. In this work, internal gears are investigated. The properties of the considered internal gears are computed and analysed using the method described in section 3 and using the FEM. An exemplary graph which shows the influence of the parameter u associated with sharpening of gear-shaper cutter on parameters of internal gears machined with such tool is given in Fig. 14.

In the described case it is assumed that the tool regrinding rate is defined by the following conditions: $u = u' = 10$ and $u = u'' = -10$. Section I–I (Fig. 14) corresponds to a new cutter ($u = 10$). Section II–II corresponds to the same cutter but in the almost worn-out state, i.e. to the cutter which has reached its regrinding limit ($u = -10$). Such a tool can still be used but cannot be used any longer after the next sharpening.

From Fig. 14 it follows that successive cycles consisting of gear generation of several gears and the following of sharpening of the gear-shaper cutter lead to enlarging

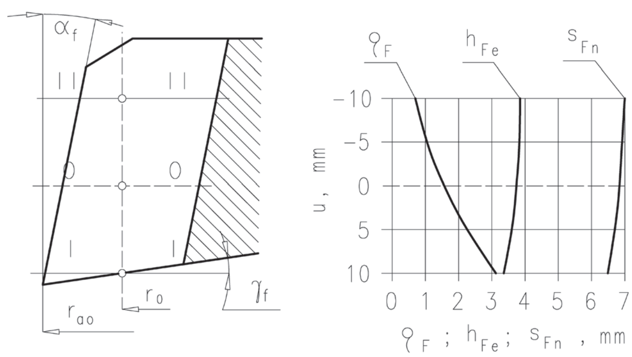

Fig. 14 Variation in the parameters of the critical section of an internal gear with the tool regrinding rate of a gear-type tool: I–I, section of the new tool (before the first sharpening); O–O, initial section ($u = 0$); II–II, section of the tool after the last sharpening. The parameters are calculated according to the method given in reference [**11**]

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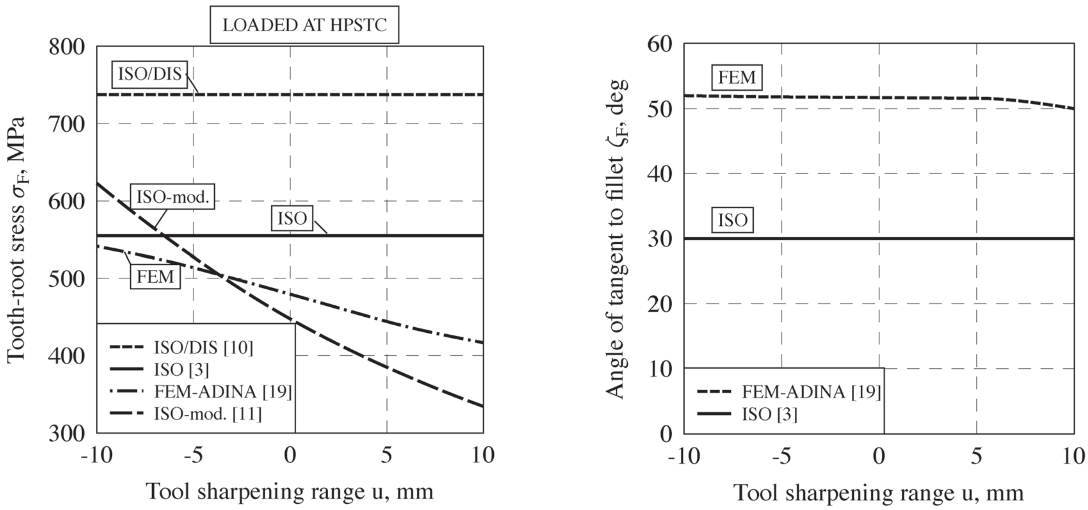

Fig. 15 Influence of the tool regrinding rate of the gear-type tool (parameter u) on the maximum stress at the tooth root of the internal gear (left) and on the angle ζ_F between the tooth centre-line and the tangent to the fillet at the critical point F (right). The main tool parameters are $z_0 = 28$, $\rho_{a0} = 0.5$ mm and u variable. The main gear parameters are $m_n = 2.75$ mm, $\alpha_n = 20^\circ$, $\beta = 0^\circ$, $z_1 = 73$ and $s_1 = 4.265$ mm

tooth thickness at the critical section of the internal gear machined with this type of tool. This should result in increasing gear tooth strength. However, simultaneously, the bending moment of the arm, h_{Fe}, increases and the radius of curvature of the fillet, ρ_F, decreases. Therefore, a significant increase in the maximum stress at the tooth root, resulting from these changes, is observed (Fig. 15, left).

These conclusions follow from analysis of the results of finite element computations and from the modified ISO 6336-3:1996 method. For example, the maximum stress computed using the FEM for the internal gear machined with a gear-shaper cutter that has already reached its regrinding limit, shown in Fig. 15, right, is approximately 23 per cent larger than the analogous maximum stress obtained in the case of a completely new gear-shaper cutter. From Fig. 15, right, it also follows that sharpening of the tool has a negligible influence on the location of the critical section.

5.5 Number of teeth of the gear-type tool, z_0

The shape of the tooth in the internal gear machined with a gear-type tool (gear-shaper cutter) depends also on the number of teeth, z_0, of such a tool. Its influence on the parameters of the critical section is shown in Fig. 16, left.

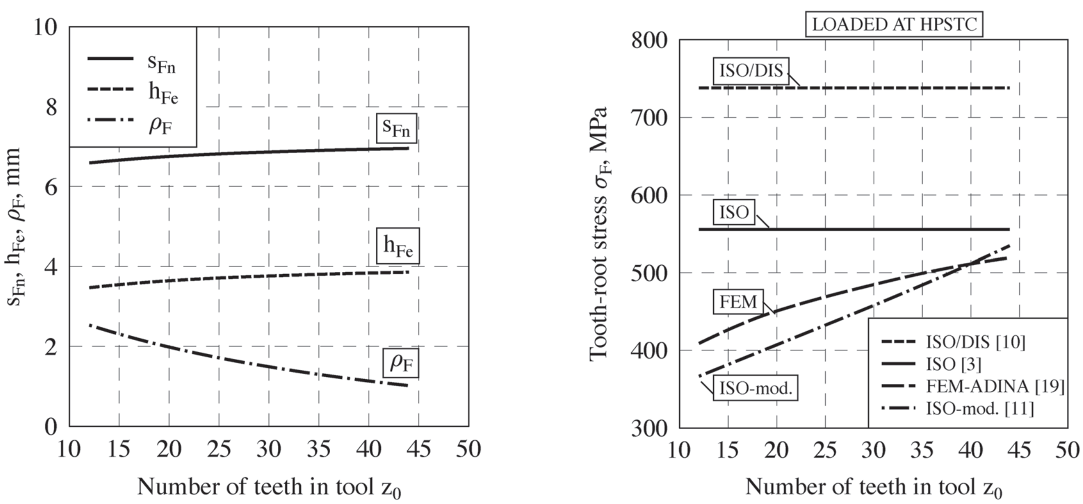

Fig. 16 Influence of the number of teeth of the gear-type tool, z_0, on the parameters of the critical section (parameters are calculated according to the method given in reference [**11**]) (left) and on the maximum stress at the tooth root of the internal gear (right). The main tool parameters are $s_0 = 4.32$ mm, $\rho_{a0} = 0.5$ mm and z_0 variable. The main gear parameters are $m_n = 2.75$ mm, $\alpha_n = 20^\circ$, $\beta = 0^\circ$, $z_1 = 73$ and $s_1 = 4.265$ mm

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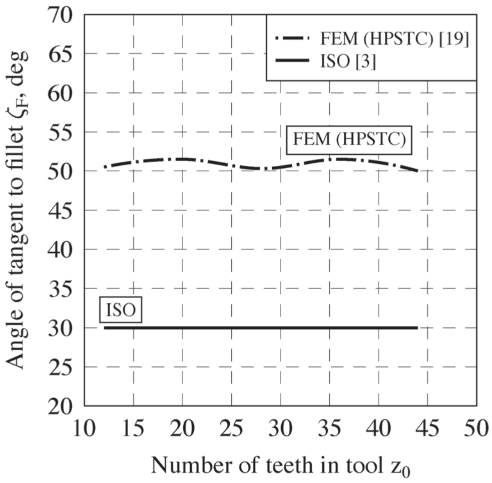

Fig. 17 Influence of the number of teeth of the gear-type tool, z_0, on the angle ζ_F between the tooth centre-line and the tangent to the fillet at the critical point F. The main tool parameters are $s_0 = 4.32$ mm, $\rho_{a0} = 0.5$ mm and z_0 variable. The main gear parameters are $m_n = 2.75$ mm, $\alpha_n = 20°$, $\beta = 0°$, $z_1 = 73$ and $s_1 = 4.265$ mm

With increasing number of teeth of the gear-type tool, z_0, the tooth thickness at the critical section, s_{Fn}, and the bending moment of the arm h_{Fe} also increase a little, but the radius of curvature of the tooth fillet ρ_F significantly decreases. The reduction in ρ_F causes an increase in the maximum stress at the tooth root of the considered internal gear calculated using the modified ISO 6336-3:1996 method. Finite element computations also lead to the same conclusion (Fig. 16, right). A change in the number of teeth of the gear type tool, z_0, has no significant effect on the location of the critical section (Fig. 17).

6 LOCATION OF THE CRITICAL SECTION

The critical section is the most dangerous section at the tooth root as far as tooth strength for bending is concerned. It is the section in which stresses caused by bending reach a maximum. Precise determination of the location of the critical section requires computation of the distribution of stresses at the tooth root. It can be obtained either in experimental investigations or as a result of the finite element computations of precise model of gear. The ISO 6336-3:1996 [**3**] and DIN 3990-3:1987 [**6**] standards make use, generally, of a simplified method of solving this task. In both standards the critical section is determined by the point at which a straight line forming the angle $\zeta_F = 30°$ to the tooth centre-line is a tangent to the fillet (Fig. 1). This principle works quite well in the case of external gears but not in the case of internal gears. In the case of internal gears the angle ζ_F has larger values and varies over wide limits, depending on the geometric parameters of machined gear and applied tool [**1**].

Results of the computations described in section 5 lead to the same conclusions. On the basis of the distribution of stresses calculated in finite element calculations the angle ζ_F is also determined for each load case and for each gear model. It considers the location of the maximum tooth-root stress. The obtained results show that mainly two parameters have significant effects on the angle ζ_F. They are the tip radius of the tool, ρ_{a0} (Fig. 12), and the number of teeth of the internal gear, z_1 (Fig. 7). On increasing either the radius ρ_{a0} or the number of teeth, z_1, the angle ζ_F significantly decreases. The other gear and tool parameters considered, i.e. the tooth addendum coefficient (Fig. 9, right), sharpening of tool (Fig. 15, right) and the number of teeth of the tool, z_0 (Fig. 17), influence the angle ζ_F to a much smaller extent. The location of the critical section is also influenced by the location of the applied load. In the case when the load is applied at the HPSTC the angle ζ_F reaches larger values than in the case when the load is applied at the tooth tip (Figs 7 and 12). In all analysed cases of internal gears the angle ζ_F is significantly larger than the value $\zeta_F = 30°$ assumed in the ISO 6336-3:1996 and DIN 3990-3:1987 standards. It varies between $\zeta_F = 45°$ and $\zeta_F = 62°$, depending on the parameters of the machined gear and applied tool.

7 CONCLUSIONS

According to the ISO 6336-3:1996 standard [**3**], the FEM is the most accurate of the numerical methods applied in the strength analysis of gears. Calculations of tooth-root stresses of the considered gears were made using the developed modified ISO 6336-3:1996 method and using the FEM. Analysis of the obtained results leads to the following conclusions:

1. The magnitude of stresses at the tooth root depends on several geometric parameters, but the most significant influences of all the parameters considered in this work are due to the following parameters of applied tool: the tip radius of the tool, ρ_{a0}, sharpening of the tool (associated with the parameter u) and the number of teeth of the tool, z_0. The influences of the number of teeth of the machined gear, z_1, and its tooth addendum coefficient x_1 on the maximum tooth-root stress are negligible. In the analysis performed, neither the module m_n of the machined gear nor its pressure angle α_n are considered, because their influences on the tooth-root strength is generally well known.
2. The ISO 6336-3:1996 method described in reference [**3**] does not consider the influences of the above-mentioned parameters on tooth-root stress, because of simplifications assumed in this method. Stresses calculated using this method do not depend on these parameters and are significantly larger than stresses computed using the FEM.

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3. Tooth-root stresses calculated using the modified ISO 6336-3:1996 method (section 3) for the parameters of the critical section determined for a real and not a virtual tooth profile are much closer to the results of accurate finite element computations than stresses calculated using the conventional ISO 6336-3:1996 method [**3**]. The differences between the results of the modified ISO 6336-3:1996 method and the FEM are larger only in the case of a very small tip radius of the tool, ρ_{a0}.
4. The angle ζ_F which determines the location of the critical section is, in the case of internal gears, much larger than that assumed in the ISO 6336-3:1996 standard [**3**] ($\zeta_F = 30^\circ$). In all models of internal gears considered in this work this angle varies in a wide range $\zeta_F \in \langle 45^\circ, 62^\circ \rangle$. This angle depends mainly on the tip radius of the tool, ρ_{a0}, and on the number of teeth of the machined gear, z_1. The influences of the three other parameters, namely the tooth addendum coefficient x_1 of the machined gear, the number of teeth of the tool, z_0, and the sharpening of tool (associated with parameter u) on the angle ζ_F are negligible.
5. According to the ISO 6336-3:1996 standard, the location of the critical section is determined by the angle between the tooth centre-line and the tangent to the fillet at the critical point $\zeta_F = 30^\circ$. Results of computations made for the numerous models of internal gears show that the maximum tooth-root stresses obtained for the angle $\zeta_F = 30^\circ$ significantly differ from results of the finite element computations of the relevant models. The developed method of calculation of the tooth-root stress of internal gears allows analyses to be performed for arbitrary values of the angle ζ_F. The performed investigations show that the minimum differences, compared with the finite element computations, are obtained for the angle $\zeta_F = 40^\circ$. As far as the risk of gear damage caused by concentration of tooth-root stresses is concerned, the maximum tooth-root stresses calculated this way for the angle $\zeta_F = 40^\circ$ are on the safe side, because they are larger than those obtained with the finite element computations (Fig. 11).

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