My Homework. This is a pre-requisite study for Laplace Transforms in circuit analysis. Source of study material: Electric Circuits 6th Ed., Nahvi & Edminister. Engineering Circuit Analysis, Hyatt & Kimmerly 4th Ed. McGrawHill.

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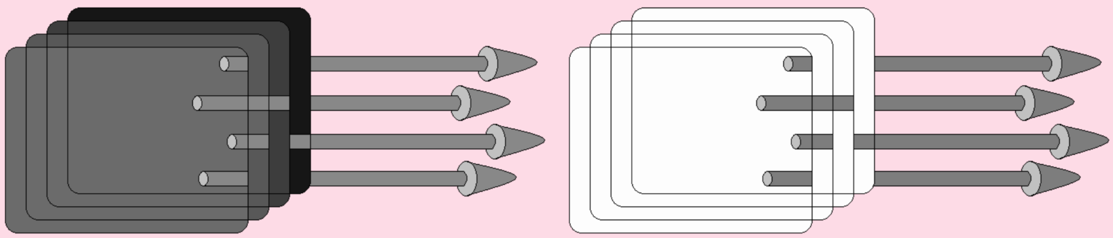
Part 1.
3 Short Chapters.
Basic To Intermediate.

Circuiting PrerequisitesTo Laplace Transform Electric Circuits.



April 2020.

Engineering college year 2 course of 4 year program OR year 1 of 3 year program. Re-fresher OR Self Study. Graduate Study Review.
May be used in New Zealand, US, Malaysia, India, Pakistan, UK, and other Common Wealth Country engineering colleges.
Any errors and omissions apologies in advance.

Chapter 1: The 's' In Electric Circuit Analysis and Laplace.

I personally do not know anyone in the work place that uses Laplace Methods in engineering problem solving. That does NOT mean its not an 'in-demand subject'. It is in great demand.

From my understanding of those who apply Laplace methods in engineering are engineers with a 'strong technical knowledge' in their subject matter. Heavy duty serious engineers. *My experience is more into electrical engineering for various building construction types; commercial to industrial.*

Fourier Analysis is not an easy subject for signals, comparatively Laplace is much harder. Fourier Analysis application you can directly get thru by pulling out your engineering textbook, whereas Laplace requires several different areas of advanced mathematics to solve related problems. For example it starts with differential equations with their boundary conditions, this itself is a tough subject for me. Various forms and solutions of differential equations exist. You have to be familair with them to select the suitable one for the solution. Then several other methods need to be applied for example partial fractions being one of them! There is lots here leading to Bode plots. Obviously not a one stop get all done.

Serious engineering work require's use of Lapalce for serious engineers.

Primarily used in Control Systems in Electrical Engineering if you are asking which courses uses it most. Also used in Signals and Systems. Other disciplines such as Chemical Engineering, Mechanical Engineering use it too but not at the Electrica level.

Its been around for decades gives it further credibility.

HERE the aim is not on theory. Objective is to work through electrical circuits examples, low on theory, progressively build up circuit problem solving skill. Get to where I, and maybe you, can get to? That is Laplace applications in electrical circuits. One side of this task is getting thru a mathematical process leading to a solution, the other to understand and interpret the solution. Both I find HARD. *SO LETS GET THE CIRCUITS A LITTLE STRAIGHT SO WE UNDERSTAND THE OUTCOME.* For easy problems maybe there are easy solutions that too can have a tricky turn, so maybe not easy. Higher level of difficulty comes with complexity of the electrical circuits.

I got into this, recently or now, knowing I do NOT plan to beat this subject, NOR reach to a level of heavy duty electrical engineering. Rather just to gain the tools for applying Laplace and intepretating the results. In short build a skill set.

Again, not here on how to study Laplace for electrical circuits, but to get the skills required and ready to get into Laplace for electrical circuits.

Fortunatley this subject Laplace is used with Laplace Tables. Similar to differential or integral tables. That makes things a little simpler but not necessarily always for all.

Complex Frequency In Engineering Electrical Circuits.

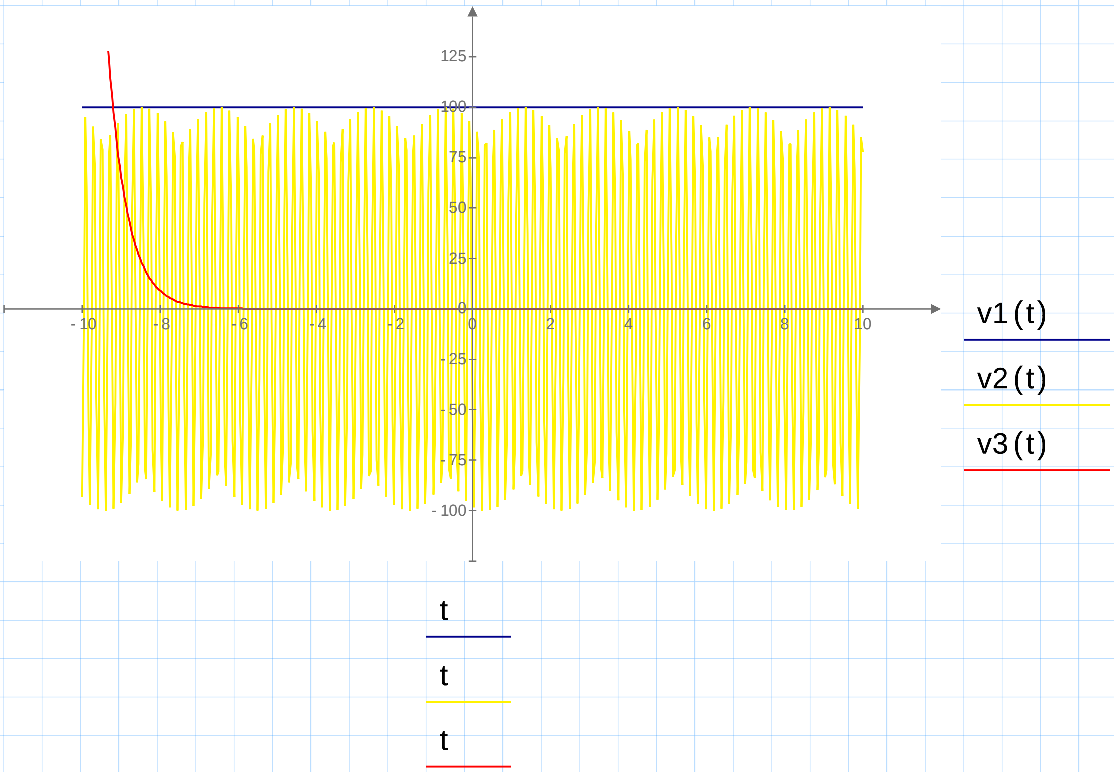
In real world electrical circuits there is a device or machine generating a voltage. In a power system we have the generator producing a sinusoidal voltage. This we are familiar with, the simplest case. However, the form of voltage can take various shapes, be it at generation or at a point in the circuit. You seen.... them from triangle, square, pulse,...........luckily the sinusoidal was not that impossible but it too has its characteristics. At the simpler level we just refer to the power supply provided by the power company 60 or 50 Hz ac power.

We want to create an expression where the following forms of waveshapes can be found in one wave function:
1. Constant voltage: 100V dc, 20kV dc,.....
2. Sinusoidal voltage: 100 cos(100t + 30 deg)V ac where 30 degs the phase angle
3. Exponential voltage: 1 x (10^-6) x (e^-2t) V

$v1(t) := 100 \quad V$

$v2(t) := 100 \cdot \cos\left(2 \cdot t + \frac{2 \cdot \pi}{3}\right) V \;=\; v2(t) := 100 \cdot \cos(500 \cdot t + 120 \cdot \mathbf{deg}) V$

$v3(t) := 1 \cdot 10^{-6} \cdot e^{-2 \cdot t} \quad V$



These functions chosen first, v1 v2 and v3, so we get plots we see on this page.
Next how do we create wave functions like these waves and what do they mean?

So how we do this is something Euler's function.......?
Euler to the rescue.

$A \cdot e^{j \cdot (\omega \cdot t + \phi)} = A \cdot (\cos(\omega t + \phi) + j \cdot \sin(\omega t + \phi))$ A is a contant or magnitude.

We know the cosine term is real and the sine term is imaginary.
Does it make a difference which one has priority?
Maybe, usually we go for the real part first.
This has been my experience from lectures, textbooks,....power systems,..... Why fool with the imaginary part? *If I remember correctly the imaginary part is a convenience to solve the math.*

$A \cdot e^{j \cdot (\omega \cdot t + \phi)} = Re \cdot (A \cdot \cos(\omega t + \phi)) + Im \cdot (j \cdot \sin(\omega t + \phi))$

$\cos(\omega t + \phi) = Re \cdot e^{j \cdot (w \cdot t + \phi)}$ we can drop the 'Re' because we know the cosine term is real.

$\cos(\omega t + \phi) = e^{j \cdot (w \cdot t + \phi)}$

$A \cdot e^{j \cdot (\omega \cdot t + \phi)} = A \cdot \cos(\omega t + \phi)$

Next the 'horse before the cart OR the cart before the horse'?
This is about s = sigma + (omega)t.
Cart before the horse because in the later topics here maybe, hope to, we see why we need it in this format. I maybe wrong in the choice of phrase.

$s = \sigma + j \cdot \omega$ j-omega unit is 1/sec, sigma has to be same, since as they are added.
$\sigma = s - j \cdot \omega$ sigma is known as Neper frequency Np/s.
omega since the beginning of time was radian/sec.

We merely want to pull in the expression sigma-t into the cosine term.
Just multiply it in, we'll see the 'beauty and elegance' of the math later.

$A \cdot e^{\sigma \cdot t} \cdot e^{j \cdot (\omega \cdot t + \phi)} = A \cdot e^{\sigma \cdot t} \cdot \cos(\omega t + \phi)$ Pull out your math book? Agreed.

Lets concentrate on the LHS.

$A \cdot e^{\sigma \cdot t} \cdot e^{j \cdot (\omega \cdot t + \phi)} = A \cdot e^{\sigma \cdot t \cdot j \cdot \omega \cdot t + \sigma \cdot t \cdot j \cdot \phi}$

$= A \cdot e^{j \cdot \phi} \cdot e^{(\sigma + j \cdot \omega) \cdot t}$ rearranging

Substituting $s = \sigma + j \cdot \omega$

$= A \cdot e^{j \cdot \phi} \cdot e^{s \cdot t}$ <--- the form needed.

Variables/Constants: $j := \sqrt{-1}$ *Math book has it as i, in EE its j not to mix-up for current i.*

Table below of Functions their Complex Frequency - s, and Amplitude/Constant A:

$s = \sigma + j \cdot \omega$

f (t)	s	A	
100	$0 + j \cdot 0$	100	s = 0 and w = 0, f(t) is a constant
$100 \cdot \cos\left(2 \cdot t + \frac{2 \cdot \pi}{3}\right)$	$0 + j \cdot 2$	100	s = 0 and w = 2 (in *w* is in *w*t), A max: 100, f(t) is a constant.
$100 \cdot \cos(2 \cdot t + 120\ deg)$	$0 + j \cdot 2$	100	s = 0 and w = 2, A_max = 100
$1 \cdot 10^{-6} \cdot e^{-2 \cdot t}$	$-2 + j \cdot 0$	$1 \cdot 10^{-6}$	s = -2 and w = 0, (here *s* in *s*t) exponential decay
$2 \cdot e^{-5\ t} \cdot \cos(2\ t - 120\ deg)$	$-5 + j \cdot 2$	2	s = -5 and w = 2 , f(t) damped cosine
$2 \cdot e^{-3\ t} \cdot \cos(30\ t + 30\ deg)$	$-3 + j \cdot 30$	2	s = -3 and w = 30, f(t) damped cosine

Go through the table above, got the idea. Match them to them. Tricky on the s in exp term is st, while w in cosine term is wt, and both? Multiplied to t.

Requirement / Caution / Note:

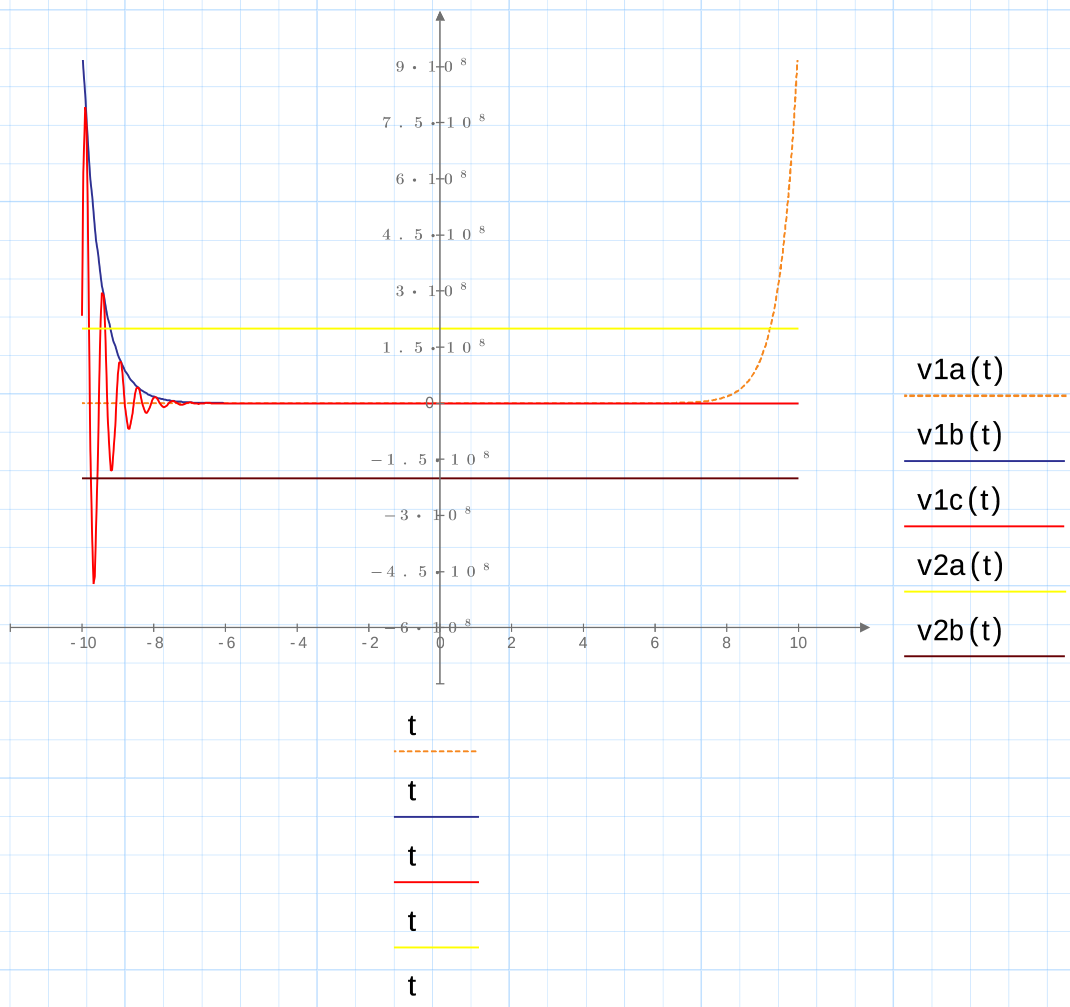
1). Only NEGATIVE values of s are considered, may be zero but NOT positive.
2). s and w - both non-zero, function is a damped cosine (pos or neg but not zero).
3). s and w - both zero, function is a constant.
4). s and w - where s = non-zero, and w = 0 function is an exponential decay function.

Some memorisation may help but better you have your textbook or notes for the above 4 cases or situations.

Continued on next page.

Plots of functions with sigma and omega values (s = sigma + j_omega):

$v1a(t) := 2 \cdot e^{2 \cdot t}$
$v1b(t) := 2 \cdot e^{-2 \cdot t}$
$v1c(t) := 2 \cdot \left(e^{-2 \cdot t}\right) \cdot \cos(300 \cdot t - 90)$
$v2a(t) := 2 \cdot 10^{8} \cdot e^{0 \cdot t}$
$v2b(t) := -2 \cdot 10^{8} \cdot e^{-0 \cdot t}$



Plot Notes:
When sigma = 0: Results in maximum amplitude, V2a(t)=+2x10^8 and V2b(t) -ve.
No damping. e^0t = 1, maximum constant amplitude. CORRECT.
When omega = 0: Results in exponential decay with initial values Vm.
V1a(t) positive sign on exponent WRONG.
V1b(t) negative sign on exponent CORRECT.
When sigma = 0 and omega = 0 : Results in a damped cosine - V1c(t) CORRECT.

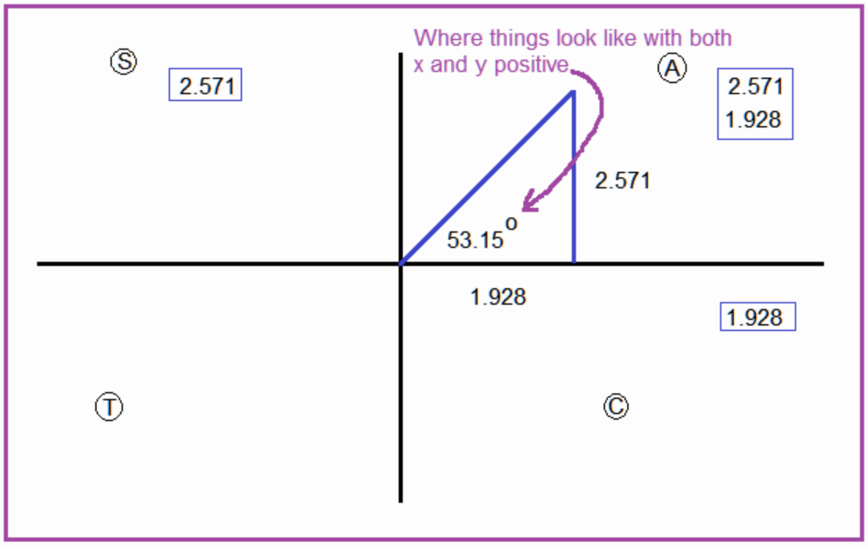
Practice Example 1 (Schaum's Outline: Electric Circuits. Nahvi & Edminister):
Given time function i(t) or v(t).
Provide corresponding amplitude and phase angle?
Mathcad_Prime command for polar angle: [shift][ctrl][p] after the magnitude and enter angle.
$t := 1$ $' := 1$ $\omega := 1$ $\tau := 1$ <--*Declaration for text edititng purpose.*

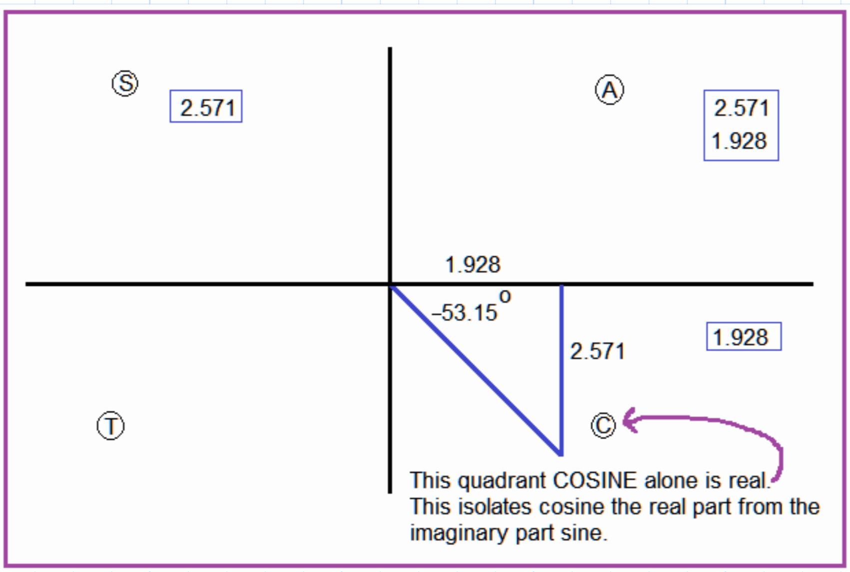
Time_Function	Amplitude_PhaseAngle $(A\angle deg)$	s_ComplexFreq $\left(\frac{1}{s}\right)$	Units
$86.6\ (A)$	$86.6\angle 0°$	0	N/A
$15.0 \cdot e^{-2\cdot 10^3 \cdot t}$ A	$15.0\angle 0°$	$2\cdot 10^3$	Np/s
$25.0\cdot\cos(250\cdot t - 45°)$ V	$25.0\angle -45°$	$j\cdot 250$	rad/s
'	'	$-j\cdot 250$	and rad/s
$0.5\cdot\sin(250\cdot t + 30°)$ V	$0.5\angle -60°$	$j\cdot 250$	rad/s
'	'	$-j\cdot 250$	rad/s
$5.0\cdot e^{-100\cdot t}\cdot\sin(50\ t + 90°)$ A	$5.0\angle 0°$	$-100 + j\cdot 50$	1/s
'	'	$-100 - j\cdot 50$	1/s
$3\cdot\cos(50\ t) + 4\cdot\sin(50\ t)$ A	$5\angle -53.13°$	$j\cdot 50$	rad/s
'	'	$-j\cdot 50$	rad/s

Explanation Attempted:
Study the units - Right most column. Complex frequency s has -/+ signs.
$0.5\cdot\sin(250\cdot t + 30°)$ V
Trig function is sine this is Imaginary so we take the cosine.
$\sin(30\ \mathbf{deg}) = 0.5$ $\cos(-60\ \mathbf{deg}) = 0.5$

$5.0\cdot e^{-100\cdot t}\cdot\sin(50\ t + 90°)$ A
There are 2 t's, both multiplied to each other, resulting in an add.
-100t + 50t OR -100t - 50t
For the phase angle, same trig thing from sine to cosine; Im to Re.
$\sin(90\ \mathbf{deg}) = 1$ $\cos(0\ \mathbf{deg}) = 1$ <---we see 0 degs.

$3\cdot\cos(50\ t) + 4\cdot\sin(50\ t)$ A
Amplitude = 5 from Pythogeras Triangle sides 3 4 and 5.
Calculate angle; cos 50=+ve value 1st & 4th quadrant, sin 50 +ve 1st 2nd quadrant.
So its either +/- angle. Schaum's Outline(Series) has it as - 53.13 deg. So make y -ve.
$x := 3\cdot\cos(50°) = 1.928$ $y := 4\cdot\sin(40\ \mathbf{deg}) = 2.571$ $\mathrm{atan}\left(\frac{y}{x}\right) = 53.13\ \mathbf{deg}$
Continued next page with figures.





You may have a better explanation on the reason for the -ve angle.

Check this with your local engineer.

Practice Example (Schaum's Outline: Electric Circuits. Nahvi & Edminister:
Given amplitude and phase angle in column 1, and s in column 2.
Determine the **time function**.
Mathcad_Prime command for polar angle: [shift][ctrl][p] after the magnitude and enter angle.

$A\ \phi$ (A∠deg)	s	Time_Function	Comments
$10\angle 0°$	$j \cdot 120\ \pi$	$10 \cdot \cos(120\ \pi \cdot t)$	'None
$2\angle 45°$	$-j \cdot 120 \cdot \pi$	$2 \cdot \cos(120\ \pi \cdot t + 45°)$	'Why_not_–ve_120_pi_t?'
$5\angle -90°$	$-2 + j \cdot 50$	$5 \cdot e^{-2\ t}\ \cos(50\ t - 90°)$	'
$5\angle -90°$	$-2 - j \cdot 50$	$5 \cdot e^{-2\ t}\ \cos(50\ t - 90°)$	'–j50_results_same_as_above
$15\angle 0°$	$-5000 + j \cdot 1000$	$15 \cdot e^{-5000\ t}\ \cos(1000\ t)$	'
$15\angle 0°$	$-5000 - j \cdot 1000$	$15 \cdot e^{-5000\ t}\ \cos(1000\ t)$	'same_for_negative'
$100\angle 30°$	0	$86.6\angle 0°$	$100 \cdot \cos(30\ deg)$

Comments:
Check on the negative j, its the same as positive j in the 2nd function above.

No where in electrical engineering you seen the first cosine or sine term having a negative sign, the first term is positive sign (120 Pi t*). I can be corrected, but textbook wise I have not seen it.* This sets the path of the signal from left to right, travelling in the positive direction. But the +j and -j results in the same, all in the first quadrant where sine, cosine, and tangent are all positive AND in 4th quadrant where cosine alone is positve and real.

Maybe All this has to do with phase shift and time shift.

Short notes, chapter, on s.

Next on to those waveforms, really interested in their appearance, how to plot, and some information that can be captured from them.

Chapter 2: Waveforms and R, L and C.

$\omega := 1 \quad \tau := 1$ <--- *Declaration for the text content not for calculations or plots.*

Time Shift and Phase Shift.

(Schaum's Outline: Electric Circuits. Nahvi & Edminister).

If the signal described by a function v(t) cos wt is delayed by tau seconds, how do we write it?........We write it like this:

$$v(t-\tau) = \cos\cdot\omega(t-\tau) = \cos\cdot\omega(t-\theta)$$

where $\theta := \omega\cdot\tau$

Some confusion is created when we say the plot of the delayed function or signal is shifted to the right in the graph. We assumed it would be on the left in comparison to v(t), because it started late,like in a race, but its actually ahead to the right. Why? Because the function v(t) sets the time duration t, and a shift-delay would be time t minus tau, (t - tau), so if we set t at 5 seconds, and delay tau at 2 seconds, the delay plot will start at t = 2, whereas original v(t) starts at 0 and runs through t = 5. So, that is why its seen to the right. You would expect it behind v(t) as in to the left. Its just how plots are graphed. Now if it was not a delay rather a lead, then its v(t + tau) which sets the lead plot to the? Left. Yes! You got it. The lead appears before the main signal of concern. To the left. Lead was ahead. Lag? its taken 2 seconds in the positive direction before it appeared, plus quanity 2 seconds, so now we see it lagging.

You start at t=0, and run thru the time, the function is set for where t = 5 seconds for example or t = T one period, or t = 2 pi one cycle.

The delay shifts the graph v(t) to the right by an amount of tau seconds, which corresponds to a phase lag of theta = (omega)(tau) = 2 pi f tau.

$$\theta = \omega\,\tau = 2\,\pi\cdot f\cdot\tau$$

A time shift of tau seconds to the left on the graph produces (vt + tau), resulting in a leading phase angle.

Conversely, a phase shift of theta corresponds to a time shift of tau. Therefore, for a given phase shift, the higher the frequency, the smaller the required time shift. --- Page 119 Schaum's Outline. See example below.

Example ---> $f1 := 50 \qquad f2 := 400 \qquad \theta := 30\ \mathbf{deg}$

$$\tau_1 := \frac{\theta}{2\cdot\pi\cdot f1} = 0.0017 \qquad \tau_2 := \frac{\theta}{2\cdot\pi\cdot f2} = 0.0002$$

Lower f1 higher tau 1. Higher f2 smaller tau 2.

Tau 1 > Tau 2.

Continued next page.

Clear (ω, τ) <--- *Clear variables.*

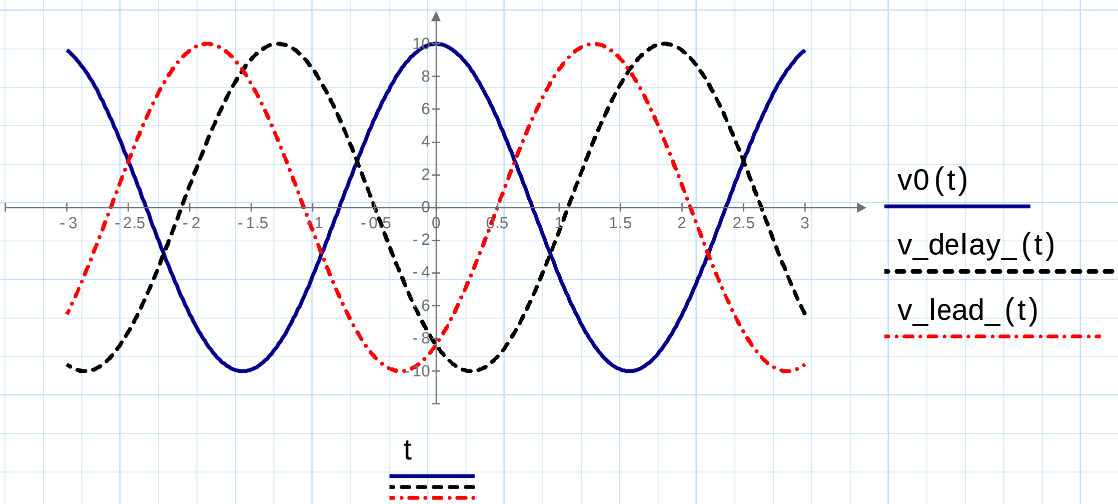
We have 3 functions, the original function, then one with a time delay, and last one with a time lead. The graph is plotted on the -ve and +ve side. See below.

$v0(t) := 10 \cdot \cos(2\ t)$

$v\_delay\_(t) := 10\ \cos(2\ t - 10)$ Appearing later to the right; NEGATIVE 10.

$v\_lead\_(t) := 10\ \cos(2\ t + 10)$ Appearing early to the left; POSITIVE 10

To capture the waveform in its delay and lead, look at when the waves first pass the 0 axis and then cross the x-axis. Notice now, the blue solid curve plot crosses the x-axis at close to 0.75, the delay as we were told to its right close to 1 in the black dashed curve, lastly the lead which we were told will be on the left of the original signal shown as red plot the dash dot curve close to 0.5.



Lets try to plot the functions, i(t) and v(t), from Chapter 1.
May not be easy. Lets give it a try.
These were the functions in the table where we searched for s, given function i(t) or v(t) with amplitude and phase angle.

Study each graph's waveform pattern, identify a period, you may measure of the x-axis which is the time axis t. Exponential plots may not show a cycle or a period, here observe the plot's gradient; steep or gradual, ...etc.
s = sigma + j(omega), this may not be easy to capture, but what we are looking at here in our examples is the exponential terms.

You got the period you inverse it for the frequency. Then you may calculate omega, 2 x PI x f. You do not normally see a period in Exponential terms because as you will come to know this function dies out. DIES OUT before it can do any pattern, they are beautiful in appearance their plots, but no cycle. Critical math function in? Electrical circuit analysis and? Design.

$i1(t) := 86.6$ A.

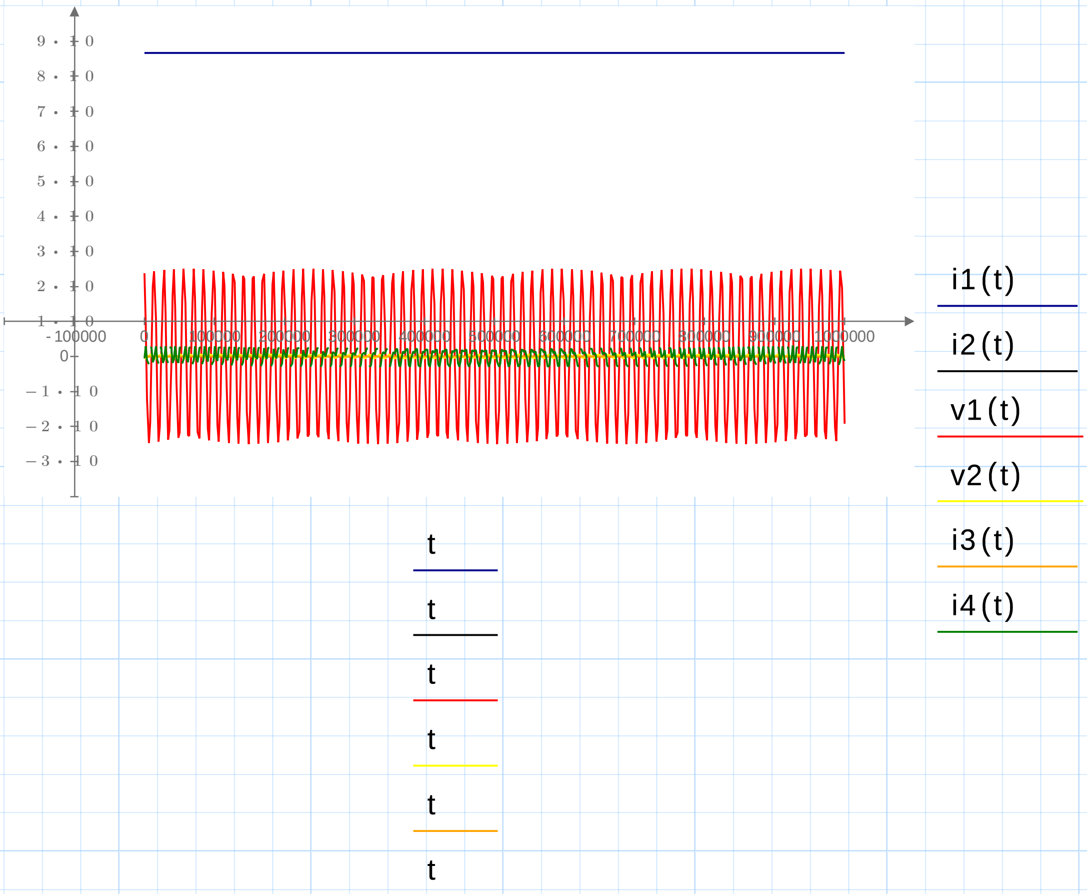
$i2(t) := 15.0 \cdot \left(e^{-2 \cdot 10^{3} \cdot t}\right)$ A.

$v1(t) := 25.0 \cdot \cos(250 \cdot t - 45)$ V.

$v2(t) := 0.5 \cdot \sin(250 \cdot t + 30)$ V.

$i3(t) := 5.0 \cdot \left(e^{-100 \cdot t}\right) \cdot \sin(50 \cdot t + 90)$ A.

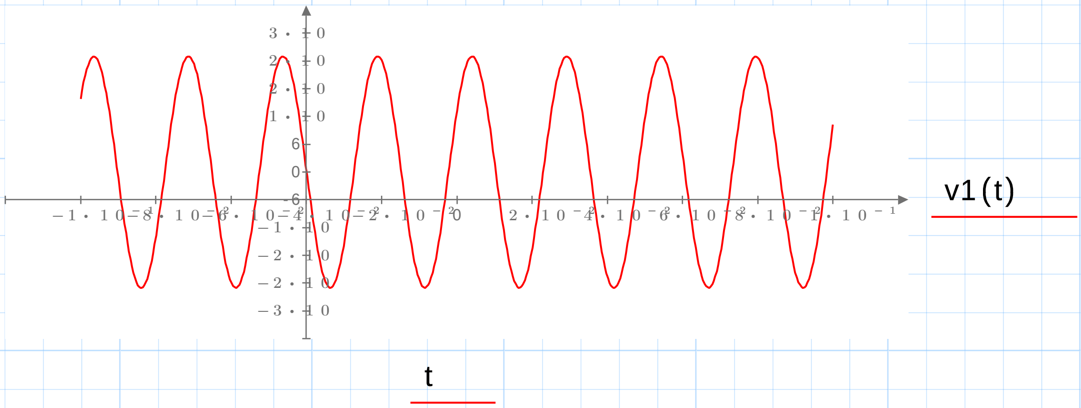
$i4(t) := 3 \cdot \cos(50 \cdot t + 4 \cdot \sin(50 \cdot t))$ A.

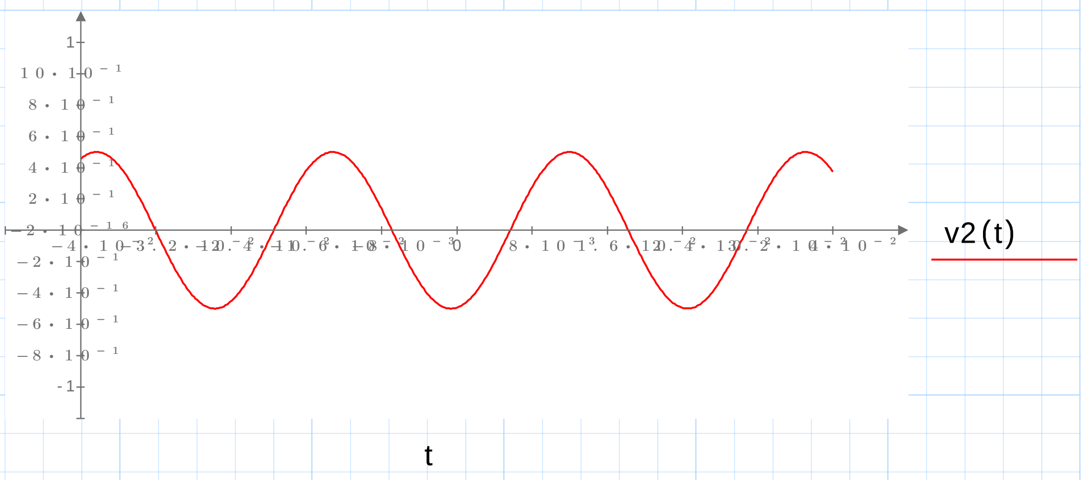


Its difficult to see much in this graph. Worthless it is NOT, nor the software, just the setup with regards to the time scale. Plot individually or if possible in groups.
x-axis scale needs to suit each function's plot. You can do this in Excel? Yes.

$v1(t) := 25.0 \cdot \cos(250 \cdot t - 45)$ V. NEGATIVE -45? delay.

$v2(t) := 0.5 \cdot \sin(250 \cdot t + 30)$ V. POSITIVE +30? lead.



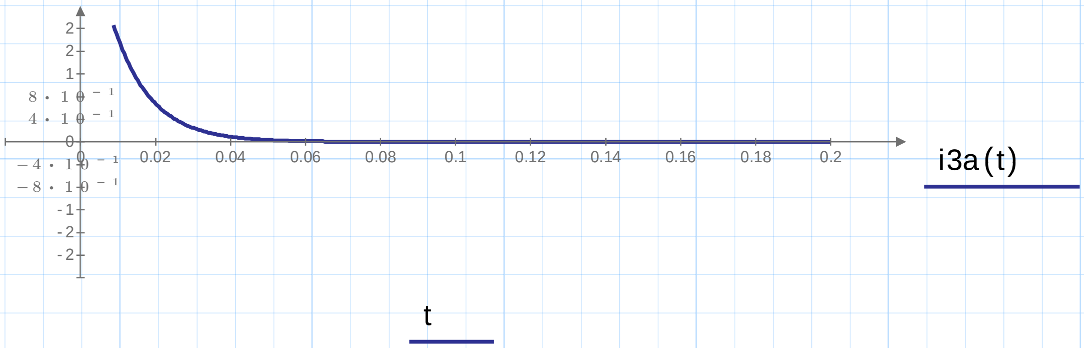


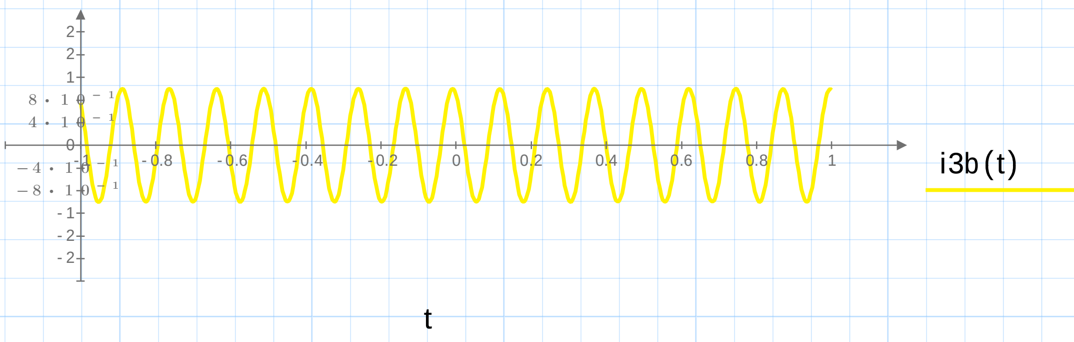
We split i3(t) into its exponential and sinusoidal term. Plot these two plots. Then plot i3(t). So you multiply i3a(t) to i3b(t) you see the final i3(t). Otherwise its not easy to capture the plot on the graph by going straight to i3(t).

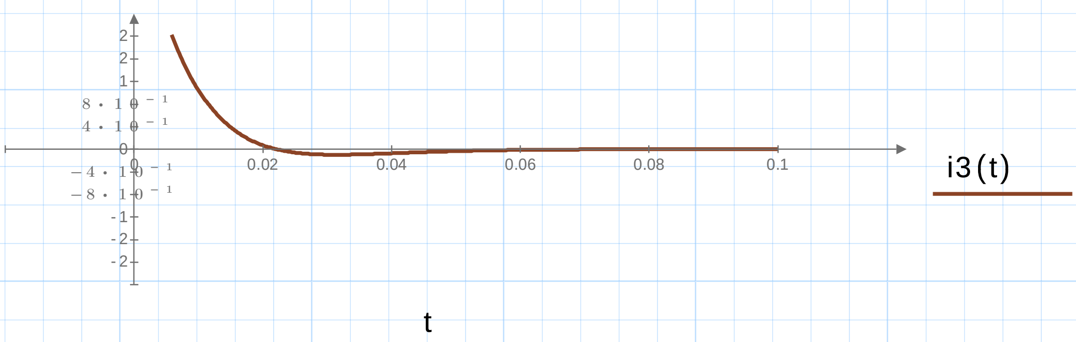
$i3(t) := 5.0 \cdot \left(e^{-100 \cdot t}\right) \cdot \sin(50 \cdot t + 90)$ A.

$i3a(t) := 5.0 \cdot \left(e^{-100 \cdot t}\right)$ A. <----Exponential term

$i3b(t) := \sin(50 \cdot t + 90)$ A. <----Sinusoidal term





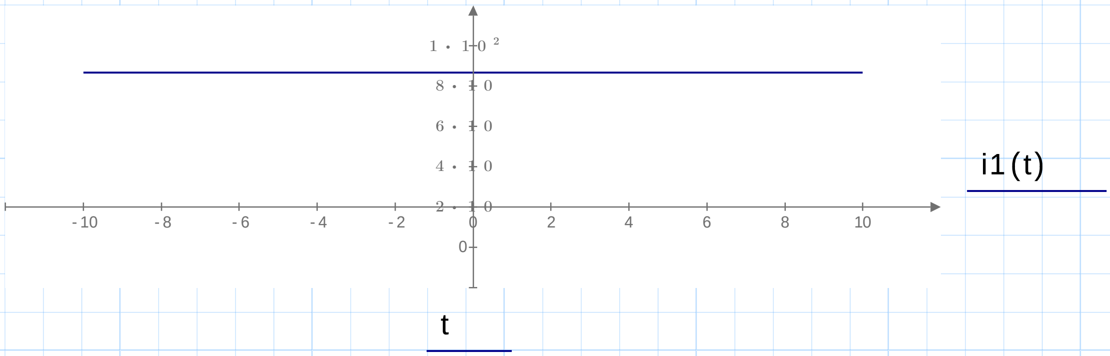


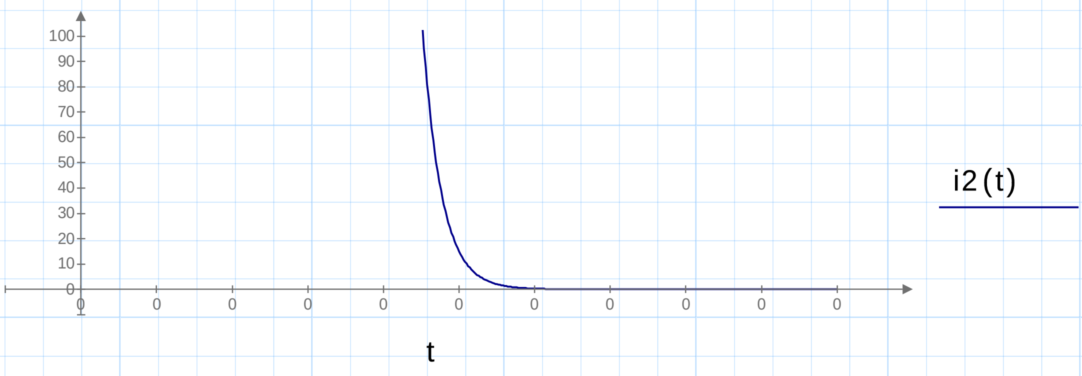
Sinusoidal term i3b(t) does not show its behaviour/pattern/shape in the summation function i3(t). *Maybe weak in comparison to exponential function! Observation!*

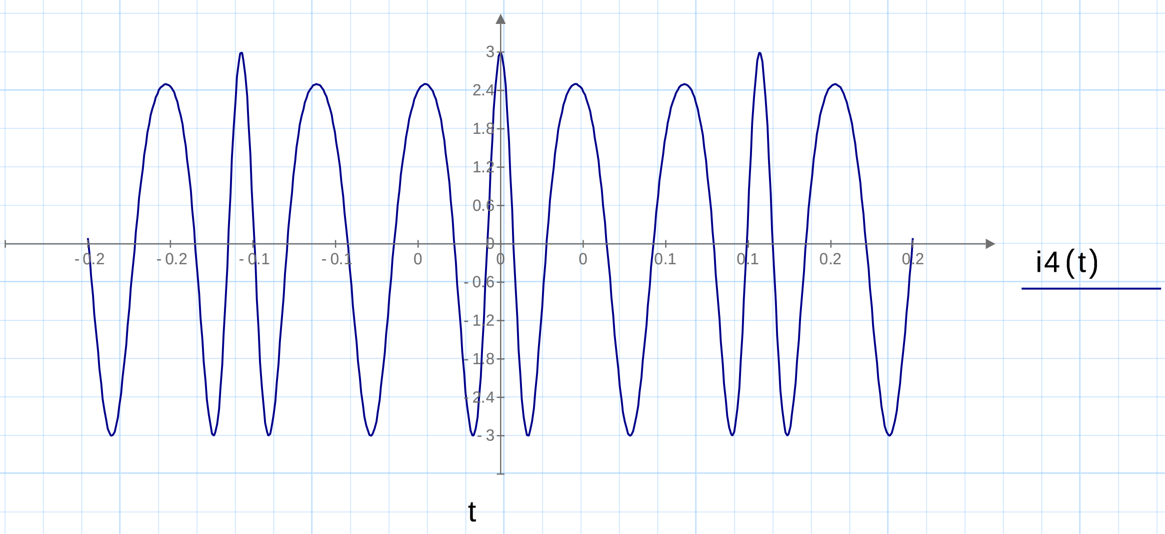
$i1(t) := 86.6$ A.

$i2(t) := 15.0 \cdot \left(e^{-2 \cdot 10^{3} \cdot t}\right)$ A.

$i4(t) := 3 \cdot \cos(50 \cdot t + 4 \cdot \sin(50 \cdot t))$ A.







Waveforms: Various Voltage/Power Source Inputs To Resistor, Inductor, and Capacitor.

Inductor's and Capacitor's current and voltage waveform, amplitude, and phase angle BEFORE switch closing, AT CLOSING of switch, and AFTER closing of switch?

The above statement is one major obstacle in engineering. Its about setting the initial conditions. Hard topic.

There are many different sources of voltage and current into the circuit, and we can't remembers them all, so some bacis tools can help guide what to expect.

Some encouragement:

An expert is looking it at everyday and will be good at it, so this is NOT a set back.

How many engineers actually carry a digital oscilloscope, multimeter, or any other measuring instrument daily? Almost none except for those working in a laboratory in a design or manaufacturing capacity, even then its a select few at each location.

Not to fuss here.
Experts? Not us! We got Schaums Outline.

Continued next page.

Circuit Component's Waveform: Inductor

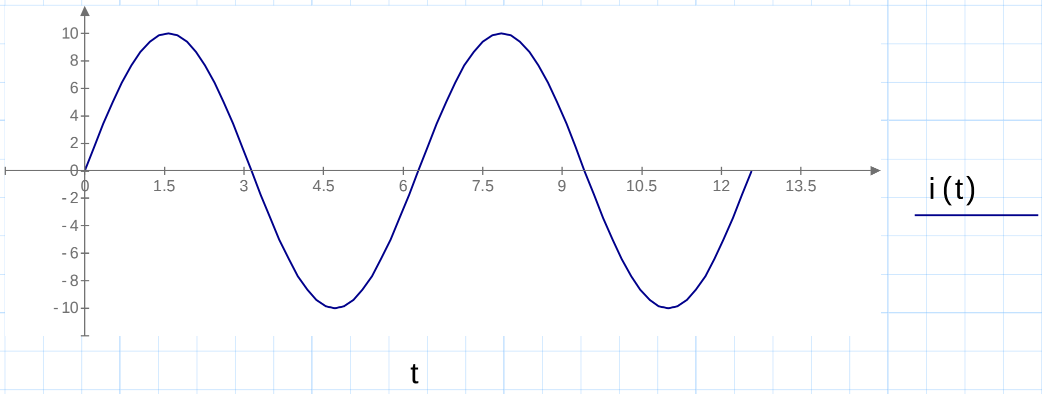
clear (t)

$i(t) := 10.0 \cdot \sin(t)$

So we sense the amplitude is 10A, thats when t =0, because sin 0 deg is 1, thats maximum, the remaining would be oscillating dependent on sine value's contribution to the curve.

$t := 0, \frac{\pi}{18} .. 4\,\pi$

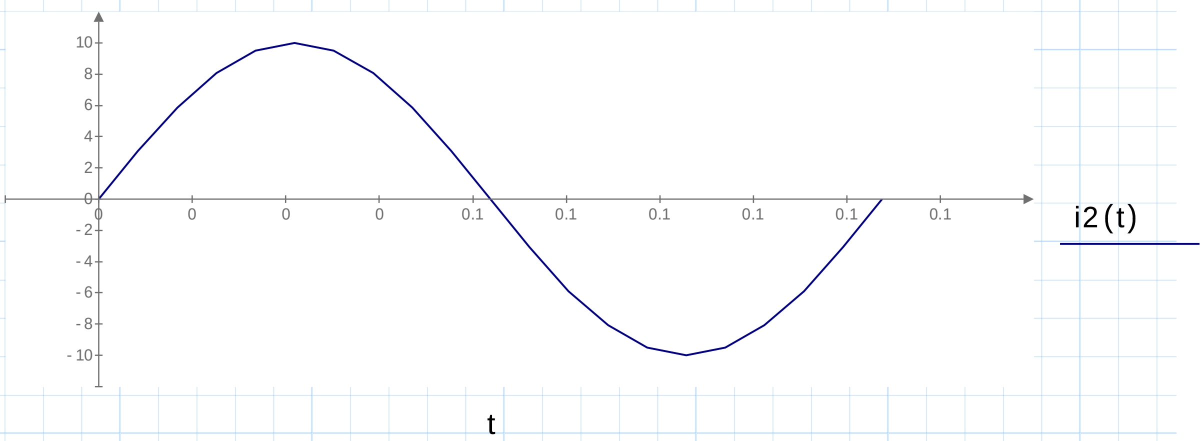
Reading the plot i(t) below: Amplitude is +/-10 A depending on cycle. PI/18 = 180/18 = 10 degree interval, and 4 PI = 2 cycles, 1 cycle is 2 PI. x-axis is correct at 2PI = 3.14 x 2, and end of wave at 4 x 3.14.



clear (t, f, T)

$i2(t) := 10.0 \cdot \sin(50 \cdot t)$ $f := \frac{50}{2 \cdot \pi} = 8$ $T := \frac{1}{f} = 0.1$ $\frac{2 \cdot \pi}{50} = 0.1257$

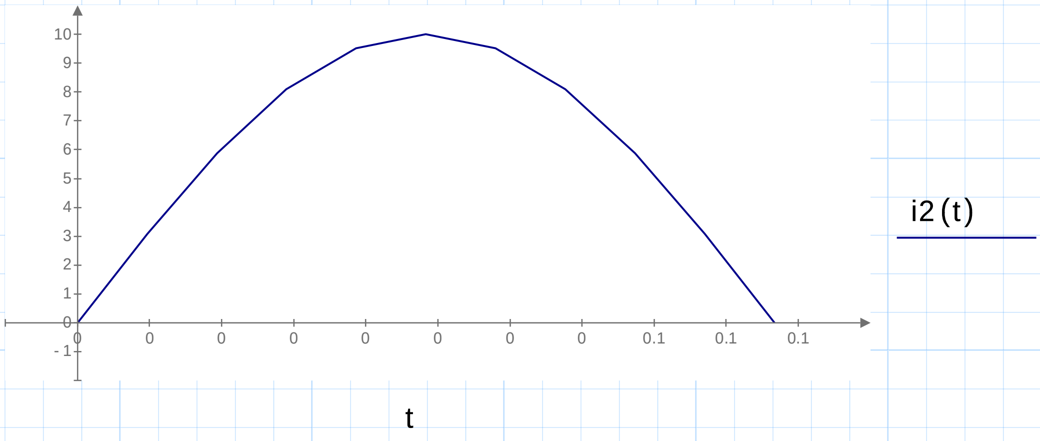
$t := 0, \frac{\pi}{500} .. \frac{2\,\pi}{50}$ Plot interval: $\frac{\pi}{500} = 0.0063$ Period of cycle see x-axis at 0.1257



Multiplied t by 50, previous to it was just 10 sin (t), did some changes as seen in plot above. *Electrical Engineering is **filled with this.** Its troubling to begin .....start loving it so it becomes? Friendly.*

clear(t) $\quad i2(t) := 10.0 \cdot \sin(50 \cdot t) \quad f := \frac{50}{2 \cdot \pi} = 8 \quad T := \frac{1}{f} = 0.1 \quad \frac{2 \cdot \pi}{50} = 0.1$

$t := 0, \frac{\pi}{500} .. \frac{\pi}{50}$ $\quad$ Plot interval: $\frac{\pi}{500} = 0.0063$



Here we made the plot frame smaller ended at PI/50. Which happens to be half cycle because full cycle was 2 (PI /50). So, so far we got some things going in the right direction. Amplitude is 10 amp, wave is in the top half so we say its got a positive area only.

Question:
Suppose a 30 mH inductor has a current i(t) = 10.0 sin 50t (A).
Plot the voltage, power, and energy (work in Joules):

L-voltage: L (di/dt). So differentiate i(t) then multiply to L.

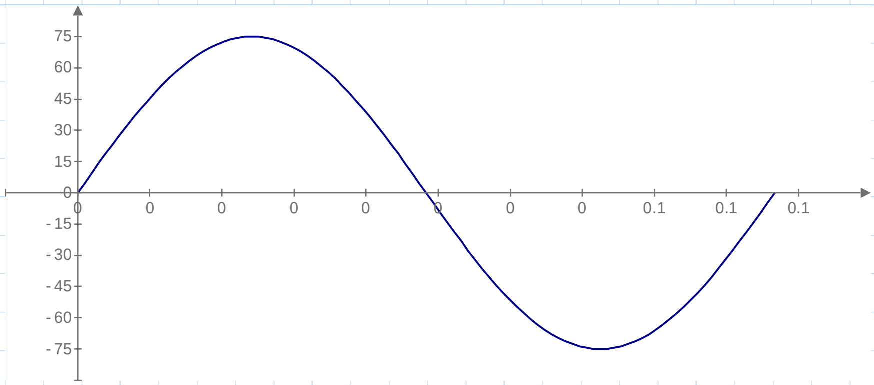
i = 10.0 sin (50t) $\quad i_L(t) := 10 \cdot \sin(50 \cdot t)$
di/dt: $= 50 \cdot 10 \cdot \cos(50 \cdot t)$
$L := 0.030$ H, 30 mH.
$0.030 \cdot 50 \cdot 10 = 15$
$v_L = 15 \cdot \cos(50 \cdot t) \quad v_L(t) := 15 \cdot \cos(50 \cdot t)$ V. Voltage across inductor.

Capacitor power p = vi
Trig identity:
sin(a) cos (b) = 1/2 sin(a + b) + 1/2 sin(a - b) ......Equation 1.
Now what? See the plot above this page. Its half a cycle.
So if we were to plot half a cycle we only need one half of Equation 1 above.
$p_L = \left(\frac{1}{2}\right) \cdot 10 \cdot 15 \cdot \sin(50 + 50) \cdot t = 75 \cdot \sin(100 \cdot t)$

We shall return to this shortly, why we dropped half the expression off.

$p_L(t) := 75 \cdot \sin(100 \cdot t)$

$t := 0, \frac{\pi}{5000} .. \frac{\pi}{50}$ Plot interval increased for smoother plot: $\frac{\pi}{5000} = 0.0006$



$p_L(t)$

t

Lets go to the full cycle instead of half the trignometric expression.

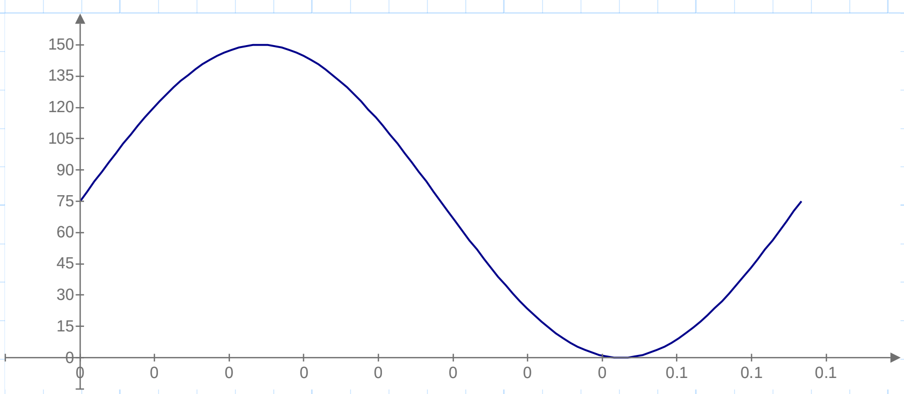
$$p_L = \left(\frac{1}{2}\right) \cdot 10 \cdot 15 \cdot \sin(50+50) \cdot t + \left(\frac{1}{2}\right) \cdot 10 \cdot 15 \cdot \cos(50-50) \cdot t$$

$p_{L\_total} = 75 \cdot \sin(100 \cdot t) + 75 \cdot \cos(0 \cdot t)$

$p_{L\_total}(t) := 75 \cdot \sin(100 \cdot t) + 75 \cdot \cos(0 \cdot t)$

2nd half: Has cos(0) this equals 1 which equals the amplitude of the term which is 75.
Looks like a constant 75 for all values of t.
Is it worth plotting or drop this half? Lets plot.
We need to double the range on the plot to one full cycle? Not necessary.

$t := 0, \frac{\pi}{5000} .. \frac{\pi}{50}$ Plot interval increased for smoother plot: $\frac{\pi}{5000} = 0.0006$



$p_{L\_total}(t)$

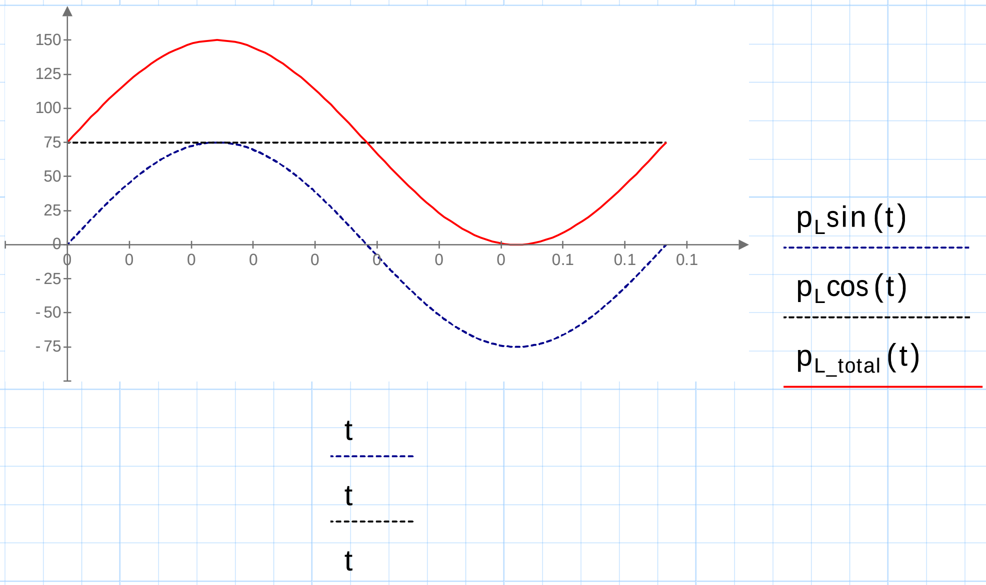
t

We added 75 to each point in the plot. Shifted it up above t-axis = 0.
Achievement? Yes, but lets plot each term separately. NOT over yet.

$p_{L\_total}(t) := 75 \cdot \sin(100 \cdot t) + 75 \cdot \cos(0 \cdot t)$

$p_L sin(t) := 75 \cdot \sin(100 \cdot t)$
$p_L cos(t) := 75 \cdot \cos(0 \cdot t)$

$t := 0, \frac{\pi}{5000} .. \frac{\pi}{50}$ Plot interval increased for smoother plot: $\frac{\pi}{5000} = 0.0006$



Textbook took up the first term the sine term leaving out the cosine term BECAUSE the input is a sinusoidal sine wave hence the output across the inductor should similarly behave in a sinusoidal manner, sine wave, NOT a flat line constant 75 V contribution to the output. A changing current with respect to time t, di/dt, will result in a changing voltage with respect to the same time t.
Answer is the first term or the first half term that is the sine term plot. CORRECT.

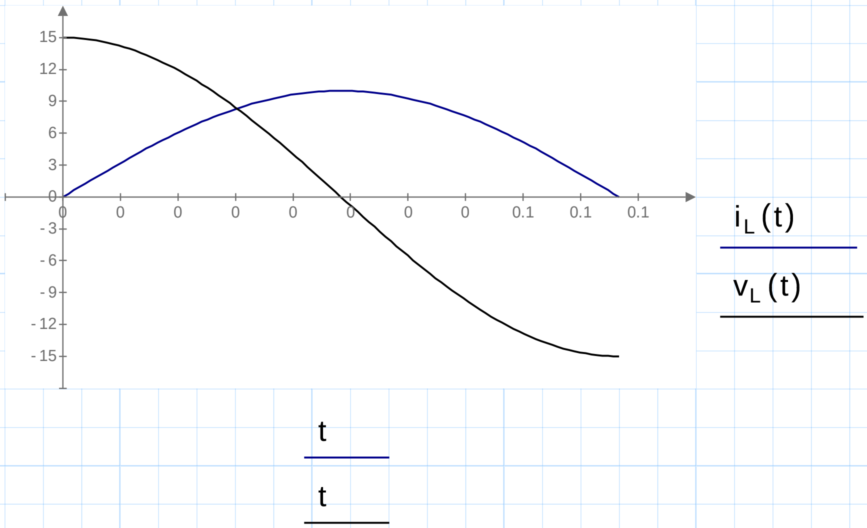
Maybe most textbooks dont explain this. Maybe engineer or student saw it.

Clearly some blanks are there so the instructor/lecturer has his/her ability to perform in the teachig process. Now having some insight in this and how things work in circuits, it maybe best to ask your lecturer/instructor/tutor.

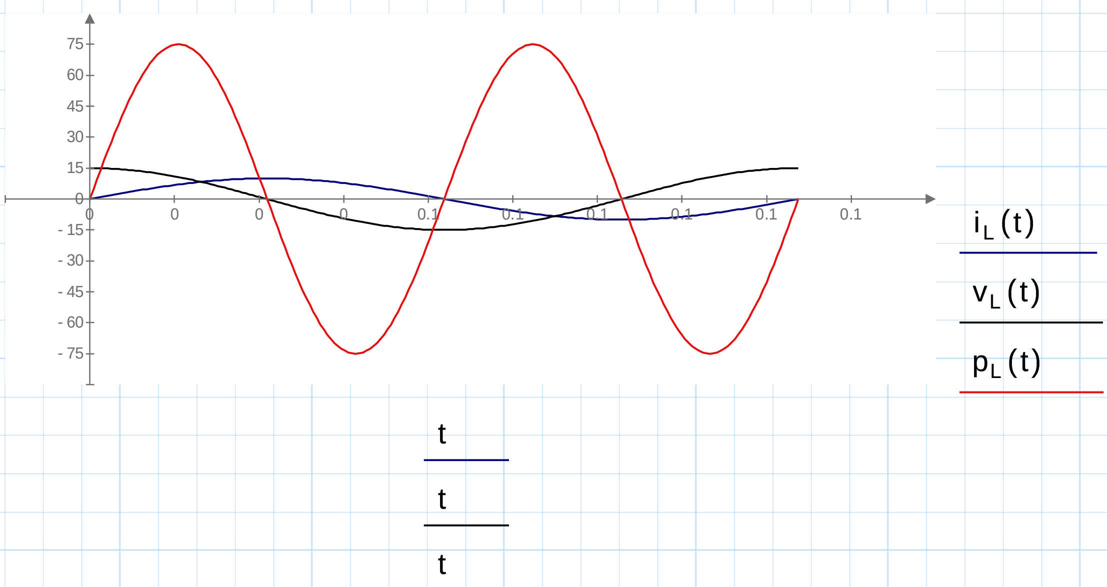
Next plot for the inductor's current, and voltage.
Since we had not plotted current and voltage.

$i_L(t) := 10 \cdot \sin(50 \cdot t)$
$v_L(t) := 15 \cdot \cos(50 \cdot t)$
$p_L(t) := 75 \cdot \sin(100 \cdot t)$

$t := 0, \frac{\pi}{5000} .. \frac{\pi}{50}$ Plot for HALF cycle/period.



$t := 0, \frac{\pi}{5000} .. \frac{2\pi}{50}$ Plot for FULL cycle/period.



CORRECT!

Continuing with the energy OR work plot:

W_L = (1/2) L i^2.......better shown in integral form because its over a time period.
Looks more like **something dc**.

$$W_L(t) := \int_{t1}^{t2} p(t)\,dt \quad = \int_{t1}^{t2} L \cdot i\,di \quad = \left(\frac{1}{2}\right) \cdot L \cdot \left(i_{final}^{2} - i_{initial}^{2}\right) \quad = \quad \left(\frac{1}{2}\right) \cdot L \cdot i^{2}$$

Since we are plotting, we set the time range in the plot, that takes care of the time dt.

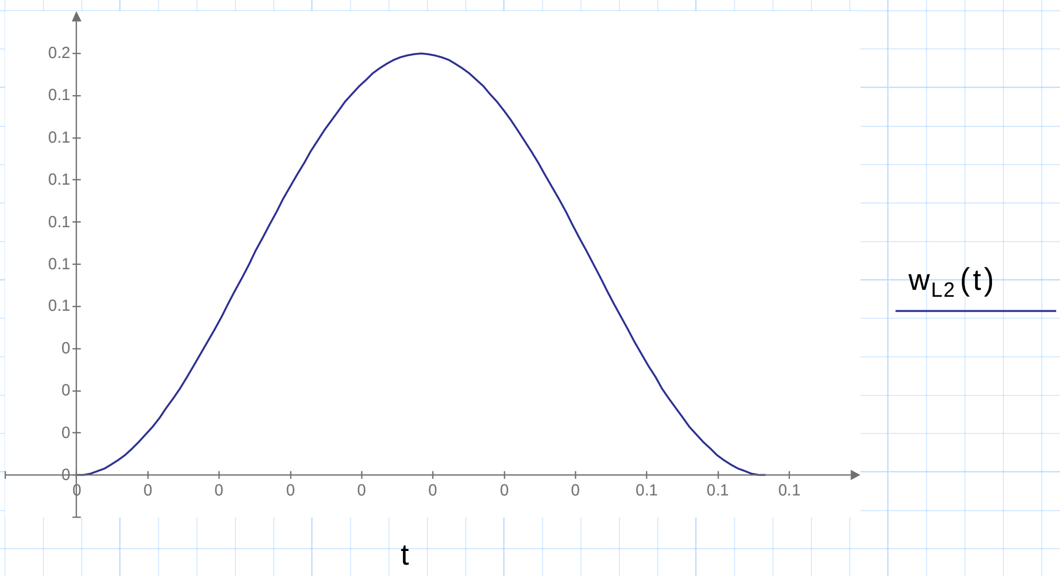
$w_L(t) := \left(\frac{1}{2}\right) \cdot 0.03 \cdot \left(i(t)^{2}\right) \qquad w_L(t) := 1.5 \cdot 10^{-3} \cdot \left(i(t)^{2}\right)$ **NO!** we will try it first.

We have i(t) so we square it and multiply to (1/2)L? **NO.**
The sinusoidal term 10 sin (50t) we cant square it we need a trig identity.
But we will try it with a term that is composed of i(t) x i(t) first.

$$i_{squared}(t) := i_L(t) \cdot i_L(t)$$

$$w_{L2}(t) := 1.5 \cdot 10^{-3} \cdot i_{squared}(t)$$

$t := 0, \frac{\pi}{5000} .. \frac{\pi}{50}$ Plot for one cycle/period for a better picture on the waveform.



Almost the same shape, similar but the amplitude or maximum is not. **WRONG!**
Where in the problem?
In the intepretation of the expression.

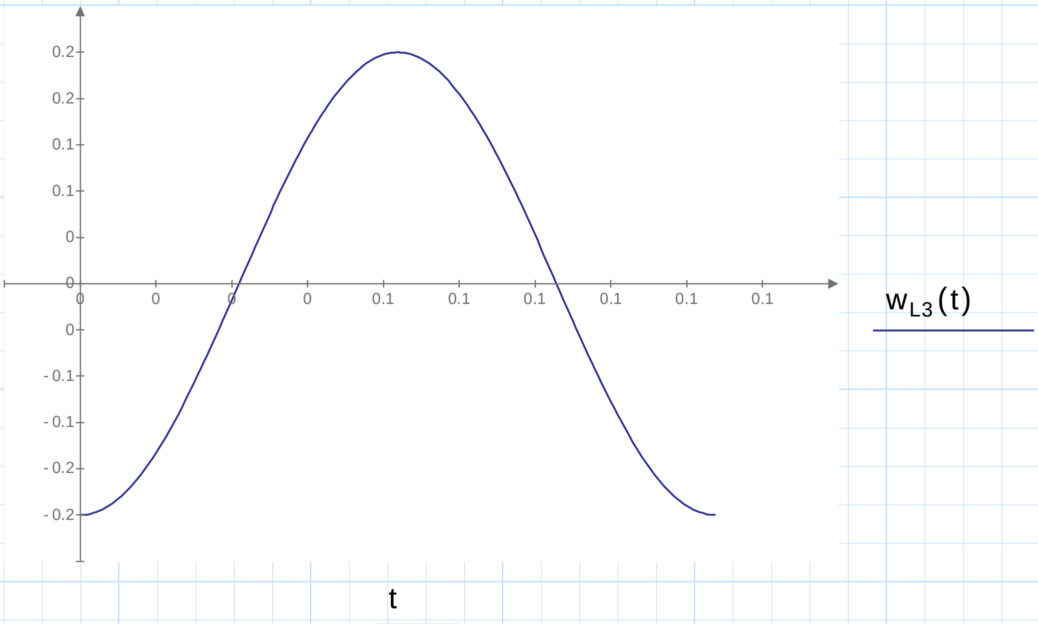
Work of inductor L = $L \cdot \int i\, di$

$\int 10 \cdot \sin(50\ t)\, dt = -\left(\frac{1}{50}\right) \cdot 10 \cdot \cos(50\ t)$ Lets try.

$w_{L3}(t) := -\left(\frac{1}{50}\right) \cdot 10 \cdot \cos(50\ t)$

Got it? Trig identity for the squared term! <---Next try.

$t := 0, \frac{\pi}{5000} .. \frac{2\ \pi}{50}$ We doubled the plot range to one cycle because only half of what is shown below was obtained.



Wrong! The amplitude is off.

Now we do the last option or so I/we think.; ie i(t) multiply i(t)

10 sin (50t) x 10 sin (50t) = 100 sin**^2**(50t).

$100\left(\sin(50\ t)^2\right)$ <---This is what we are looking at, it has a solution from trig identities.

sin^2(t) = **(1/2) (1 - cos 2(t))** <---The trig identity.

sin^2(50t) = (1/2) (1 - cos (**2 x 50t**))
= (1/2) (1 - cos (100t))

Next multiply it by L and 100 the amplitude

w_L= (1/2)*(L)* 10^2*((1/2) (1 - cos (100t))).....Get formula right (1/2)L then the i(t) term squared with the amplitude.

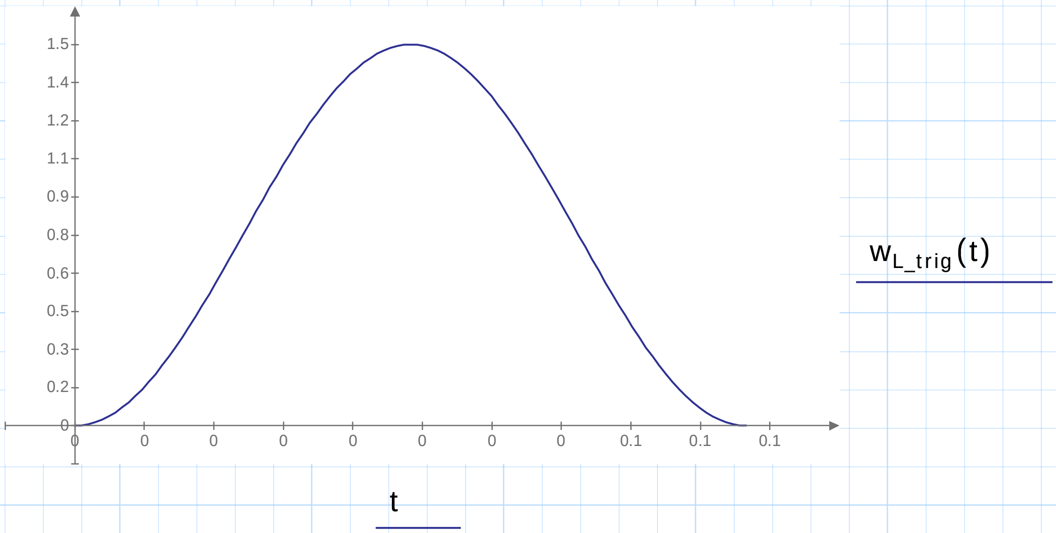
w_L= (0.5) (0.03) (10x10) ((1/2) (1 - cos (100t)))

Amplitude = $0.5 \cdot 0.03 \cdot 10^2 \cdot \left(\frac{1}{2}\right) = 0.75$

$w_{L\_trig}(t) := 0.75 \cdot (1 - \cos(100\ t))$

$t := 0, \frac{\pi}{5000} .. \frac{\pi}{50}$

We halved the plot range (2Pi/50 to Pi/50), to one cycle because really only half of what is shown below was obtained.



CORRECT!

With some frustration and continued effort got the plots done same as Schaums Outline Nahvi & Edminister.

Its not about getting the plot done 1st time rather the **'path'** on **'how to'** get to the correct plots.

Discussion on Inductor:

1. There is no voltage across an inductor if the current through it is not changing with time.
2. A finite amout of energy can be stored in an inductor even if the voltage across the inductor is zero, such as when the current through it is constant.
3. It is impossible to change the current through an inductor by a finite amount in zero time, for this requires an infinite voltage across the inductor. It will be advantageous later to hypothesize that such a voltage may be generated and applied to an inductor, but for the present we shall avoid such a forcing function or responce. *(GOOD).*
   An inductor resists an abrupt change in the current through it in a manner analogous to the way a mass resists an abrupt change in its velocity.
4. The inductor never dissipates energy, but only stores it.
   Although this is true for the mathematical model, it is NOT true for a physical inductor.

......reference from page 123 Engineering Circuit Analysis 4th ed Hyatt & Kemmerly.

Please check. Similar notes on Capacitor available here.

The notes are good but do not do much for me/us concerning inductor conditions before switch is closed time -t, then at time t =0 when switch is closed, we all seem to have the general idea here, and then at t+ when its past a few seconds or miliseconds. Its a struggle each time, of course if you do this everyday............... For example note 4 above, its ok for the math but in reality its not true. Give us an example?

First reading its hard. Its a few chapters before I see it straight. You maybe faster....maybe.

Inductor Example 1:

An inductance of 2 mH has a current i = 5.0(1 - e^-5000t) A.
Find the corresponding voltage and maximum stored energy?

clear (t)

$i_L(t) := 5.0 \cdot \left(1 - e^{-5000 \cdot t}\right)$

$i_L(t) := 5.0 - 5 \cdot e^{-5000 \cdot t}$

$$\frac{di}{dt} = 5000 \cdot 5 \cdot e^{-5000 \cdot t}$$

$$= 25 \cdot 10^3 \cdot e^{-5000 \cdot t} \quad A$$

$i_{di\_dt}(t) := 25 \cdot 10^3 \cdot e^{-5000 \cdot t}$ A.

$L := 2 \cdot 10^{-3}$ H

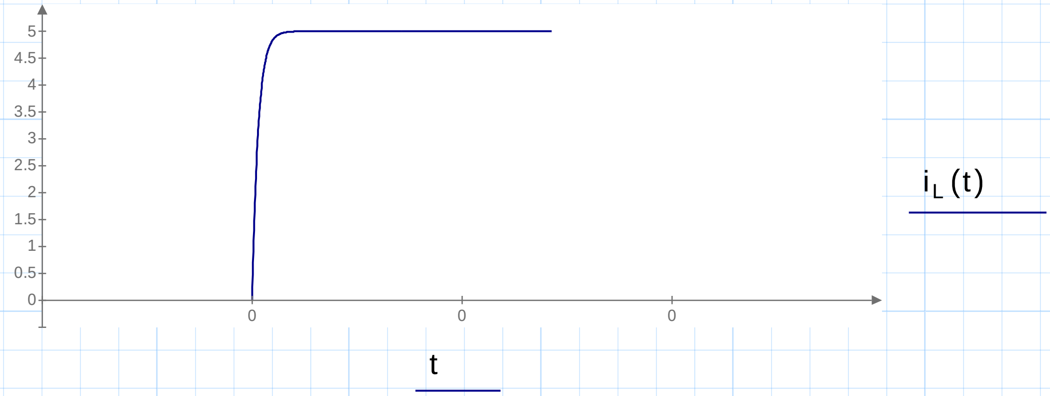
$$v_L = L \cdot \frac{di}{dt}$$

$$v_L = \left(2 \cdot 10^{-3}\right) \cdot 25 \cdot 10^3 \cdot e^{-5000 \cdot t}$$

$$v_L = 50 \cdot e^{-5000 \cdot t}$$

$v_L(t) := 50 \cdot e^{-5000 \cdot t}$

$t := 0, 0.00001 .. 0.01$



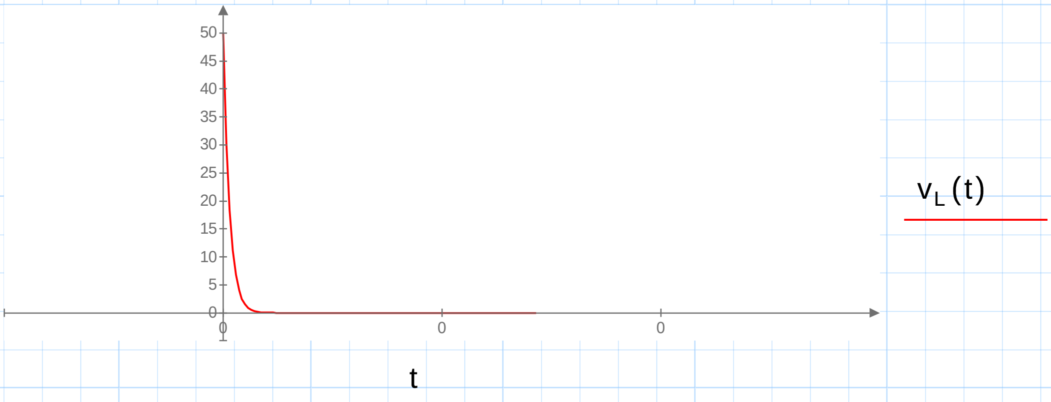
The plot of i(t) shows the maximum current Imax = 5 A.
It does not mean it stays there forever, given the mathematical expression thats its plot.

$I_{max} := 5$

$W_L = \left(\frac{1}{2}\right) \cdot L \cdot I_{max}^2 = 0.025$ J (Joules) Answer.

$v_L(t) := 50 \cdot e^{-5000 \cdot t}$ clear (t)

$t := 0, 0.0001 .. 0.01$



Voltage drops from a maximum of 50V down to zero.

Continued on next page.

Inductor Example 2:

An inductance of 3 mH has a voltage that is described as follows:
for 0 < t < 2 ms, V = 15 V
and
for 2 < t < 4 ms, V = -30.0 V.
Obtain the corresponding current and sketch v_L and i for the given intervals?

Solution:

Given 2 time inervals, the mid-point between the intervals is 2 ms.

For the voltage across the inductor L:

$$v = L\,(di/dt)$$

$$\frac{v}{L} = (di/dt)$$

Old math comes in....from differentiation to? Integration.

$$\frac{v}{L}\,dt = di$$

$$\int_0^t \left(\frac{v}{L}\right) dt = \left(\frac{1}{L}\right)\cdot\int_0^t (v)\,dt = \int_0^t i\,dt = i(t)$$

So that helped, but that don't solve the initial value conditions merely gets a numerical result. Given the voltage and inductance RHS in expression above seems to have some potential for computation.

For 0 < t < 2 ms, V = 15 V. $L := 3\cdot 10^{-3}$ H

Note: t is LESS than 2ms not at 2 ms.

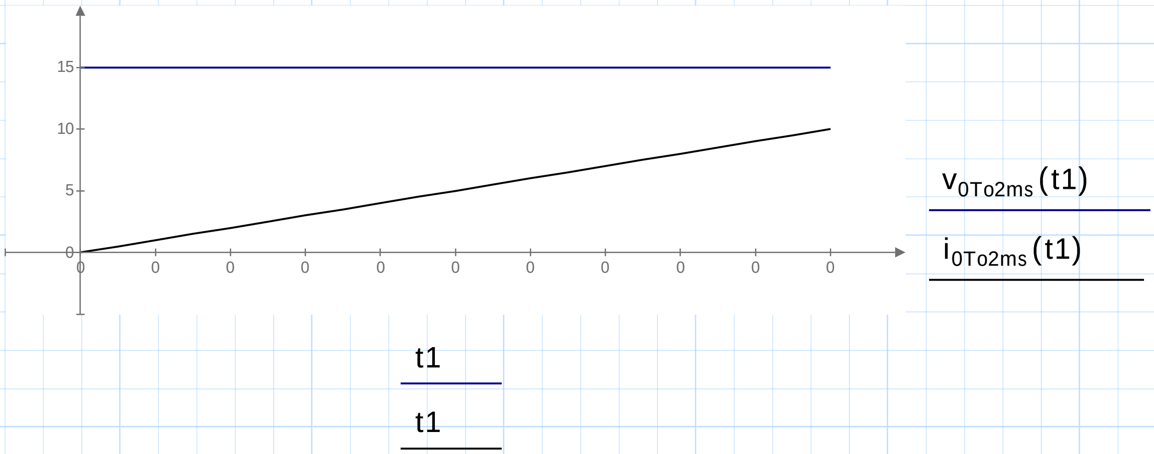
$$i = \left(\frac{1}{L}\right)\cdot\int_0^t 15\,dt = \left(\frac{1}{3\cdot 10^{-3}}\right)\cdot(15\ t) = \left(5.10^{3}\right)\cdot t \quad A$$

<--Leave it in this form so we can plot from 0 to ...2ms.

Lim t = 0 to 2 ms   Lim t = 0 to 2 ms

$i_{0To2ms}(t1) := 5\cdot 10^{3}\cdot t1$ <--- plot function. Answer.

$v_{0To2ms}(t1) := 15.0 \quad i_{0To2ms}(t1) := 5 \cdot 10^{3} \cdot t1 \quad t1 := 0, 0.0001 .. 0.002$



Plot above for the first time interval.

Discussion:

Time travelling from 1.9 2.0 2.1 ms so if we have 2.0001 ms its in, but not at 2.0.
What if we have an overlap on the 2ms? Would this overlap level out, cancel may not be the appropriate word for it, so it balances the output? Obviously now the graphing has to take into consideration some attention at 2ms.
So the problem is how we graph it, thats what we want to overcome not so the exact time when it turns from 1.9 --> 2.0 --> 2.1 we know where this is and how the limits stretch.
Take the fear of making errors out, of course we cant have 2 values at time t = 2.0 ms. That we know, so is it a matter of 'finese, class, show-man-ship, trick,.......? May be.

Both the intervals do NOT hit 2 ms so which has the right to be on 2 ms.
i). 0 < t < 2 ms
ii). 2 < t < 4 ms
At time t =2 belongs to which interval? That is the question here.

I had a lengthy discussion here, it was attempting most possible combinations or options, then reasoning thru them to strike the wrong ones out. Problem was it got difficult to keep track of things which were wrong and the reaons why. *THIS YOU CAN DO AND THEN CHECK THE SOLUTION.*

If we are going to plot over an interval of time, the expression need to be a function of time. This you know.

So, next we go straight to the path of the solution. Its not the inductor initial conditions problem we are given the intervals, need to evaluate/calculate given values.

At exactly t = 2ms, ie we say t in the middle, we have BOTH 15V and -30V contributing to the current i(t):

Both voltage expressions contributing to the current at time t = 2ms.
Limits from 0 to 2 to 4; arranged in time inverval (0-2) + (2-4)

$$\left(\frac{1}{L}\right)\cdot\int_{0}^{2\ ms} 15\, dt \quad = \quad \left(\frac{1}{L}\right)\cdot 15\ t$$

$$\text{i\_L for (t = 2ms)} = \left(\frac{1}{3\cdot 10^{-3}}\right)\cdot\left(0-\left(15\cdot 2\cdot 10^{-3}\right)\right) = -10 \quad A$$

$$\left(\frac{1}{L}\right)\cdot\int_{2\ ms}^{4\ ms} -30\, dt \quad = \quad -\left(\frac{1}{L}\right)\cdot 30\ t$$

$$\text{i\_L at (t=2ms)} = -\left(\frac{1}{3\cdot 10^{-3}}\right)\cdot\left(30\cdot 2\cdot 10^{-3} - 30\cdot 4\cdot 10^{-3}\right) = 20 \ \ A$$

Sum of current at t = 2ms

$i(2ms) = -10 + 20 = 10$ A. Answer

Next to the last interval between 2-4 ms.

At t > 2ms and less than 4ms :

This would be the integral $\displaystyle\int_{2\ ms}^{t}\left(\frac{v}{L}\right) dt = \left(\frac{1}{L}\right)\cdot\int_{2\ ms}^{t}(v)\, dt = \int_{2\ ms}^{t} i\, dt = i(t).$

Why is the upper limit t not 4 ms?
Because the problem states t < 4ms so we use 't' then that does not imply 4 ms.

$$i = \left(\frac{1}{L}\right)\cdot\int_{'>2\ ms}^{'<4\ ms} v\, dt$$

Because V = -30V here it reflects on 2 < t < 4 ms.

$$= \left(\frac{1}{L}\right)\cdot\int_{2\ ms}^{4\ ms} v\, dt \quad = \quad \left(\frac{1}{L}\right)\cdot\int_{2\ ms}^{t} v\, dt$$

We are sure of the lower limit, the upper limit maybe left as 't' so it will form our expression.

$$i = \left(\frac{1}{L}\right)\cdot\int_{2\ ms}^{t} -30\, dt = \left(\frac{1}{L}\right)\cdot(-30\ t) \quad = \quad \left(\frac{1}{L}\right)\cdot(-30)\cdot\left(t-2\cdot 0^{-3}\right)$$

Limits 2ms to 't'

$$i = \left(\frac{1}{3\cdot 10^{-3}}\right)\cdot(-30)\cdot\left(t-2\cdot 0^{-3}\right) = -10\cdot 10^{3}\cdot\left(t-2\cdot 0^{-3}\right) = -10\cdot 10^{3}\cdot t + 20 \quad \text{A.}$$

$$i = 20 - 10\cdot 10^{3}\cdot t \quad \text{A.}$$

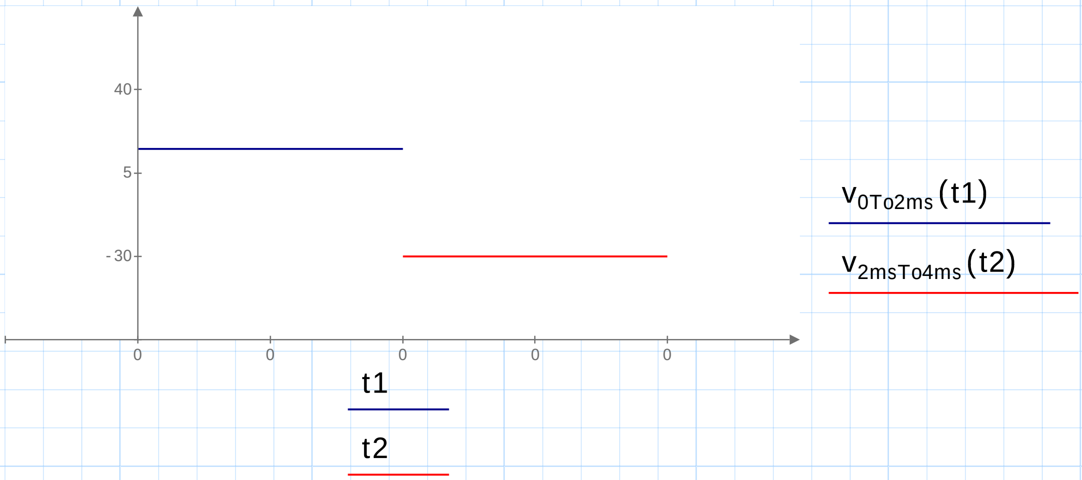
$i(2<t<4) = 10 + 20 - 10\cdot 10^{3}\cdot t = 30 - 10\cdot 10^{3}\cdot t$ A Answer.

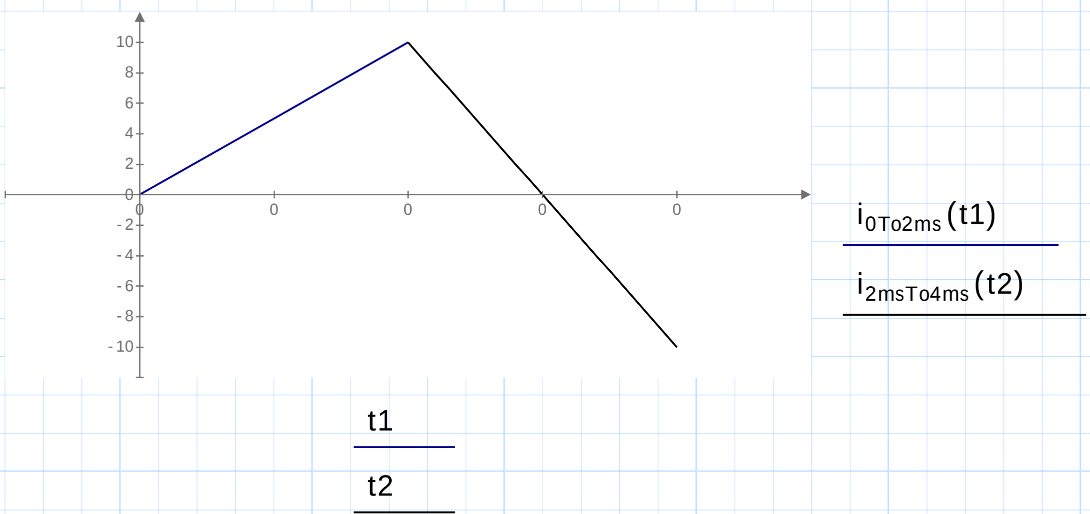
Plot of voltage and current shown on two separate graphs. Clear $(t1, t2)$

$t1 := 0, 0.001 .. 0.002$ $\qquad$ $t2 := 0.002, 0.0021 .. 0.004$

$v_{0To2ms}(t1) := 15.0$ $\qquad$ $v_{2msTo4ms}(t2) := -30.0$

$i_{0To2ms}(t1) := 5\cdot 10^{3}\cdot(t1)$ $\qquad$ $i_{2msTo4ms}(t2) := 30 - \left(10\cdot 10^{3}\right)\cdot(t2)$



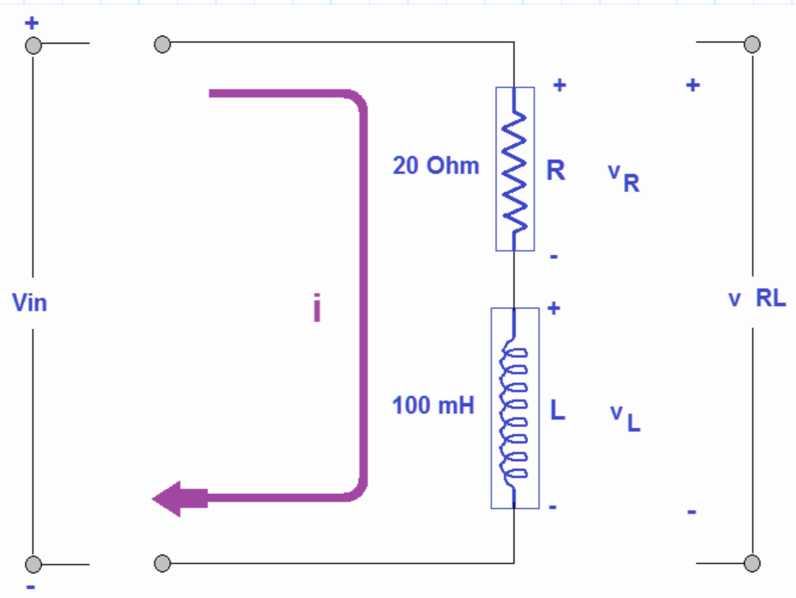


Correct plots above. Much discussion can come out of this worthy example only because of mistakes, its hard to place it in proper steps, and the limits concerned. Learning example.

clear (t1 , t2 , t3 , t4 , t5 , t6 , t7)

Inductor Example 3:

A 100mH inductor in series with the 20 Ohm resistor.
Has a current i thru it.
Find and plot the voltages across R, L, and RL.



clear (t11 , t22 , t33)
Graphs on next page, see equations for each plot for the currents and voltage.

Plot are not continued vertically i.e. the vertical drop or rise lines are not shown it can be done but would take extra time and effort. You can see where the lines are not connected as the current or voltage continues you may assume they are. These are shown in Schaums Outline.

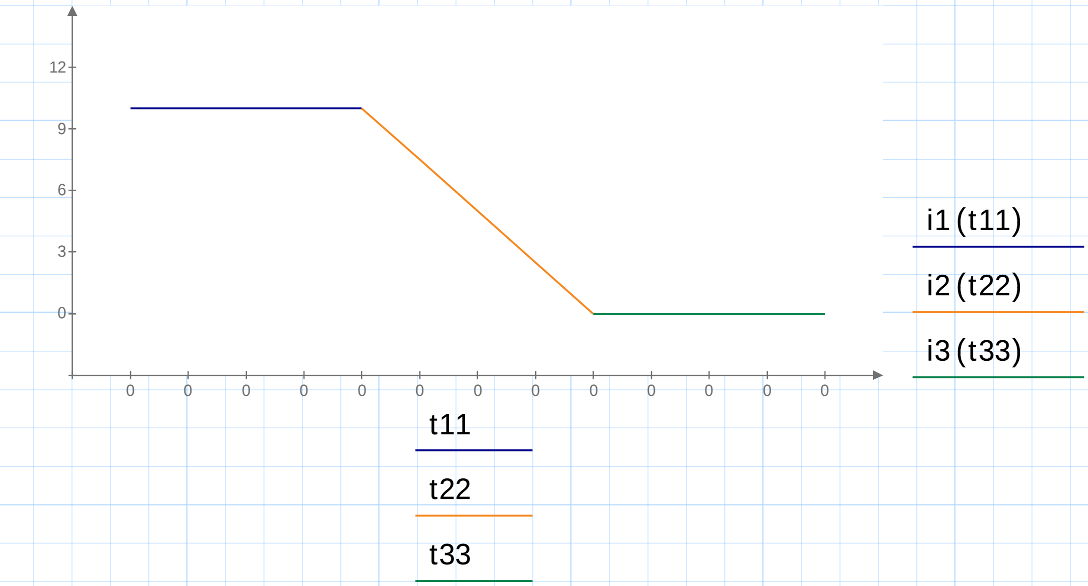
This is not an initial condition example though worthy.

Current i waveform:

$t11 := 0, -0.00025 .. -0.001$ $\quad i1(t11) := 10$

$t22 := 0, 0.00025 .. 0.001$ $\quad i2(t22) := 10 - (10^4 \cdot t22)$

$t33 := 0.001, 0.00125 .. 0.002$ $\quad i3(t33) := 0$

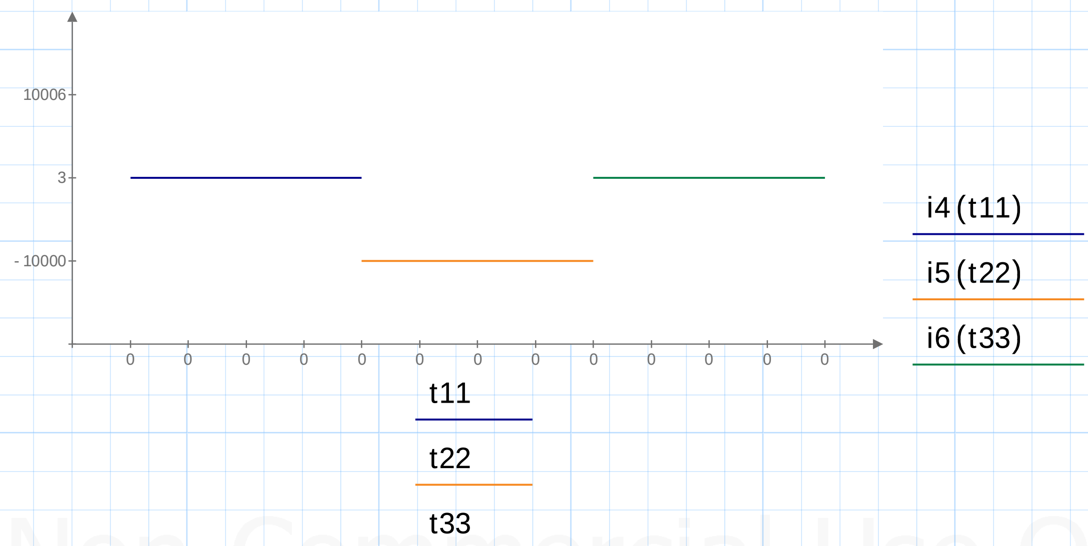


Current (di/dt) waveform:

$t11 := 0, -0.00025 .. -0.001$ $\quad i4(t11) := 0$

$t22 := 0, 0.00025 .. 0.001$ $\quad i5(t22) := -(10^4)$

$t33 := 0.001, 0.00125 .. 0.002$ $\quad i6(t33) := 0$

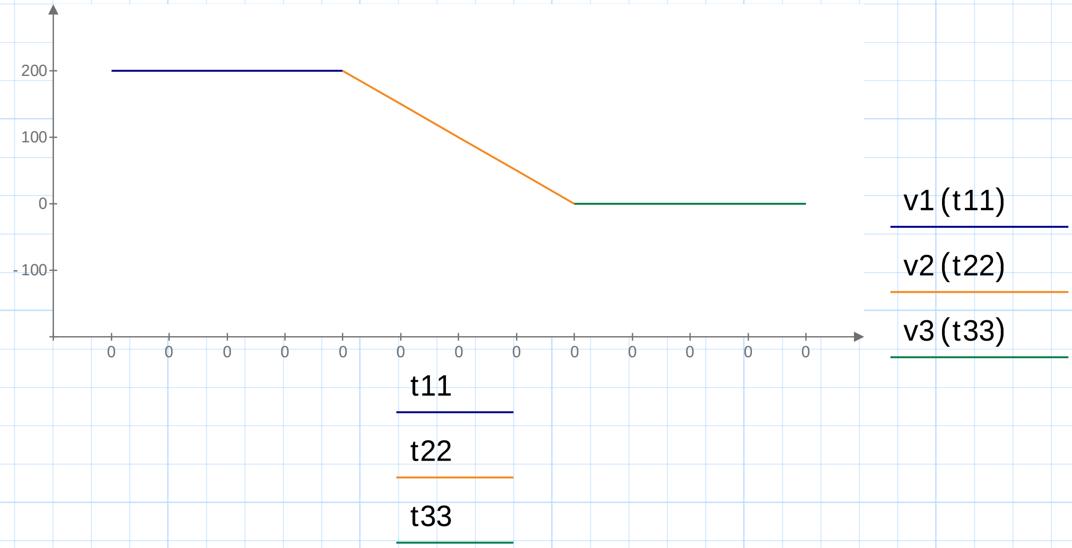


Voltage across resistor (20 ohms) Ri(t):

$t11 := 0, -0.00025 .. -0.001$ $\quad v1(t11) := 10 \cdot 20$

$t22 := 0, 0.00025 .. 0.001$ $\quad v2(t22) := \left(10 - \left(10^4 \cdot t22\right)\right) \cdot 20$

$t33 := 0.001, 0.00125 .. 0.002$ $\quad v3(t33) := 0$

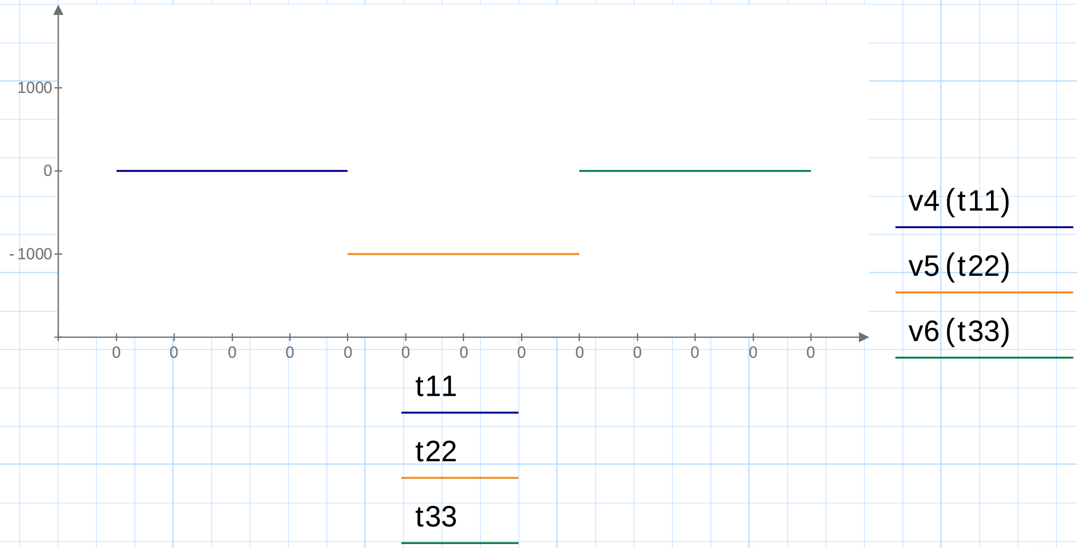


Voltage across inductor 100 mH vL =L (di/dt):

$t11 := 0, -0.00025 .. -0.001$ $\quad v4(t11) := \left(100 \cdot 10^{-3}\right)\ 0$

$t22 := 0, 0.00025 .. 0.001$ $\quad v5(t22) := \left(100 \cdot 10^{-3}\right) \cdot \left(-\left(10^4\right)\right)$

$t33 := 0.001, 0.00125 .. 0.002$ $\quad v6(t33) := 0$

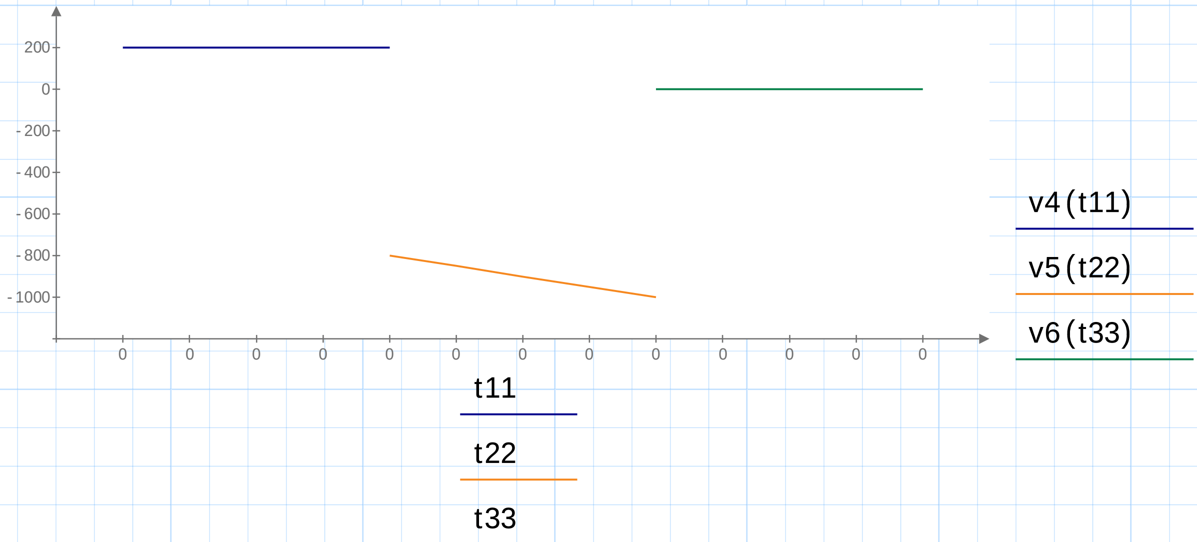


The resistor (200 Ohms) and inductor (100 mH) are in series.
So their voltages are? Added.
vRL = vR + vL.

$t11 := 0, -0.00025 .. -0.001$ $\quad v4(t11) := v1(t11) + v4(t11)$

$t22 := 0, 0.00025 .. 0.001$ $\quad v5(t22) := v2(t22) + v5(t22)$

$t33 := 0.001, 0.00125 .. 0.002$ $\quad v6(t33) := v3(t33) + v6(t33)$



Resistor voltage has the same shape as the source current.
The scaling, ratio of one to the other, by a factor of 20. An observation made in Schaums.

Continued on next page.

Circuit Component's Waveform: Capacitor

Discussion on Capacitor:

1. The current through a capacitor is zero if the voltage across it is not changing with time. A capacitor is therefore an open circuit to dc.
2. A finite amount of energy can be stored in a capacitor even if the current through through the capacitor is zero, such as when the voltage across it is constant.
3. It is impossible to change the voltage across a capacitor a finite amount in zero time, for this requires an infinite current through the capacitor. It will be advantageous later to hypothesize that such a current may be generated and applied to a capacitor, but for the present we shall avoid such a forcing function or response.
   A capacitor resists an abrupt change in the voltage in a manner analogous to the way a spring resists an abrupt change in its displacement.
4. The capacitor never dissipates energy, but only stores it.
   Although this is true for the mathematical model, it is NOT true for a physical inductor.

......reference from page 128 Engineering Circuit Analysis 4th ed Hyatt & Kemmerly.
Please check.

Capacitor Example 1:

A capacitor of 60uF has a voltage described as follows:
0 < t < 2ms,
v = 25.0 x (10^3)t V.

Sketch i, p, and w for the given interval and find Wmax.

Solution:

$C := 60 \cdot 10^{-6}$ F

$v(t) = 25 \cdot 10^{3} \cdot t$ V $\qquad \frac{dv}{dt} = 25 \cdot 10^{3}$

$i_c(t) = C \cdot \left(\frac{dv}{dt}\right)$

$i_c(t) = \left(60 \cdot 10^{-6}\right) \cdot \left(25 \cdot 10^{3}\right) = 1.5$ A. Answer.

$i_c(t) := 1.5$ Expression for current through capacitor.

$p = v \cdot i = 25 \cdot 10^{3} \cdot t \cdot 1.5 = 37.5 \cdot 10^{3} \cdot t$ W. Answer.

$$w_C = \int_0^t p\,dt = \int_0^t (37.5 \cdot 10^3 \cdot t)\,dt$$

$$= \frac{37.5}{2} \cdot 10^3 \cdot t^2$$

$$w_C = 18.75 \cdot 10^3 \cdot t^2$$ mJ. Answer.

$w_c(t) := 18.75 \cdot 10^3 \cdot (t^2)$ Expression for work.

A little deceptive or tricky, where would max work come from?
Its the time t when its maximum at 2 ms. Plug it in the equation above.

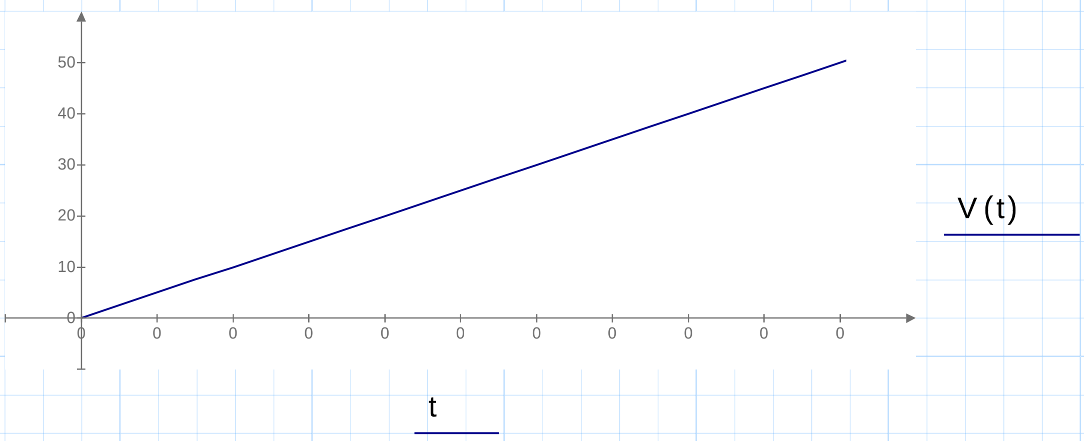
$W_{max} = (18.75 \cdot 10^3) \cdot (2 \cdot 10^{-3})^2 = 75 \cdot 10^{-3}$ J or 75 mJ. Answer.

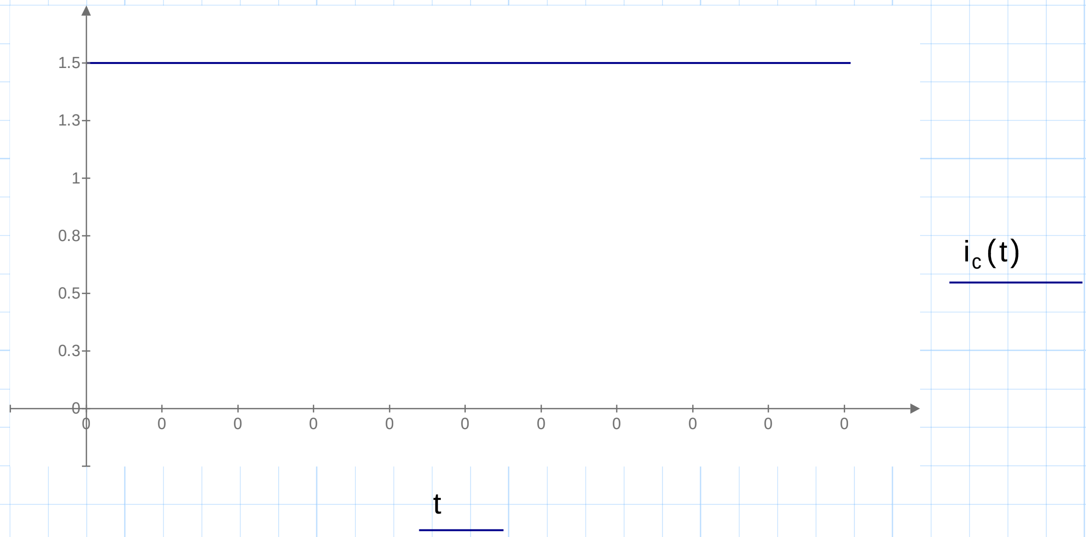
OR using the capacitor work expression (1/2 x C x V^2max):

$V = v \cdot t = 25 \cdot 10^3 \cdot t$ $\quad V(t) := 25 \cdot 10^3 \cdot t$

$V_{max} = v(t) \cdot t_{max} = 25 \cdot 10^3 \cdot (2 \cdot 10^{-3}) = 50$

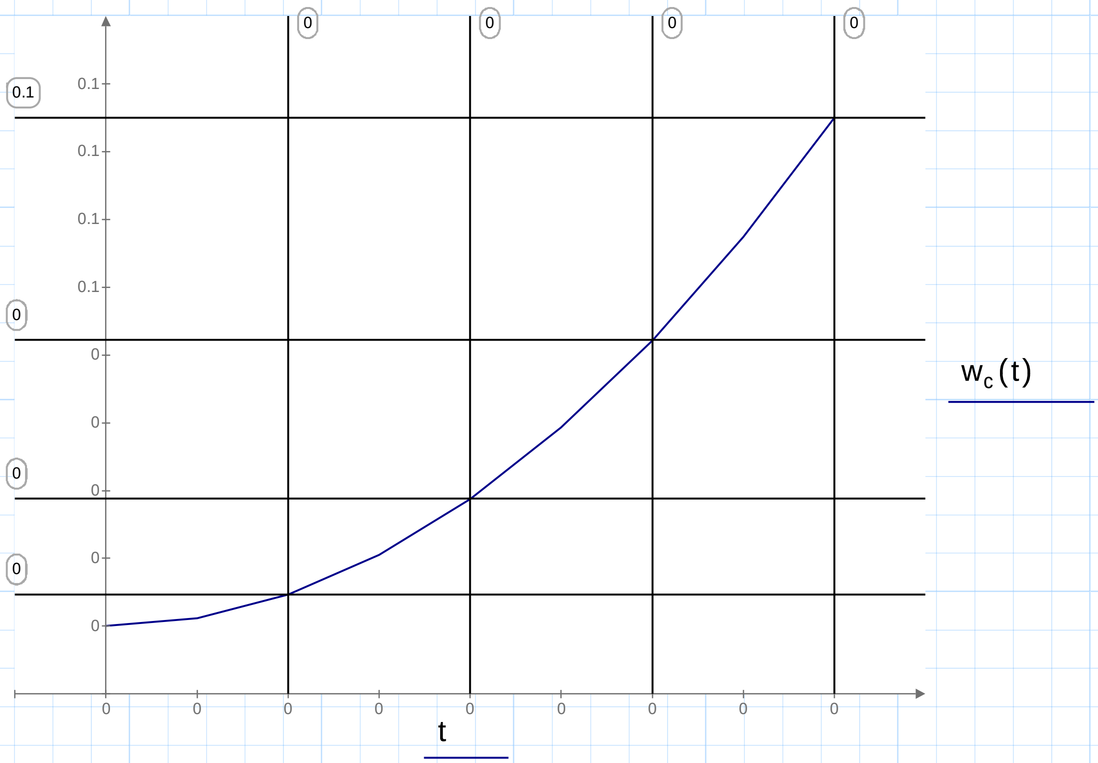
$W_{max} = \left(\frac{1}{2}\right) \cdot (60 \cdot 10^{-6}) \cdot (50^2) = 75 \cdot 10^{-3}$ J or 75 mJ. Answer.





clear(t)

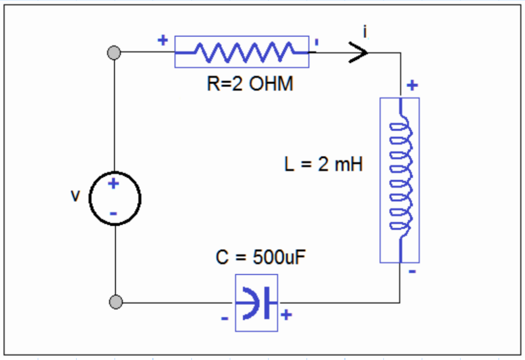
t := 0.0, 0.00025 .. 0.002

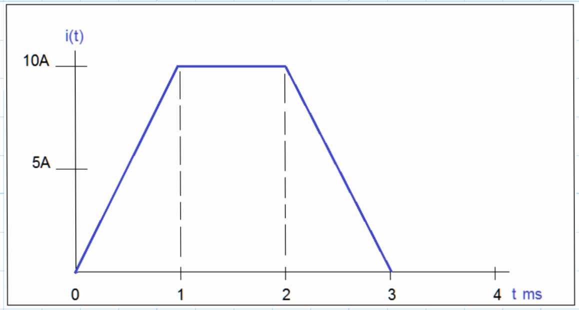


All plots completed.

Example Capacitor 2:

A series R L C circuit has R = 2 ohm, L = 2mH, and C = 500uF.
Has a current which increases linearly from 0 to 10A in the interval 0<= t <=1 ms,
remains at 10A for 1 ms<= t <=2 ms,
and decreases linearly from 10A at t =2 ms to 0 at t = 3ms.
Sketch v_R, v_L, and v_C.





Solution:

$R := 2$ Ohm $\quad L := 2 \cdot 10^{-3}$ H $\quad C := 500 \cdot 10^{-6}$ F

Hint: Is the voltage across the resistor dependent on anything other than v = iR? Yes, the current here is changing over a period, so the voltage would reflect the current over the interval. v_R is not dependent on time like L and C are di/dt and dv/dt respectively. The maximum voltage in any time interval would be the maximum voltage across the resistor, since regardless of interval v_R = iR.

Maximum current during interval 1 - 2 ms which is 10A.

$i_{max} := 10$ A.
$V_{max}R := i_{max} \cdot R = 20$ V.

Interval 0<= t <= 1 ms:

Lets construct the current, di/dt, through the inductor L:

$di := 10 - 0 = 10$ A. $\quad dt := 1 \cdot 10^{-3} - 0 = 0$ s

$$\frac{di}{dt} = \frac{10}{1 \cdot 10^{-3}} = 10000$$ A/s. Makes sense by definition. Creative, thought there was no way of getting some values with respect to time!

We may now calculate the voltage across the inductor for this interval:

$v\_L = L \cdot 1 \cdot 10^4 = 20$ V.

Capacitor voltage:

$$v\_C = \left(\frac{1}{C}\right) \cdot \int i \, dt$$

$$\frac{di}{dt} = 1 \cdot 10^4 \quad A$$

$$di = 1 \cdot 10^4 \cdot dt$$

$$\int 1 \, di = \int_0^t 1 \cdot 10^4 \, dt$$ <----plugs in the v_C equation

$$i = 1 \cdot 10^4 \cdot t$$

$$v\_C = \left(\frac{1}{C}\right) \cdot \int i \, dt$$ <----plugs in here

$$v\_C = \left(\frac{1}{C}\right) \cdot \int_0^1 1 \cdot 10^4 \cdot t \, dt$$

$$v\_C = \left(\frac{1}{2\,C}\right) \cdot 1 \cdot 10^4 \cdot t^2 = 1 \cdot 10^7 \cdot t^2 \quad V$$

At end of 1 ms what is the voltage across the capacitor?
Plug in 1ms for t^2.

$$v\_C = \left(\frac{1}{2 \cdot 500 \cdot 10^{-6}}\right) \cdot 1 \cdot 10^4 \cdot \left(1 \cdot 10^{-3}\right)^2 = 10 \quad V$$

Interval 1<= t <= 2 ms:

Here the current is constant. So we know we cant get any value from differentiating, because its 0. How now?

$$\frac{di}{dt} = 0$$

Voltage across inductor v_L = 0 = L (di/dt).

$v\_L = 0$ V

How do we work on the capacitor voltage?
Capacitor? Was there an existing charge built up before this time interval? No.
We just calculated 10V between 0-1 ms.

d1/dt in 1-2 ms when current is constant 10A.

$v\_C = \left(\frac{1}{C}\right) \cdot \int i\, dt$

$i = 10$ A

$v\_C = \left(\frac{1}{C}\right) \cdot \int_{1}^{t} 10\, dt$ <----plugs in the v_C equation
Keep upper limit t so that helps build the equation, no doubt the plot is not to exceed 2 ms for this expression.

$v\_C = \left(\frac{1}{C}\right) \cdot 10\ t$

Lim t: 1- t ms

$v\_C = \left(\frac{1}{C}\right) \cdot 10 \cdot (t) \cdot 10^{-3} = \left(2 \cdot 10^{3}\right) \cdot 10 \cdot (t-1) \cdot 10^{-3}$

Lim t: 1- t, with ms in the expression — Lim included (t-1)

$= 20 \cdot 10^{3}\ (t-1) \cdot 10^{-3}$

$= 20 \cdot 10^{3}\ \left(t - 1 \cdot 10^{-3}\right)$ <----Correct.

However, coming in from interval 0-1ms we have a capacitor voltage 10V, this needs to be added in to the interval 1-2 ms which is here now **1-t ms**.
Correct.

$v\_C_{1\_2ms} = 20 \cdot 10^{3}\ \left(t - 1 \cdot 10^{-3}\right) + 10$ V.

$v\_C_{1ms} = 20 \cdot 10^{3}\ \left(1 \cdot 10^{-3} - 1 \cdot 10^{-3}\right) + 10 = 10$ V.

$v\_C_{1.5ms} = 20 \cdot 10^{3}\ \left(1.5 \cdot 10^{-3} - 1 \cdot 10^{-3}\right) + 10 = 20$ V. at 1.5 ms

$v\_C_{2ms} = 20 \cdot 10^{3}\ \left(2 \cdot 10^{-3} - 1 \cdot 10^{-3}\right) + 10 = 30$ V. at 2 ms

This interval ends at 2 ms.

Interval 2<= t <= 3 ms:

This interval the values are in the opposite direction to interval 0-1 ms.
0-1 incline, 2-3 deciline at same gradient.
So could a minus sign on (0-1ms) treat the last interval? Should. According to Math but the capacitor voltage is higher at 2 ms than it was at 1 ms, we need to study this further. Its does not look like a straight opposite use of -ve sign.

Lets construct the current, di/dt, through the inductor L:

$di := 0 - 10 = -10$ A. $\quad dt := (3-2) \cdot 10^{-3} = 0$ s

$\frac{di}{dt} = \frac{-10}{1 \cdot 10^{-3}} = -10000$ A/s. Negative sign for declining gradient/slope.

We may now calculate the voltage across the inductor for this interval:

$v\_L = L \cdot -1 \cdot 10^{4} = -20$ V. Negative magnitude compared to 0-1 ms. Correctt.

Capacitor voltage:

$v\_C = \left(\frac{1}{C}\right) \cdot \int i\, dt$

$\frac{di}{dt} = -1 \cdot 10^{4}$ A

$di = -1 \cdot 10^{4} \cdot dt$

Capacitor voltage will build up (positive) with a negative current, its the charge per time, Q/t, magnitude which determines the voltage increase. So, we make it? Positive.

$\int 1\, di = \int_0^t 1 \cdot 10^{4}\, dt$ <----plugs in the v_C equation

$i = 1 \cdot 10^{4} \cdot t$ *For the plot use negative sign for decline slope.*

$v\_C = \left(\frac{1}{C}\right) \cdot \int i\, dt$ <----plugs in here

$v\_C = \left(\frac{1}{C}\right) \cdot \int_2^t \left(1 \cdot 10^{4} \cdot t\right) dt$

$v\_C_{2\_3ms} = \left(\frac{1}{2\ C}\right) \cdot \left(1 \cdot 10^{4}\right) \cdot t^2 = \left(1 \cdot 10^{7}\right) \cdot t^2$ V

Lim t: 2-3ms Lim t: 2-3ms

At end of 2 ms in the 1-2ms interval the capacitor voltage calculated was 30V.
Carry forward 30V to the next interval v_C_2-3ms.

Lets calculate the capacitor voltage at end of 3 ms:

$$v\_C_{2\_3ms} = \left(\left(\frac{1}{2\ C}\right)\cdot\left(1\cdot10^{4}\right)\cdot\left(3.0\cdot10^{-3} - 2.0\cdot10^{-3}\right)^{2}\right) + 30 = 40 \text{ V. Correct.}$$

What about at 2.5ms? We divide the interval by 2 for half the time then add 30V.

$$v\_C_{2\_3ms} = \left(\left(\frac{1}{2\ C}\right)\cdot\left(1\cdot10^{4}\right)\cdot\frac{\left(\left(3.0\cdot10^{-3} - 2.0\cdot10^{-3}\right)\right)^{2}}{2}\right) + 30 = 35 \text{ V. Correct.}$$

Next we need to get our expressions in order so they plot!

clear (t1, t2, t3, t)   t := 0, 0.0005 .. 0.003

t1 := 0, 0.0001 .. 0.001   t2 := 0.001, 0.0011 .. 0.002   t3 := 0.002, 0.0021 .. 0.003

See plots on next page.

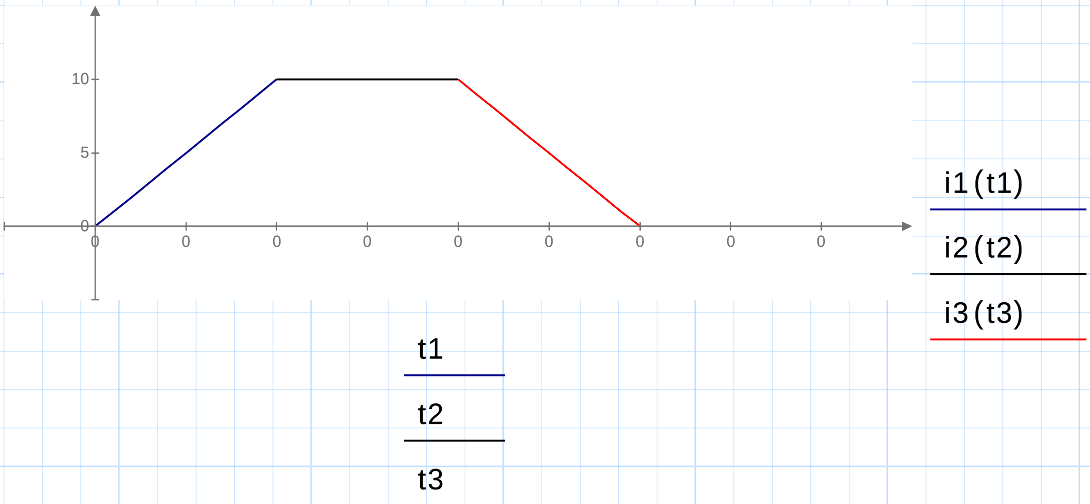
Circuit current plot:

$i1(t1) := 1 \cdot 10^4 \cdot t1 \qquad i2(t2) := 10$

We need to fix the plot expression for i3(t3) below because it starts at 10A:

$-1 \cdot 10^4 \cdot (0.002) = -20$ <---for i3(t3) we need to add 30 to get it to ?10.

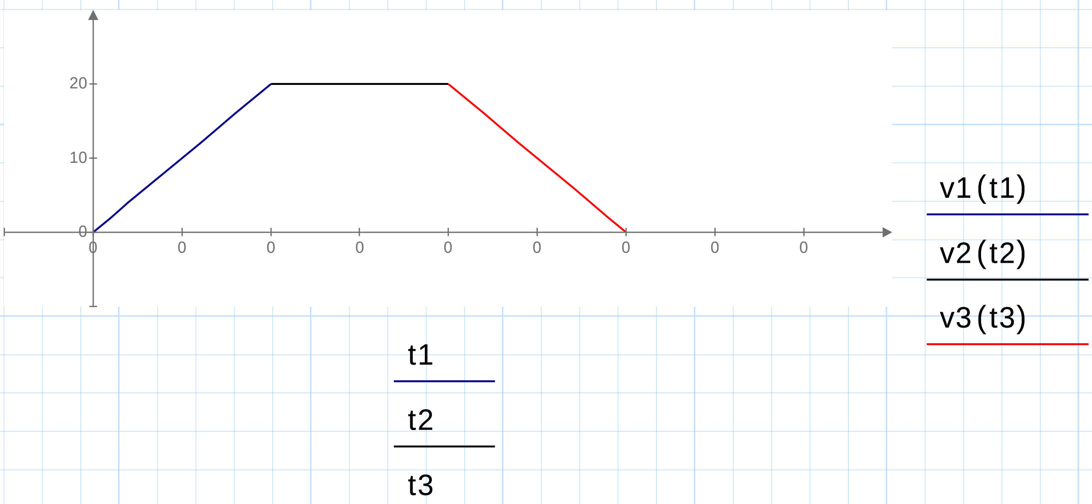
$i3(t3) := -1 \cdot 10^4 \cdot t3 + 30$



Resistor voltage plot:

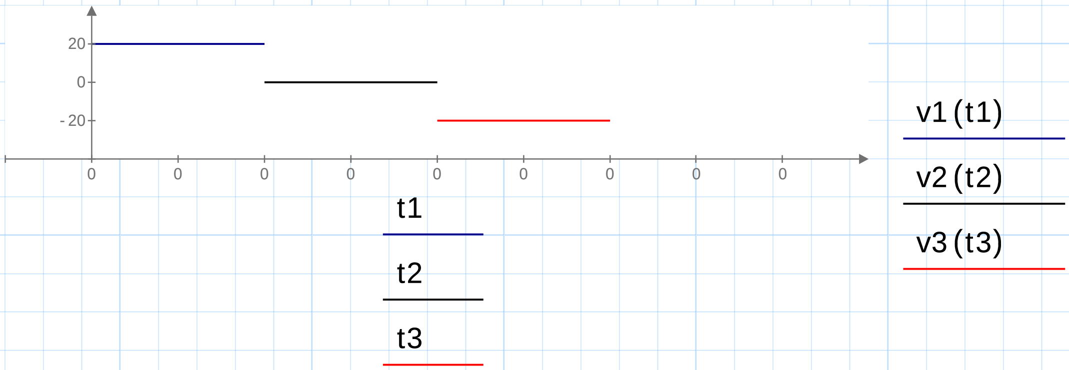
Multiply the circuit current equations by 2 ohm, as shown below.

$v1(t1) := 2 \cdot \left(1 \cdot 10^4 \cdot t1\right) \qquad v2(t2) := 2 \cdot 10 \qquad v3(t3) := 2 \cdot \left(-1 \cdot 10^4 \cdot t3 + 30\right)$



Correct. Study the two plots above.

Inductor voltage plot: $v1(t1) := 20$ $v2(t2) := 0$ $v3(t3) := -20$



Capacitor voltage plot:

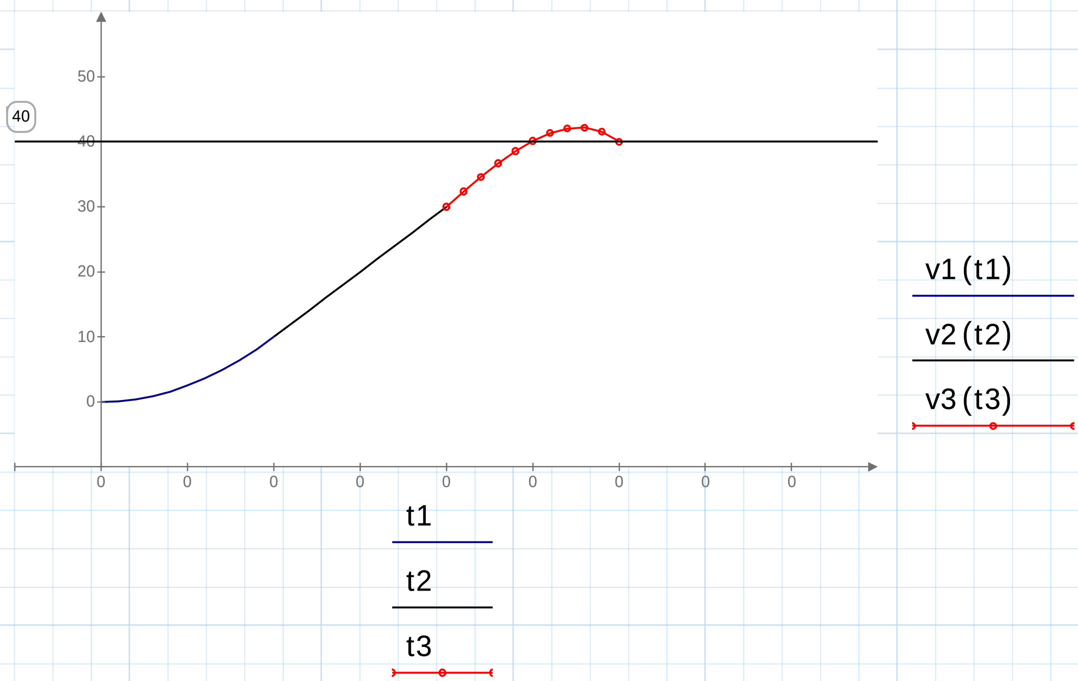
$v1(t1) := 1 \cdot 10^7 \cdot t1^2$ Next fix plot v2_C, first get the value for t=1ms then adjust.

$20 \cdot 10^3 \left(1 \cdot 10^{-3} - 1 \cdot 10^{-3}\right) = 0$ <--Need to start at 10, so we add? 10.

$v2(t2) := \left(20 \cdot 10^3 \left(t2 - 1 \cdot 10^{-3}\right)\right) + 10$ <--Add 10

$v3(t3) := \left(1 \cdot 10^7 \cdot \left(t3^2\right)\right) - 2 \cdot \left(5^{(t3 \cdot 1000) - 1.0}\right)$

This was not easy for me, because of the t3^2 term. You may do better. Not to exceed 40V peak. See next page.



How does the v_C plot look like? Below using stem plot, vertical lines, 3 points chosen.
v_C between 2 - 3 ms is a smooth curve.

$v3(t3) := \left(\left(1 \cdot 10^{7}\right) \cdot (t3)^{2}\right)$

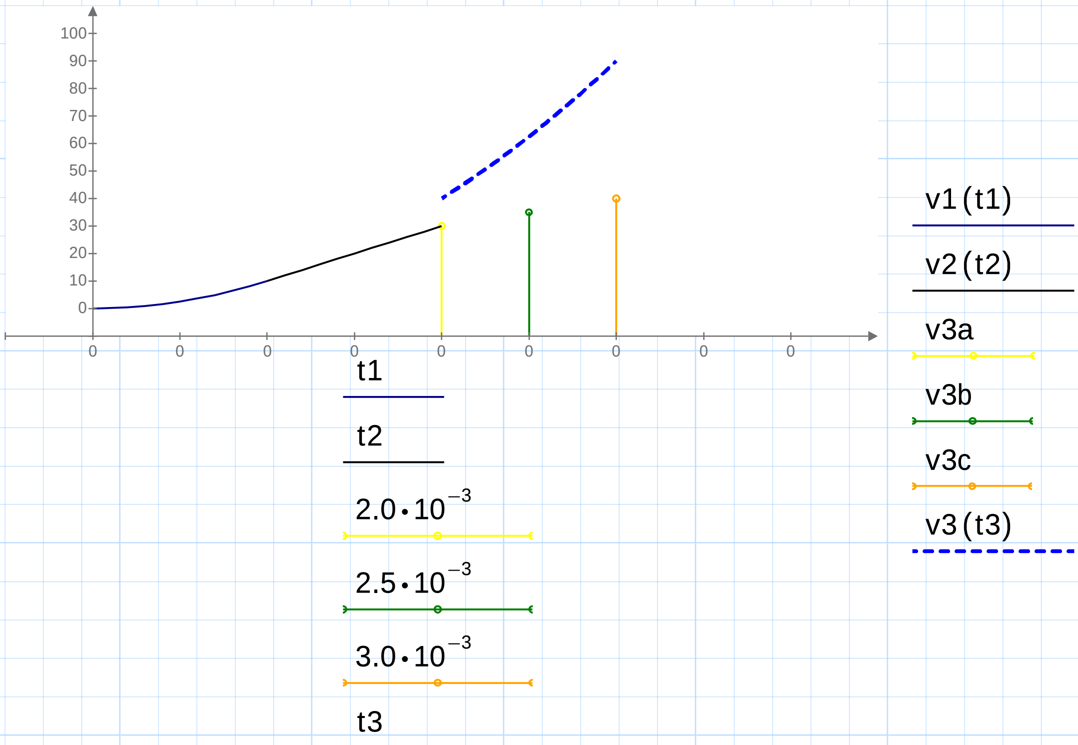
v3(t3) cannot fix with simple add, since there is a ? squared term. Shown in heavy blue dahsed. So we do a stem plot for the points.

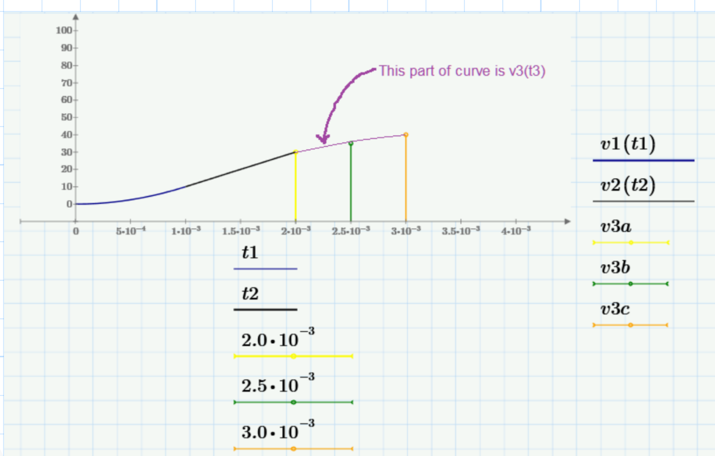
$v3a := \left(\left(1 \cdot 10^{7}\right) \cdot \left(2 \cdot 10^{-3}\right)^{2}\right) - 10 = 30$

$v3c := \left(\left(1 \cdot 10^{7}\right) \cdot \left(3 \cdot 10^{-3}\right)^{2}\right) - 50 = 40$

$v3b := \left(\left(1 \cdot 10^{7}\right) \cdot \frac{\left(\left(3 \cdot 10^{-3}\right)^{2} - \left(2 \cdot 10^{-3}\right)^{2}\right)}{2}\right) + 10 = 35$

Divided by 2, interpolate at 2.5ms

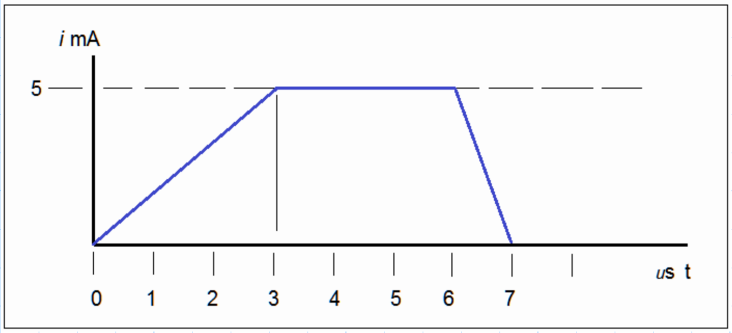




<---Textbook (Schaums).
You can manual-hand plot it using the point method.
Completed.

Example Capacitor 3 (Schaums Supplementary problem 2.21):

The current after t=0 in a single circuit element is as shown in figure below.
Find the voltage across the element at t = 6.5 us.
If the element is:
a). a resistor of 10 k ohm
b). an inductor 15 mH
c). a 0.3 nF capacitor with Q(0) = 0



Solution:

We are given the current graph over a period of 7 microseconds.
There are 3 stages of the current over 3 intervals.
It is increasing, constant, and decreasing.
We need to set or form a current expression in each stage whereby the specific element will be able to use it. This is similar to the previous. *Good exercise.*

Interval 0 - 3 ms:

$R := 10 \cdot 10^{3}$ Ohm $\quad L := 15 \cdot 10^{-3}$ H $\quad C := 0.3 \cdot 10^{-9}$ F

$i_{max} := 5 \cdot 10^{-3}$ A

$di = 5 \cdot 10^{-3} - 0 = 0$ A. or 5 mA Increasing.

$dt = 3 \cdot 10^{-6} - 0 = 0$ second or 3 us.

$\frac{di}{dt} = \frac{5 \cdot 10^{-3}}{3 \cdot 10^{-6}} = 1666.7$ A/s

$v\_R := i_{max} \cdot R = 50$ V

$$v\_L = L\cdot\left(\frac{di}{dt}\right)$$

$$= L\cdot\left(\frac{5\cdot 10^{-3}}{3\cdot 10^{-6}}\right) = 25 \quad \text{V}$$

$$v\_C = \left(\frac{1}{C}\right)\int_0^t i\,dt$$

$$\frac{di}{dt} = 1.67\cdot 10^{3} \quad \text{A/s}$$

$$di = 1.67\cdot 10^{3}\cdot dt$$

$$\int 1\,di = \int 1.67\cdot 10^{3}\,dt$$

$$i = \left(1.67\cdot 10^{3}\right)\cdot t$$ Solved for i, slope equation.

$$v\_C = \left(\frac{1}{C}\right)\int_0^t \left(1.67\cdot 10^{3}\right)\cdot t\,dt$$

$$\int_0^t \left(1.67\cdot 10^{3}\right)\cdot t\,dt = \left(\frac{1.67}{2}\right)\cdot 10^{3}\cdot t^{2} = 835\cdot t^{2}$$

$$v\_C = \left(\frac{1}{C}\right)\cdot 835\cdot t^{2} = \left(2.78\cdot 10^{12}\right)\cdot t^{2}$$

$$v\_C = \left(2.78\cdot 10^{12}\right)\cdot t^{2}$$ <--- This is plotted over the interval.

$$v\_C_{3ms} = \left(2.78\cdot 10^{12}\right)\cdot\left(3\cdot 10^{-6}\right)^{2} = 25$$ V at end of 3 ms.

Interval 3 - 6 us:

$$i_{3\_6us} = 5\cdot 10^{-3} \quad \text{A}$$

$$vR_{3\_6us} = 5\cdot 10^{-3}\cdot 10\cdot 10^{3} = 50$$ V, same current as i_max.

$$\frac{di}{dt} = \frac{0.005 - 0.005}{03\cdot 10^{-6}} = 0$$

Inductor current is constant at 5 mA, the derivative of a constant is 0, so the voltage across the inductor is 0.

$$v\_L_{3\_6us} = 0 \quad \text{V}$$

$i_{3\_6us} := 5 \cdot 10^{-3}$ A

$v\_C_{3\_6us} = \left(\frac{1}{C}\right)\int_{3\ us}^{6\ us} i\,dt = \left(\frac{1}{C}\right)\int_{3\ us}^{6\ us} \left(5 \cdot 10^{-3}\right) dt = 1.67 \cdot 10^{7} \cdot t$
Lim t: 3-6us

However, coming in from interval 0-3 *us* we have a capacitor voltage 25.02 V, this needs to be added in to the interval 3-6 *us*, at end of 6us, i.e. a 3 *us* interval.

$v\_C_{3\_6us} = 1.67 \cdot 10^{7} \cdot t + 25.02$ V.

$v\_C_{3us\_interval} = 1.67 \cdot 10^{7} \cdot \left(3 \cdot 10^{-6}\right) + 25.02 = 75.1$ V.

$v\_C_{6us} =$ 75.12 V. at end of 6 *us*.

Interval 6 - 7 ms:

$i_{max\_6us} := 5 \cdot 10^{-3}$ A

$i_{max\_6.5us} := 2.5 \cdot 10^{-3}$ at 6.5ms on the point in the graph i(6.5us) = 2.5mA

$di = 0 - 5 \cdot 10^{-3} = 0$ A. or 5 mA Increasing.

$dt = 7 \cdot 10^{-6} - 6 \cdot 10^{-6} = 0$ s or 1 us.

$\frac{di}{dt} = \frac{-5 \cdot 10^{-3}}{1 \cdot 10^{-6}} = -5000$

-5,000 A/s, -ve sign, for the capacitor is concerned with the magnitude, charge continues to build up on the capacitor plates, and is building capacitor voltage. Inductor may reveal a negative voltage, charge is not building on the inductor, rather current change di/dt. Current change with amplitude and sign.

$v\_R := i_{max} \cdot R = 50$ V same in all 3 intervals here for i_max but at 6.5 *us*? Next.

$v\_R_{6.5ms} := i_{max\_6.5us} \cdot R = 25$ V Answer.

$v\_L = L \cdot \left(\frac{di}{dt}\right)$

$v\_L_{6.5us} = L \cdot \left(-5 \cdot 10^{3}\right) = -75$ V Answer.

$v\_C = \left(\frac{1}{C}\right)\int_{6\ us}^{t} i\,dt$

Discussion: In a series circuit, the capacitor voltage is going to add with the other elements in the circuit. So polarity has to be in order, RLC series circuit would not have a negative voltage other than dependent on di/dt but would that not go back to rms? RMS later chapters, that the voltage and current are taken for their positive real values....averaged....

$\frac{di}{dt} = 5 \cdot 10^{-3}$ A/s, using absolute value, magnitude, positive sign.
If we took the di/dt from 6-6.5 same slope.

$di = 5 \cdot 10^{-3} \cdot dt$

$$\int 1\, di = \int 5\cdot 10^{-3}\, dt$$

$$i = \left(5\cdot 10^{3}\right)\cdot t$$ Solved for i, slope equation.

$$v\_C_{6\_7us} = \left(\frac{1}{C}\right)\int_{6\,us}^{7\,us}\left(5\cdot 10^{3}\right)\cdot t\, dt$$

$$\int \left(5\cdot 10^{3}\right)\cdot t\, dt = \left(\frac{5}{2}\right)\cdot 10^{3}\cdot t^{2} = 2500\cdot t^{2}$$

$$v\_C_{6\_7us} = \left(\frac{1}{C}\right)\cdot 2500\cdot t^{2} = \left(8.33\cdot 10^{12}\right)\cdot t^{2}$$ <--- This is plotted over the interval.

Add the carry forward capacitor voltage of 75.12 V.

$$v\_C_{6\_7us} := \left(8.33\cdot 10^{12}\right)\cdot\left(\left(7\cdot 10^{-6}\right)^{2} - \left(6\cdot 10^{-6}\right)^{2}\right) = 108.3$$

$$v\_C_{6.5us} := \frac{v\_C_{6\_7us}}{2} = 54.1$$

$$v\_C_{6\_6.5us} := \left(8.33\cdot 10^{12}\right)\cdot\left(\left(6.5\cdot 10^{-6}\right)^{2} - \left(6\cdot 10^{-6}\right)^{2}\right) = 52.1$$

The voltage is slightly lower in the 6_6.5us calculation.
We add the 75.12V.

$$v\_C_{final\_6.5us} := 75.12 + 52.06 = 127.2$$ V. Answer is different from Schaums their answer is 81.3V.

Discussion: In the first interval 0-3us, the C voltage was 25.02. This slope is lower in comparison to the 6-7us, here its steep. The voltage reduces to 0 but that does not reduce the voltage across the capacitor because its stored during this period. It only looses potential when the current supply is stopped. So here I calculated aproximately 52V which is twice the interval 0-3us. Maybe there is an error in the Schaums solution. Not likely so you check thru for corrections. I spent enough time and could not get that answer. Not likley from 75 to an increase of only 6 or 7 volts to 81.3V.

No plots required here. Luckily!
Looks like a stopping point on this chapter. Ok maybe 1 more exercise problem. Wait, managing the -ve sign and continuing with the reasoning can be a serious time consuming event.

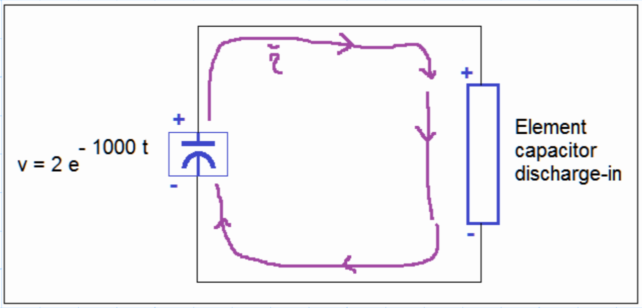
Next chapter a jump ahead to a section in chapter 8 of Schaums so we get an appreciation of the 's' complex frequency, eventually leads to Laplace circuit applications. Just a small part of chapter 8 not all of chapter 8. After which we begin with chapter 7 to finish it, and then get back on 8.

Here where I**, or we, **hope to complete the pre-requisite studies.

Example Capacitor 4
(Schaums Supplementary problem 2.27 followed by 2.28 continued):

A 10uF capacitor discharges in an element such that its voltage is v = 2e^(-1000t).
Find the current and power delivered by the capacitor as function of time.

Solution:



<---Capacitor discharges into element.

$C := 10 \cdot 10^{-6}$ F

$v(t) = 2\, e^{-1000\, t}$ V

$$v_C = \left(\frac{1}{C}\right) \int_0^t i\, dt$$

$$v_C \cdot C = \int_0^t i\, dt$$

To remove the integral on the left hand side we can differentiate both sides with respect to dt.

$$\frac{d\left(v_C \cdot C\right)}{dt} = \frac{d\left(\int_0^t i\, dt\right)}{dt} = i$$

Solving LHS should provide for i(t).

$$\frac{d\left(10 \cdot 10^{-6}\right)\left(2\, e^{-1000\, t}\right)}{dt} = (-1000) \cdot \left(10 \cdot 10^{-6}\right)\left(2\, e^{-1000\, t}\right) = i(t)$$

$$-(1000) \cdot \left(10 \cdot 10^{-6}\right) \cdot (2) = 0$$

$$i(t) = -0.02 \cdot e^{-1000\, t}$$

$i(t) = -20 \cdot e^{-1000\, t}$ mA Answer. Schaums answer was positive value.

Discussion: In our original expression, LHS is the gradient of a slope. The RHS was the area under the limits 0 to t. RHS: Integral sign with dt cancelled. RHS integral area's limits are Lim t: 0-->t. Taking it to the LHS, the reverse of it would be Lim t: t<--0; making it a negative sign on the LHS. So the LHS experiences a change of sign it beame negative. -i(t) = -20 e ^ -1000t, which by multiplying by -ve, made it positive. Otherwise the answer Schaum provided maybe in error not likely.

$i(t) = 20 \cdot e^{-1000\, t}$ mA

Power is the equation p = vi

Since we have both in time functions they can be multiplied.

$$p_{discharged}(t) = v(t) \cdot i(t)$$

$$= \left(2\, e^{-1000\, t}\right) \cdot \left(20 \cdot e^{-1000\, t}\right) \cdot 10^{-3}$$

$$= (2 \cdot 20) \cdot \left(e^{-1000\, t}\right) \cdot 10^{-3}$$

$$= 40 \cdot 10^{-3} \cdot \left(e^{-1000\, t}\right)$$

$$= 40 \cdot \left(e^{-1000\, t}\right) \quad \text{mJ}$$ Answer.

Example Capacitor 5
(Schaums Supplementary problem 2.28 continued):

Find voltage v, current i, and energy W in the capacitor of problem 2.27 at time t = 0, 1, 3, 5, and 10 ms.
By integrating the power delivered by the capacitor, show that the energy dissipated in the element during the interval from 0 to t is equal to the energy lost by the capacitor.

Solution:

Applying the expression achieved in 2.27 exmaple 4 capacitor, with the appropriate time t in ms, to solve for v and i. Then the work done by the capacitor C_w = (1/2)C (v^2) solves for the energy W, which is work lost by capacitor equal energy gained in the element. Refer to units in previous example.

t: 0

v	i	W
$2\, e^{-1000 \cdot (0)} = 2$ V	$20 \cdot e^{-1000 \cdot (0)} = 20$ mA	$\left(\frac{1}{2}\right) \cdot C \cdot 2^2 = 20 \cdot 10^{-6}$ uJ

t: 1 ms

v	i	W
$2\, e^{-1000 \cdot \left(1 \cdot 10^{-3}\right)} = 0.7358$ V	$20 \cdot e^{-1000 \cdot \left(1 \cdot 10^{-3}\right)} = 7.3576$ mA	$\left(\frac{1}{2}\right) \cdot C \cdot 0.7358^2 = 0$ uJ

Continued next page.

t: 3 ms

v

$$2\,e^{-1000\cdot\left(3\cdot 10^{-3}\right)} = 100\cdot 10^{-3}\ \text{V}$$

100 mV

i

$$20\cdot e^{-1000\cdot\left(3\cdot 10^{-3}\right)} = 0.9957\,\text{mA}$$

W

$$\left(\frac{1}{2}\right)\cdot C\cdot(0.1)^2 = 0\ \text{J}$$

0.05 uJ

t: 5 ms

v

$$2\,e^{-1000\cdot\left(5\cdot 10^{-3}\right)} = 13.5\cdot 10^{-3}\ \text{V}$$

13.5 mV

i

$$20\cdot e^{-1000\cdot\left(5\cdot 10^{-3}\right)} = 0.1348\,\text{mA}$$

135 uA

W

$$\left(\frac{1}{2}\right)\cdot C\cdot(0.0135)^2 = 9.1\cdot 10^{-10}$$

0.001 uJ
aprox.

t: 10 ms

v

$$2\,e^{-1000\cdot\left(10\cdot 10^{-3}\right)} = 90.8\cdot 10^{-6}\ \text{V}$$

91 uV

i

$$20\cdot e^{-1000\cdot\left(10\cdot 10^{-3}\right)} = 0.0009\ \text{mA}$$

0.9 uA

W

$$\left(\frac{1}{2}\right)\cdot C\cdot\left(90.8\cdot 10^{-6}\right)^2 = 4.1\cdot 10^{-14}$$

0 J
aprox.

In 10 ms the work done by the capacitor is all dissipated into the element. No work left available by the capacitor. Similarly the voltage and current decreased to micro volt and micro amp. The size of the capacitor was 10uF. Good example.

Adequate, there are many different problems available to solve in Schaums Outline, and your engineering textbook. You may continue should you choose too.

Next chapter the reason why these study notes came about, then picking up with chapter 7 and end with chapter 8 of Schaums Electric Circuits.

Chapter 3: 2nd Order Series and Parallel RLC Circuits - The S.

Where The Differential Equations Came From?

Pull out the circuits textbook, the expressions for inductor L and capacitor C have the differential and integral forms. These are in these forms, hence the circuit equations take on similar forms at 1st and 2nd order expression. The resulting equation usually is a 2nd order equation for the typical cases, same here at 2nd order.

I do not know under what condition a circuit could be of a higher order when only these elements R L and C are involved. What are commonly known as passive elements or basic electrical elements. I have not come across higher than 2nd order equations, that I remember, so lets say for now its of no concern. Sure someone can show them to you and cite the pages of the engineering textbook. But we are NOT interested. Not now.

"95% of ENGINEERS in the work place rarely solve 2nd order quadratic equations". Maybe based on my electrical construction experience NOT electronic circuit manufacturing, hi-tech circuits, so you verify.

Series and Parallel 'RLC or RC or RL or LC' circuits result with a forms of math expressions, the 's' complex frequency is applied to solve them expressions.

Electric Circuits are EXACTLY like human beings, dependent on the kind of input applied into the electric circuit a particular kind of result (behaviour) is experienced at the output. Treat you with the right action you react positively. Same for electric circuits. .....Karl Bogha. <--- Maybe one day in an electric circuits textbooks.

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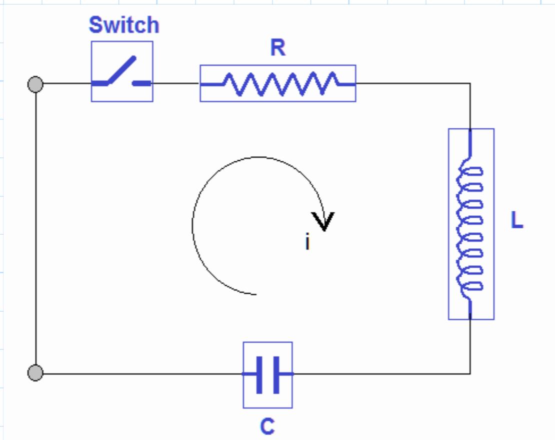
Series RLC:



There is NO voltage source in the circuit to the left.

That is obviously NOT normal.

However, we know capacitors discharge when the switch is turned off, and for a short duration discharges current into the circuit.

Though with no volt source, yet we can write applicable equations for anlysis, whilst the volt source can be inserted later.

Kirchoff conservation of voltage applied to the series RLC electric circuit:

Conservation? May not be the most appropriate choise of word, but we get tired of too many LAWS in engineering...excessive. Later we may say Norton's conservation of current at the electric circuit node. Leave it for the serious engineer to use the word law. Sounds to heave 'law'.

$$v_R + v_L + v_c = 0$$

voltage circuit; voltage loop equation.
Equal zero because there is no voltage source.

$$Ri + L\cdot\left(\frac{di}{dt}\right) + \left(\frac{1}{C}\right)\int i\,dt = 0$$

CORRECT.

$$Ri + L\cdot\left(\frac{di}{dt}\right) + \left(\frac{1}{C}\right)\int i\,dt = 0$$

differenting wrt dt

$$R\cdot\frac{di}{dt} + L\cdot\left(\frac{di^2}{dt^2}\right) + \left(\frac{1}{C}\right)\cdot i = 0$$

may look awkward but thats the result, just pull out the intergral symbol.

$$L\cdot\left(\frac{di^2}{dt^2}\right) + R\cdot\frac{di}{dt} + \left(\frac{1}{C}\right)\cdot i = 0$$

rearranging for a 2nd order equation
2nd: di^2/dt^2, 1st: di/dt, constant: i.

Above equation is good so why do we divide it by L?
Because we get R/L in the 2nd term and LC term in the 3rd term?
Or is it because the first term has unity (1) for the coefficient? Yes!
You ask your local engineer. It probably for the 1 coefficient. L multiplied by C results in nothing significant, same for R divided by L.

$$\left(\frac{di^2}{dt^2}\right) + \left(\frac{R}{L}\right)\cdot\frac{di}{dt} + \left(\frac{1}{L\cdot C}\right)\cdot i = 0$$

dividing by L

Differential Equation (DE) has a solution for the above form of expression.

$$\left(\frac{di^2}{dt^2}\right) + \left(\frac{R}{L}\right)\cdot\frac{di}{dt} + \left(\frac{1}{L\cdot C}\right)\cdot i = 0$$

For the equation above we want to solve for i.

$A1 := 1 \quad A2 := 1 \quad s1 := 1 \quad s2 := 1 \quad t := 1$ *<---Declaration for text editing not computing else we see red rectangle over the text/variable.*

$$i := A1\cdot e^{s1\cdot t} + A2\cdot e^{s2\cdot t}$$

How do we bring in the circuit equation into the solution for i?

DE has that next step, pull out Maths textbook. If need.
Not necessary to re-view just look over.
Done it once should not need going over steps proofs and truths every time and time again.

$$s1^2 \quad ==> \quad \left(\frac{di^2}{dt^2}\right)$$

$$s1 \quad ==> \quad \left(\frac{di}{dt}\right)$$

$$\text{Constant} \quad ==> \quad i$$

$$s^2 + \left(\frac{R}{L}\right)\cdot s + \left(\frac{1}{L\cdot C}\right) = 0$$ CORRECT.

Next plug-in, most favourite moment for the engineer, and for a good reason arriving to an answer-solution.

$$A1\cdot\left(s_1^{\ 2} + \left(\frac{R}{L}\right)\cdot s_s + \left(\frac{1}{L\cdot C}\right)\right) + A2\cdot\left(s_2^{\ 2} + \left(\frac{R}{L}\right)\cdot s_2 + \left(\frac{1}{L\cdot C}\right)\right) = 0$$ CORRECT.

What DE is saying is

$$s_1 = s_1^{\ 2} + \left(\frac{R}{L}\right)\cdot s_1 + \left(\frac{1}{L\cdot C}\right)$$

$$s_2 = s_2^{\ 2} + \left(\frac{R}{L}\right)\cdot s_2 + \left(\frac{1}{L\cdot C}\right)$$

In other words s1 and s2 are the roots of: $s^2 + \left(\frac{R}{L}\right)\cdot s + \left(\frac{1}{L\cdot C}\right)$ Right on the answer.

Remember we are dealing with complex frequency,
NOT a typical or usual environment in circuits in electrical construction engineering.

Next we see something like a solution to a quadratic equation

$$s_1 = -\left(\frac{R}{2\ L}\right) + \sqrt{\left(\frac{R}{2\ L}\right)^2 - \left(\frac{1}{LC}\right)} = -\alpha + \beta$$

$$s_2 = -\left(\frac{R}{2\ L}\right) - \sqrt{\left(\frac{R}{2\ L}\right)^2 - \left(\frac{1}{LC}\right)} = -\alpha - \beta$$

Where

$$\alpha = \left(\frac{R}{2\ L}\right)$$

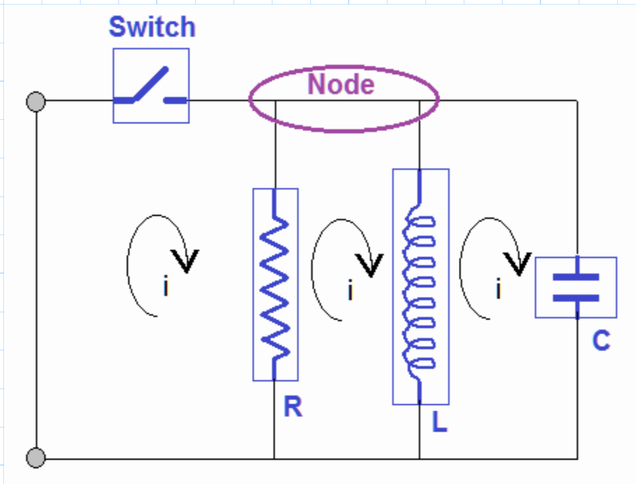
$$\beta = \sqrt{\alpha^2 - (\omega_0)^2}$$

$$\omega_0 = \left(\frac{1}{\sqrt{LC}}\right)$$

End of the case of Series RLC electric circuit.
Please note the other engineering discipline like Process Chemical.....also use the same technique for solving their variables.

Next page the Parallel RLC circuit.
Maybe some steps need not be explained as they maybe similar.

Parallel RLC:



Parallel RLC circuit, here we want to solve for voltage because the voltage is the same across the parallel branches. At the Node the voltage would be the same where all three passive elements R L and C are connected.

We use the Norton's node equation for conservation of current!

When the switch is closed, current flowing into the node generates a voltage, at node identified in circuit, the voltage is the same. Sum of current of each branch of the three elements would sum to total circuit current.

$$v_{node} = v$$

$$i_R + i_L + i_C = i$$

$$\left(\frac{v}{R}\right) + \left(\frac{1}{L}\right)\cdot\int_0^t v\,dt + (C)\cdot\left(\frac{dv}{dt}\right) = 0$$ Equal zero because there is no voltage source.

$$\left(\frac{1}{R}\right)\cdot\frac{dv}{dt} + \left(\frac{1}{L}\right)\cdot v + (C)\cdot\left(\frac{dv^2}{dt^2}\right) = 0$$ differentiating

$$(C)\cdot\left(\frac{dv^2}{dt^2}\right) + \left(\frac{1}{R}\right)\cdot\frac{dv}{dt} + \left(\frac{1}{L}\right)\cdot v = 0$$ rearranging

$$\left(\frac{dv^2}{dt^2}\right) + \left(\frac{1}{RC}\right)\cdot\frac{dv}{dt} + \left(\frac{1}{LC}\right)\cdot v = 0$$ dividing by C to make the first term coefficient 1.

$$s1^2 \;==>\; \left(\frac{dv^2}{dt^2}\right)$$

$$s1 \;==>\; \left(\frac{dv}{dt}\right)$$

$$\text{Constant} \;==>\; v$$

$$s^2 + \left(\frac{1}{RC}\right)\cdot s + \left(\frac{1}{L\cdot C}\right) = 0$$ CORRECT.

Next plug-in, most favourite moment for the engineer, and for a good reason arriving to an answer-solution.

$$A1\cdot\left(s_1^{\ 2} + \left(\frac{1}{RC}\right)\cdot s_s + \left(\frac{1}{L\cdot C}\right)\right) + A2\cdot\left(s_2^{\ 2} + \left(\frac{1}{RC}\right)\cdot s_2 + \left(\frac{1}{L\cdot C}\right)\right) = 0$$ CORRECT.

What DE is saying is

$$s_1 = s_1^{\ 2} + \left(\frac{1}{RC}\right)\cdot s_1 + \left(\frac{1}{L\cdot C}\right)$$

$$s_2 = s_2^{\ 2} + \left(\frac{1}{RC}\right)\cdot s_2 + \left(\frac{1}{L\cdot C}\right)$$

In other words s1 and s2 are the roots of: $s^2 + \left(\frac{1}{RC}\right)\cdot s + \left(\frac{1}{L\cdot C}\right)$ Right on the answer.

Remember we are dealing with complex frequency,
NOT a typical or usual environment in circuits..

Next we see something like a solution to a quadratic equation

$$s_1 = -\left(\frac{1}{2\ RC}\right) + \sqrt{\left(\frac{1}{2\ RC}\right)^2 - \left(\frac{1}{LC}\right)} = -\alpha + \sqrt{\alpha^2 - \omega_0^{\ 2}}$$

$$s_2 = -\left(\frac{R}{2\ L}\right) - \sqrt{\left(\frac{R}{2\ L}\right)^2 - \left(\frac{1}{LC}\right)} = -\alpha - \sqrt{\alpha^2 - \omega_0^{\ 2}}$$

Where

$$\alpha = \left(\frac{1}{2\ RC}\right)$$ different from Series RLC

$$\omega_0 = \left(\frac{1}{\sqrt{LC}}\right)$$ same as Series RLC

Seen the series and parallel RLC this isnt a one stop solve all circuits. For RL its got its own, same for RC....LC series or parallel. RLC maybe look like the all complete, maybe, so that is why these are usually ALWAYS shown in textbooks. Reality is ...... far too many circuits and dependent also on input source types.

Reference from Schaum's Outline and other electric circuits textbooks. Look at the textbook in your hands.

Under damped (Oscillatory), Critically damped, and Over damped:

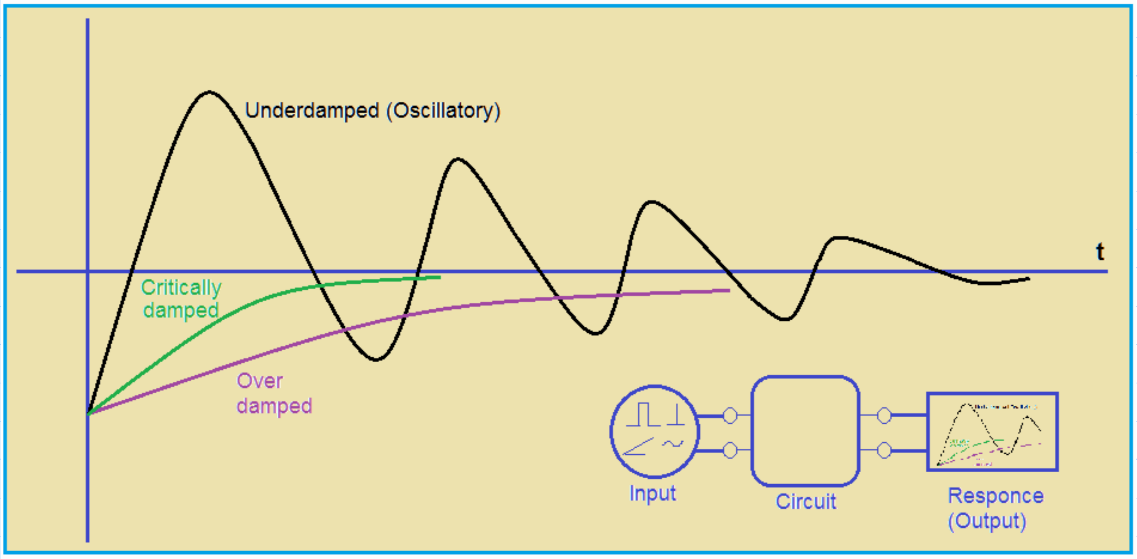


Figure above shows, dependent on input, dependent on circuit, the output responce are three cases; under, critical, and over damped.

The **CORRECT** way to read it, again, "Depending on the type of input, a particular circuit may respond in one or several ways."

Case	Series RLC	Parallel RLC
Under damped: (Oscillatory)	$\alpha < \omega_0$	$\alpha^2 < \omega_0^2$
Critically damped:	$\alpha = \omega_0$	$\alpha = \omega_0$
Over damped:	$\alpha > \omega_0$	$\alpha^2 > \omega_0^2$
α :	$\left(\frac{R}{2L}\right)$	$\left(\frac{1}{2RC}\right)$
ω_0 :	$\left(\frac{1}{\sqrt{LC}}\right)$	$\left(\frac{1}{\sqrt{LC}}\right)$

Comments: *It looks like in under damped the response is loose, not tight, up & down oscillating, maybe not reliable. Over damped its making an effort, no where near loose, but critically damped has a linear region, tighter, at the beginning before it settles to zero. Observations made purely based on the curves, application will/may decide which is suitable.*