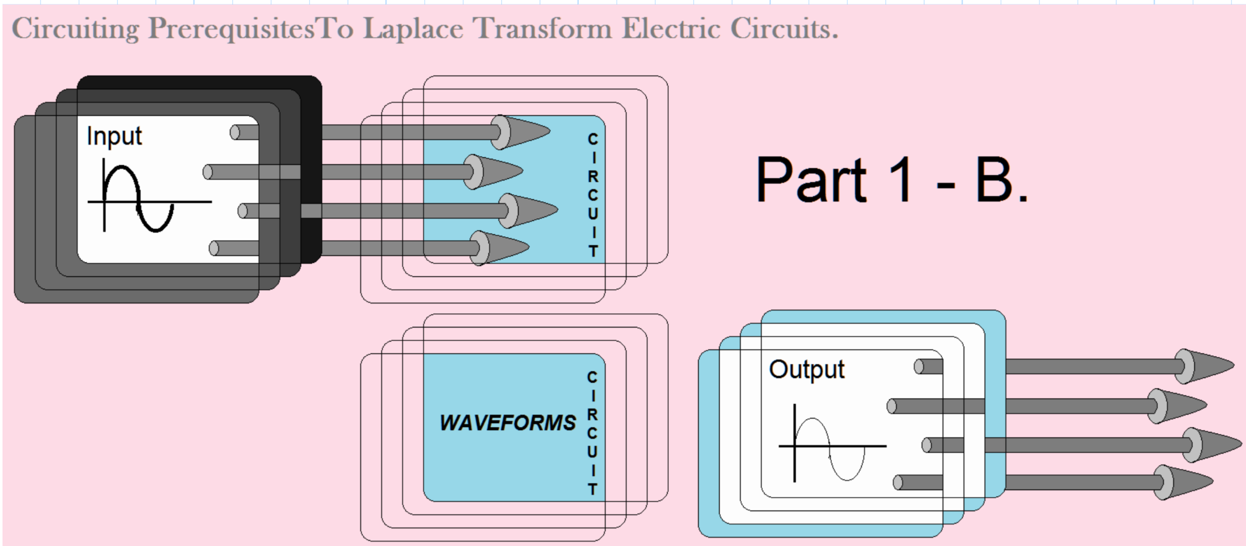
Engineering Circuits Analysis Notes And Example Problems - Schaums Outline 6th Edition.
My Homework. This is a pre-requisite study for Laplace Transforms in circuit analysis. Source of study material: Electric Circuits 6th Ed., Nahvi & Edminister. Engineering Circuit Analysis, Hyatt & Kimmerly 4th Ed. McGrawHill.
Karl S. Bogha.

Part 1 - B.
1 Chapter: Chapter 4 on circuit input waveforms and what to expect for output waveforms.
Basic To Intermediate.



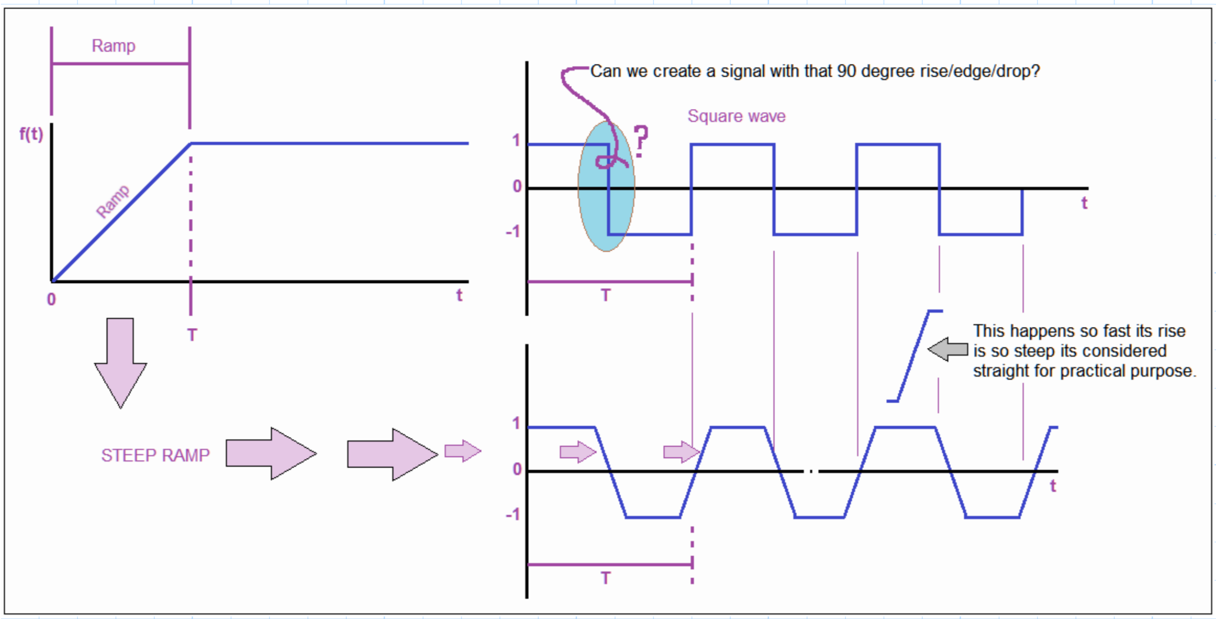
April 2020.

Engineering college year 2 course of 4 year program OR year 1 of 3 year program. Re-fresher OR Self Study. Graduate Study Review.
May be used in New Zealand, US, Malaysia, India, Pakistan, UK, and other Common Wealth Country engineering colleges.
Any errors and omissions apologies in advance.

Electric Circuit's **Input Waveforms** And What To Expect For **Output** OR **Response**.

Earlier notes (Chapters) seen the time delay (t-tau) waveform plot to the right of the base or first waveform, (t-tau) plots to the right of v(t).

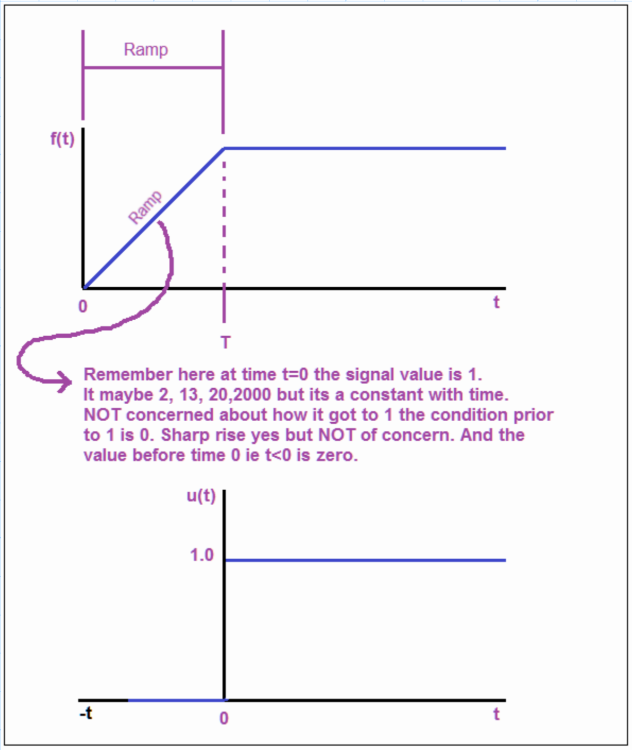
Lets revisit waveform, define the unit step,unit impulse functions.
These are the input source waveform we use here. Revisit because its in earlier chapters of textbook and maybe explained later at mid-point.
Tricky, yes, but having to remember them..... is maybe better if I/we try to build them up slowly.



Maybe if I said nothing goes from 0 to 1 without passing 0.1....0.5....0.7...0.9 that may not be sufficient but now with the figure above, the general idea is improved. So in a sense we are looking at things in a parctical sense practical value, what value is it to consider the steep rise or steep drop, because that rise or drop time period is insignificant to the circuit performance. <---Karl Bogha. *Maybe found in a textbook in the future.*

Lets review sections 6.8-6.10 so that it assists in section 7.10. The section 7.10 comes in hopefully next notes.

Continued on next page.



How we describe using the math language?

At time negative t the value is 0
At time positive t the value is 1

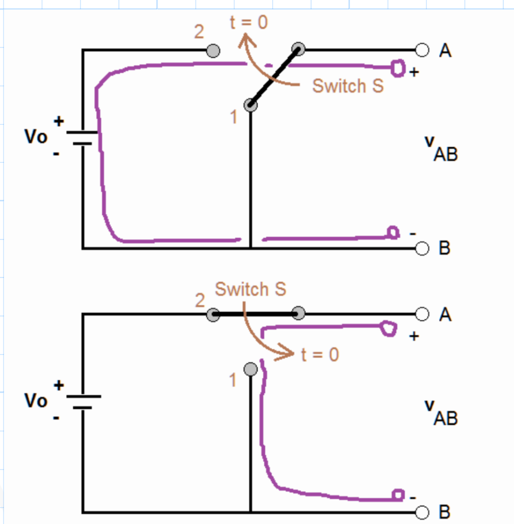
Here it has to be ONE because its UNITY STEP FUNCTION.

Notice the word 'step' its like stepping up a stairs, yes thats where it came from, make things easy. It has to be u(t) = 1 when t>0.

$$u(t) \quad \begin{array}{lll} = & 1 & \text{for } t = 0 \text{ and } t>0. \\ = & 0 & \text{for } t < 0. \end{array}$$

Lets work thru examples, 6.14, 6.15, and 6.15 all connected. Make it one example here. Just to show how this u(t) can be expressed in EE language. *I don't speak it ver well.*

Example 6.14-6.16.



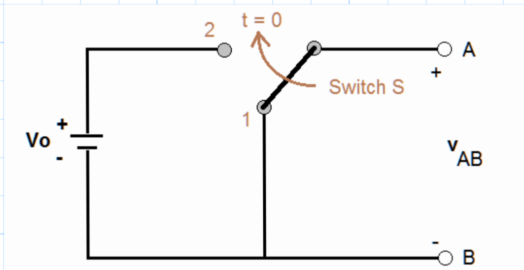
6.14.
Switch is closed in to position 2 at time t = t0. Top circuit.

Express v_AB using the step function.

t0 is NOT time t = 0.
We assumed t_0 would be time t = 0, no, not uncommon in using t0 to intent a time afer t =0.
It may read as the first action in a sequence of actions started at t0...t1....t2.....

Vo is the amplitude in the Unit Step function. If Vo = 12, then for t>0 = 1 (ON) x 12 = 12V.

Schaums page 126: The appearance of Vo across the terminals A-B is delayed until t = t0.



Before t0 the circuit is open and the voltage seen on terminals A-B is 0V.

After some time t0 the switch is closed, this is the time the circuit sees as the start of time t at t= 0.

So we have time t0 no voltage ie at t<0, and at time t=0 when circuit sees a voltage. The difference in time t=0 and t0 is the time delay.

t_0 : t <0

t : t = 0 switch closed in to position 2.

$$time\_delay = t - t_0$$

What happens in this time period? Voltage seen at A-B is 0 until its time t.

$u(t)$	=	1	for t = 0 and t>0.
	=	0	for t < 0.

Starts here at t0 with time running toward t =0

$u(t)$	=	1	for t = 0 and t>0.
	=	0	for t < 0.

Then it moves to at t=0
we have a voltage on terminals A-B.

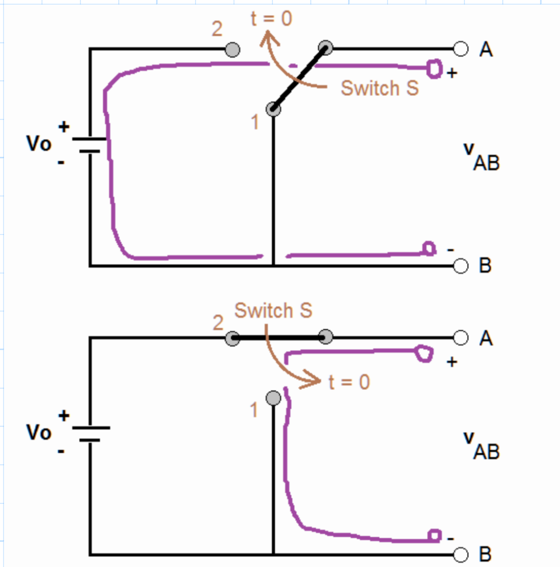
Now something is a little different enough to be a major difference.
In the second arrow case its not t>0 its just at t=0 and we have a voltage and this leads us to NOT have to include t>0 in the time delay expression above, shown below again.

$$time\_delay = t - t_0$$

This time delay results with voltage ON across terminals A-B.

So, now if we do the expression shown below, you see that it has addressed the switch in position 2 at t=0, with the time delay, and the voltage Vo appearing/manifesting across it. Since the u(t) is shown below it says pass t>0 its unit value, ON, ie Vo.

$$v_{AB}(t) = V_0 \cdot u(t - t_0)$$ Answer.



6.15

Now move down to the lower circuit in figure. At time t = 5 the switch is closed in to position 1.

Voltage ON at time t<5.
When t=5 switch is closed to position 1, there is no voltage present.

Opposite to circuit above; from ON to OFF.

How do we write the expression for the circuit voltage with respect to time?

Could we reverse the time delay? That would not account for the 5 second already passed.

$$time\_delay = t - t_0$$

$$time\_delay = t_0 - t$$ Where is the 5 seconds shown? Not there.

Discussion: Instead if we start at t =0, voltage ON, then we have at t=5 seconds later voltage OFF, we have t = 0 + 5 but this does not account for t<0. Which t<0 is OFF for the usual step function. Does it relate to the TOP CIRCUIT.

Do we need to show top circuit actions in the bottom circuit? NO.

The unit step function has two conditions to meet t<0, AND t=0 and t>0.
Does this apply? Maybe not at 0 rather at 5!

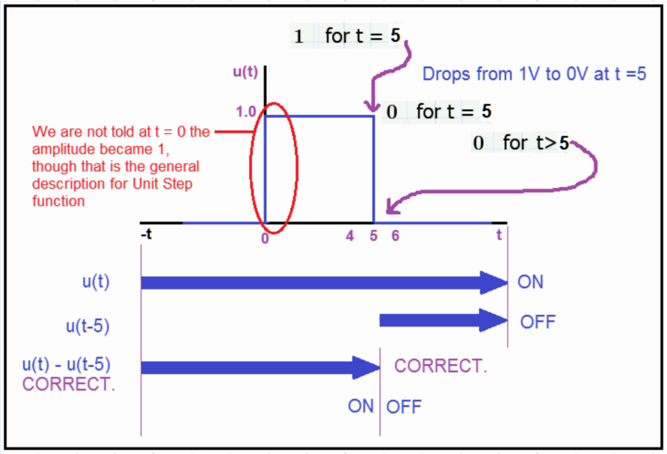
If the time of concern is 0-5 seconds is ON, 5 and onward is OFF, how do we show this?

$(t-5)$: time at and past 5 going to 6s......(t>5) - OFF

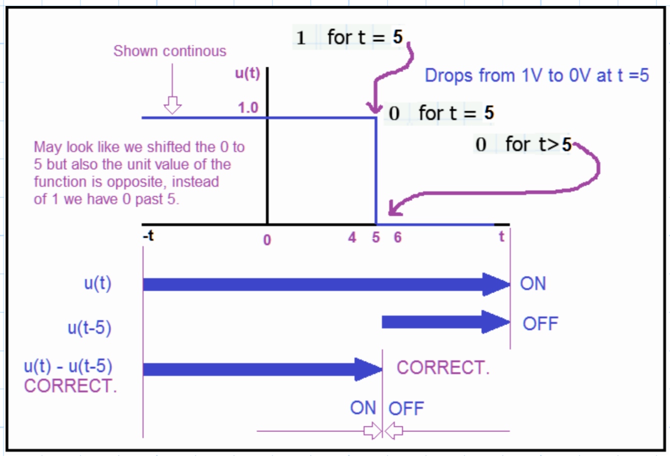
(t) : time t for all time from t< 0 to t=0, then it moves up to t = 5 --> is ON

If it were around t = 0 the usual way, <0 = OFF, > 0 = ON
now we moved past 0 to 5 and reversed the ON to OFF
so we have 5< = ON, >5 = OFF.

Continued next page with a sketch on the solution.



<---Wrong.



Top sketch NOT the same with bottom the bottom is correct. We made the assumption that it turned ON at 0 in the top, not so.

Answer provided by Schaums Outline:

$v_{AB}(t) = V_0 \cdot (u(t) - u(t-5))$ Answer.

Intepretated below:

$V_0 \cdot (u(t) - u(t-5))$:
- 1 t<5 (...and t = 5 sharp drop)
- 0 t = 5 and t >5

To continue wrestling with the expression was futile.
The answer captures what happens at t =5.
So the key or main point is focus at the time t when the switch occurs.
Not an easy solution. So refer to it again and again and again and.......again...
Its not opposite to the first circuit in the solution rather attention to t = 5 but that still would NOT SOLVE it for me I needed to study the solution and then sketch.
If thats yet WRONG correct it your selves. You wrestle/grapple/?....with it.

Lets move OR travel on to the next to the last of the 3 part.

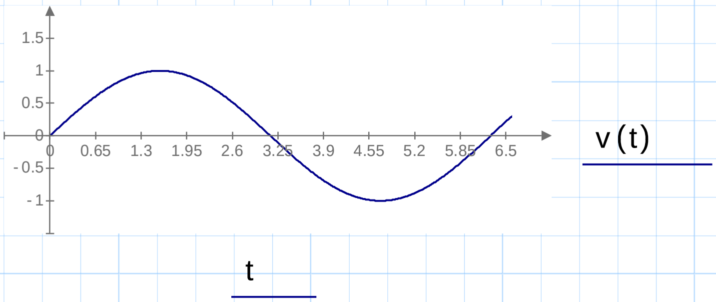
6.16

Express v(t) graphed in the figure below using the step function.

clear (t)

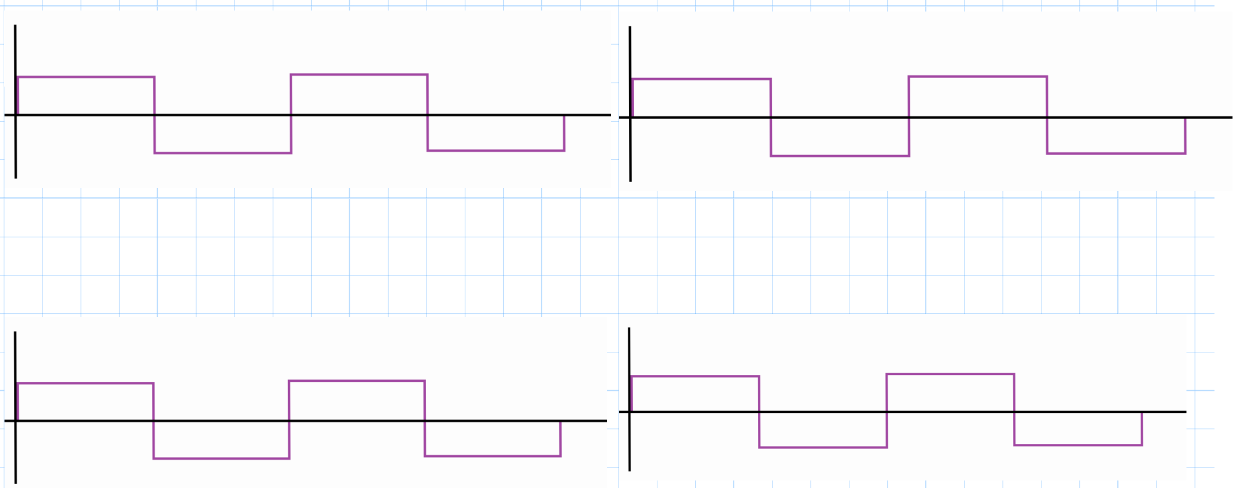
$t := 0, 0.01 .. 10\,\pi$

$v(t) := \sin(t)$



<---Graph of v(t)

Discussion with the assistance of sketches below:



Is the problem localised to the time range 0 to 2 Pi? Assume yes.

Maybe the solution is to make the step function '1' for the positive side of the wave for values from 0 to Pi and '0' OR '-1' for the remaining half.

Why trouble ourselves further, this solution nor the problem, were created in a few seconds, when they were first thought up?

Let's review the solution and work from there. Futile for me to tacke this.

Having done it I say its a really good learning outcome. Tricky yet simple!

$v(t) = (u(t) - u(t - 2\cdot\pi)\ \sin(t)):$ 1 t>? / 0 t <? <---solution provided.

This can be graphed OR maybe graphed depending on what youre looking for.

$(t - 2\ \pi)$ <--- This sets the time interval between 0 and 2 Pi? No.
Its the time past t>2 Pi.
Which is not in our consideration, we assumed it between 0 - 2 Pi.
So past 2 Pi this may be 0; u(t>2 Pi) = 0.

$u(t)$ <--- The first term on the RHS is for all t. But our function starts at t =0.
So we may assume till proven wrong that t starts at 0 to where?
At t = Pi our function changes to negative, which may be -1 0R 0.
But does that make the step function 1 or 0.
At t = 2 Pi, the first term reaches the end of the limit.

If we subtitue values for t in the functions u(t)'s below this may
lead some where and later multplied to sin (t).

Some simple 'if else' loops applied.

$t_{start} := -10\cdot\pi = -31.416$

$t_{end} := 10\cdot\pi = 31.416$

$t := t_{start} .. t_{end}$

```
u1(t) := if t ≥ 0
           || u1 ← 1
         else if t < 0
           || u1 ← 0
```

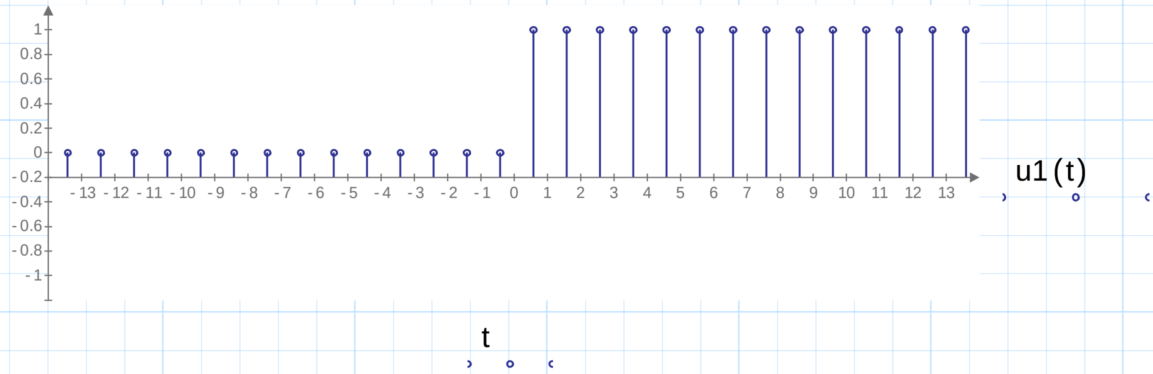
```
u2(t) := if (t − 2·π) ≥ 0
           || u2 ← 1
         else if (t − 2·π) < 0
           || u2 ← 0
```

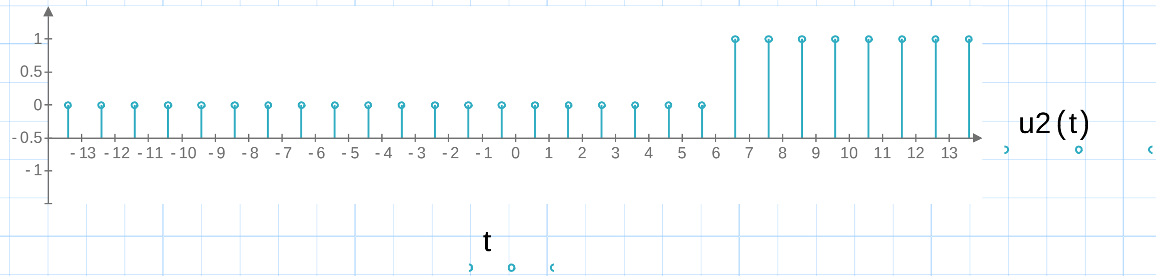
$u3(t) := (u1(t) - u2(t))$

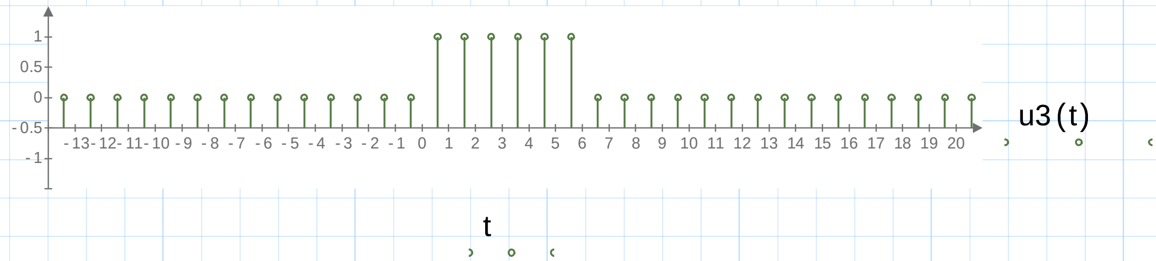
$v(t) := (u1(t) - u2(t))\cdot\sin(t)$

Lets plot each one of the functions so we can see what is occuring
these are functions that make up the solution.

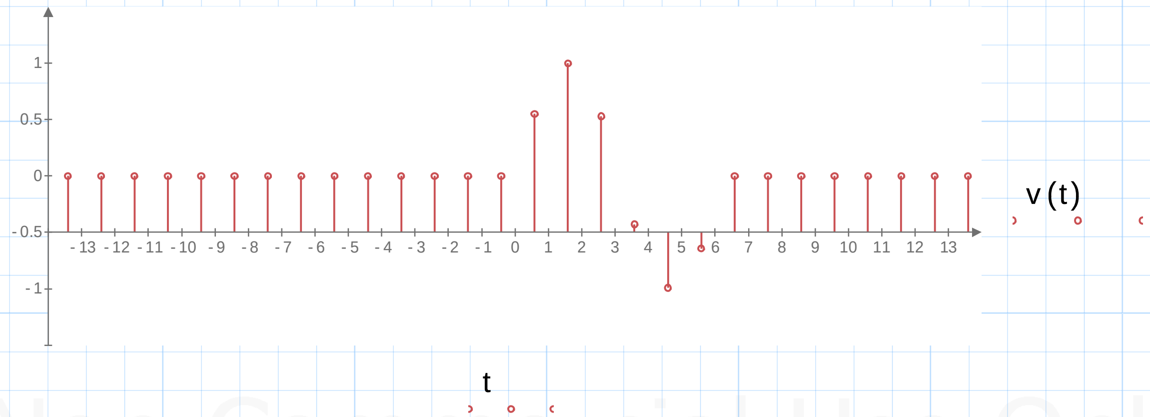
We use the stem plot, vertical line, this line appears between 2
numbers, so let say for 5 its between 5 and 6. Here, below its between
two consecutive numbers.

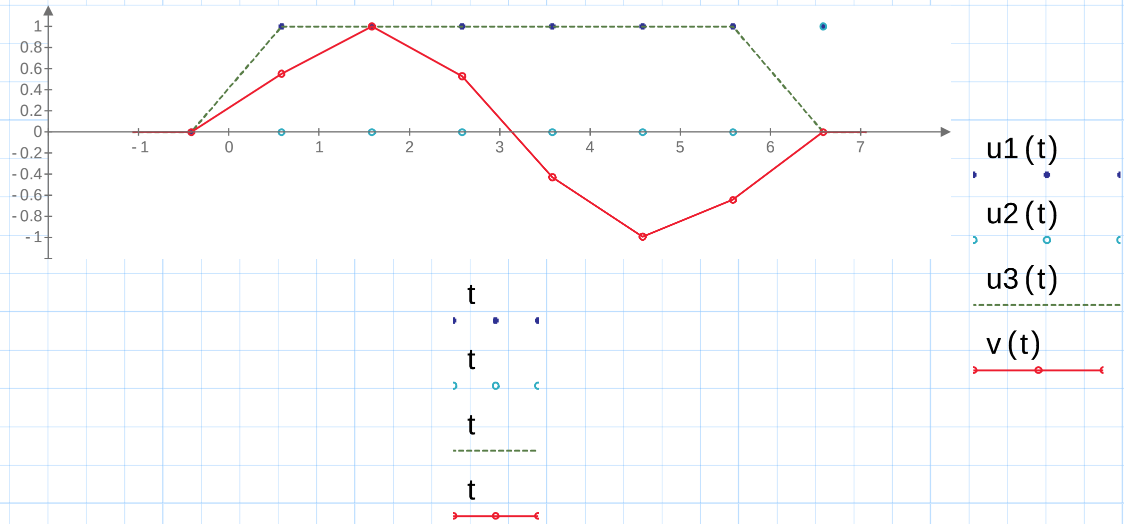






u(3) is the winner shows exactly what we were looking for, and same with v(t) the sine curve. Looks like the interval 0 to 2 Pi is the range we should be looking at.





All three plots in the graph above. The dotted slope goes from 0 to 1.
The stemp plots of u1 and u2 shows the difference we were searching for, and their subtraction leads to POSITIVE 1. u3 results in positive 1, which when multiplied to sin(t) shows the curve in the range 0 - 2Pi, elsewhere all 0. Legend is the right side of the graph the variable name, color, and symbol/line.

Trick?

$$t-(t-2\cdot\pi) \quad = \quad t-t+2\pi \quad = \quad 2\cdot\pi$$

So whatever was set to the result was always 2 Pi.
Is this logical? No, its the step function for u(t) its result being a 1 in the range 0 - 2Pi.
But, yes whatever t was its the same t in both u1 and u2 expression. t - t + 2Pi was not what led to the soution though. But is interesting to note this would apply in general.

Remember we had to consider the STEP FUNCTION of u1 and u2, 0 and 1, which if you see in the expression u3 results in 1 for the range 0 - 2Pi.

Step function did not make the sinusoidal term sin(t) equal 1 all the way through or 0, but that its curve was able to appear between 0 and Pi.

What if it was t - (t - 4 Pi)?
We would get the range stretching from 0 to 4 Pi.
It would be 2 cycles of the sine wave coming thru with the rest 0.
I think I dont want to plot it you may try. Seems its like a filter. It was able to get something thru with the rest blocked out. Maybe.

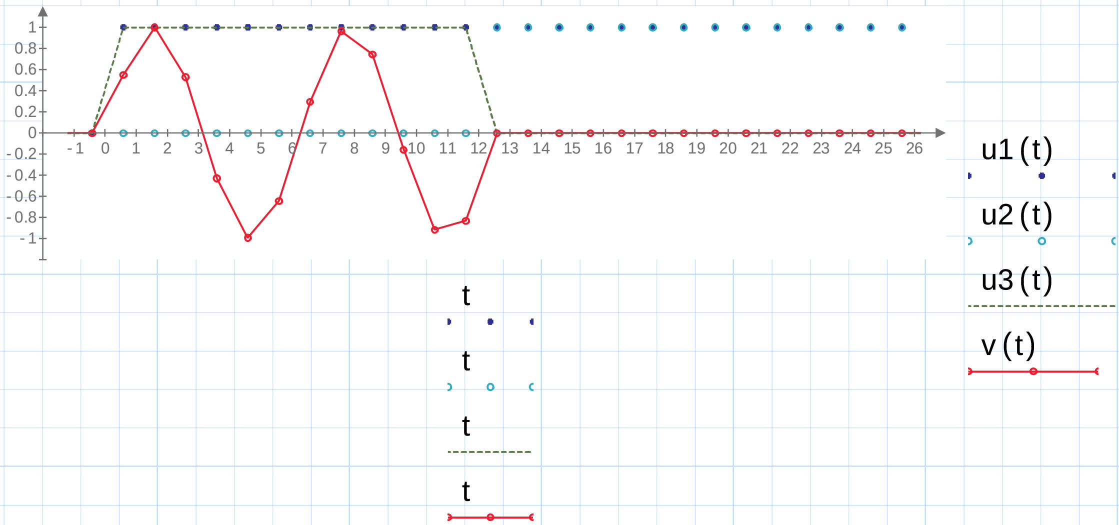
Ok the plot next page.

$$u1(t) := \begin{cases} u1 \leftarrow 1 & \text{if } t \geq 0 \\ u1 \leftarrow 0 & \text{else if } t < 0 \end{cases}$$

$$u2(t) := \begin{cases} u2 \leftarrow 1 & \text{if } (t - 4 \cdot \pi) \geq 0 \\ u2 \leftarrow 0 & \text{else if } (t - 4 \cdot \pi) < 0 \end{cases}$$

$$u3(t) := (u1(t) - u2(t))$$

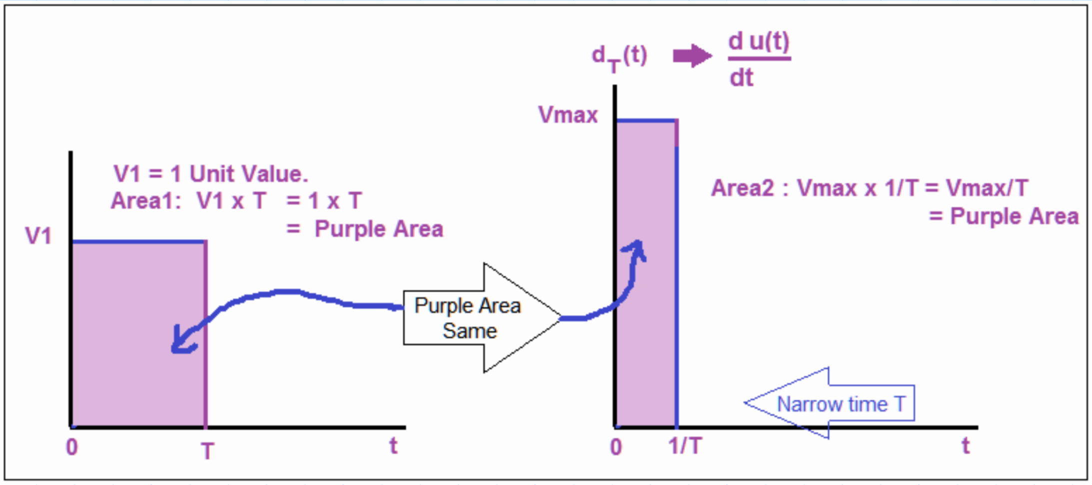
$$v(t) := (u1(t) - u2(t)) \cdot \sin(t)$$



Looks like its exactly that range 0 - 4 Pi with the rest set to 0.

Three good learning examples. They were hard for me. Seemed easy.
Going forward I may have a better attempt on the problems coming my way.

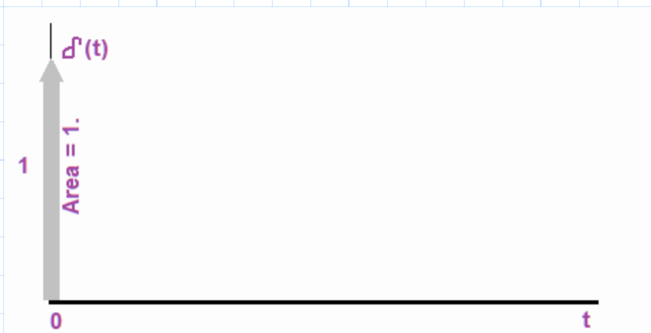
Next the Unit Impulse Function.



Important to study the figure on top right.

The graph above right derived from the left here the derivative was taken on u(t).
Area remains the same but the width of the right pulse is much narrower.

What happens when the square looking shape, pulse, on the right is squeezed so the x-axis time is reduced from T seconds to 1 second? 1/T....0.001 s which is 1 ms....1 us... Its going to get much thinner and taller in height. The shaded area would have to remain the same, area need remain equal 1. u(t) y-axis equal 1, and dT(t) much higher many times over say infinite. This then is called Unit Impulse OR Unit Delta function.



Unit Impulse
OR
Unit Delta

An impulse which is the limit of a narrow pulse with an area A is expressed by A d(t).

The magnitude A is sometimes called the strength of the impulse. A unit impulse which occurs at t = t0 is expressed by *d* (t - t0). *t0 is not 0 some value on the time axis.* Expression defining impulse/delta function:

$$d_T(t) = \begin{cases} 0 & \text{for } t < 0 \\ \frac{1}{T} & \text{for } 0 < t < T \\ 0 & \text{for } t > T \end{cases}$$

Sudden jump instantaneously, you can see its going to help in many of the sudden change events with a very high amplitude. Appears too many times in EE higher courses. So review it again and again and again....again.

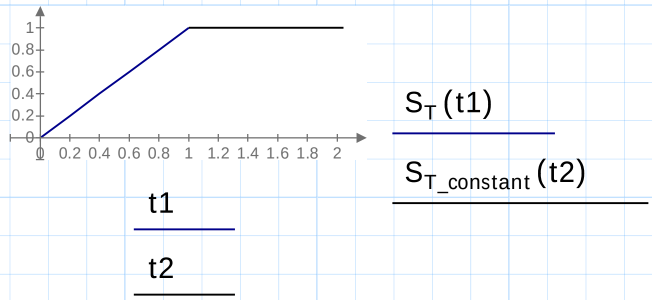
Example 6.17:

The voltage across the terminals of a 100-nF capacitor grows linearly, from 0 to 10V.
Taking the shape of the functions S_T(t) in the figure shown below.

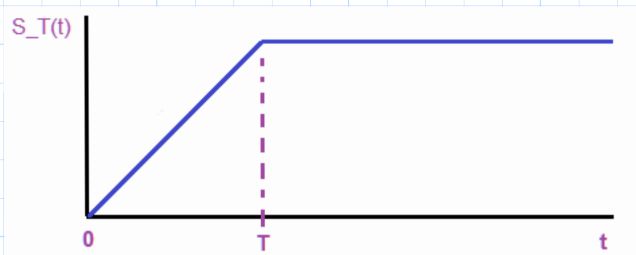
Find
(a). The charge across the capacitor at t = T
(b). Current i_C(t) in the capacitor for T = 1s, T = 1ms, and T = 1 us.

$S_T(t1) := t1$ $\qquad$ $S_{T\_constant}(t2) := 1$

$t1 := 0, 0.2 .. 1.0$ $\qquad$ $t2 := 1, 1.2 .. 10$



For this example graph on the left, time axis t1 = 1 = T.
So at T = 1, S_T(t1) = 1.



<--- Notes on top right are trying to say this.
<---- This is the voltage! increasing linearly.

Solution:

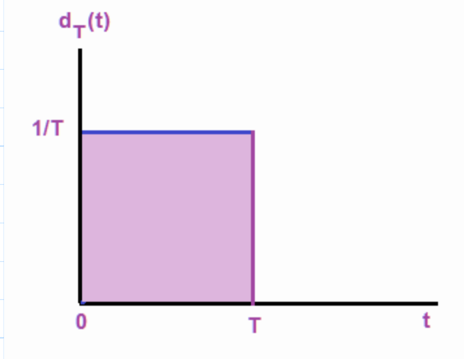
$C1 := 100 \cdot 10^{-9}$ F $\qquad$ $v_{C\_final} := 10$ V

a). C = Q/v_c, capacitance = charge across capacitor plates/voltage across capacitor.

$$Q := C1 \cdot v_{C\_final} = 1 \cdot 10^{-6} \quad \text{C (Coulomb)}$$

b). Q = C v_C, by differentiating Q, becomes charge per unit time, that is current i(t),
so we differentiate both sides.

$$i(t) = C \cdot \frac{d(v_C)}{dt}$$



We seen this figure before.
The differentiated graph.

$$i(t) \quad = \quad C \cdot \frac{d(v_C)}{dt}$$

<--The differentiated voltage multiplied by C in a time period t = 1s = T.

We set T1 = 1 s:

$T1 := 1$

$$Q\_1s := \frac{C1 \cdot v_{C\_final}}{T1} = 1 \cdot 10^{-6}$$ Coulomb per second = 1micro Amp. Answer.

For T2 = 1 ms:

$T2 := 0.001$

$$Q\_1s := \frac{C1 \cdot v_{C\_final}}{T2} = 0.001$$ Coulomb per second = 1mA. Answer.

For T3 = 1 us:

$T3 := 10^{-6}$

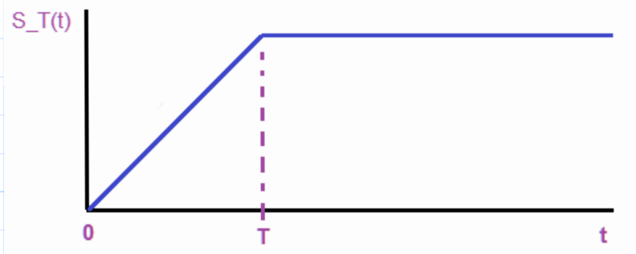
$$Q\_1s := \frac{C1 \cdot v_{C\_final}}{T3} = 1$$ Coulomb per second = 1 Amp. Answer.

What we said in the 3 solutions above was when using t = T = 1:

$$Q \quad = \quad \int_0^T i_C(t)\,dt \quad = \quad I_0 \cdot T \quad = \quad 1 \cdot 10^{-6} \quad \text{Coulomb.}$$

Note: At end of time T in the integration the amount of charge Q was still the same we had calculated prior i.e. Q=C1_vC_final. So we can say that charge Q is independent of time T = 1s, 1 ms, and 1 us. Time T did not impact the charge build-up, was independent of time.

Comment:
One of the things was differention took it to the other side then by integration turned back. Sounds like a joke, they do this regularly in EE, you dont like it as much as I do, what about the constant C in the integration above? Lets try the constant C after integration, set it equal 0 because the intial condition of the capacitor was zero. What initial condition? When we started there was no charge built up on the capacitor plates, lets say t <0, then with the swith on at t =0, charge built up. Stuff in the coming chapter.



For <0 t < T

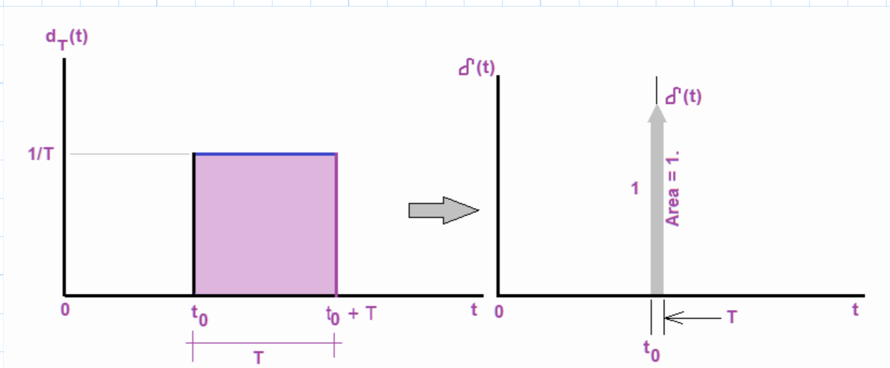
$$i_C(t) = \begin{cases} 0 & \text{for } t<0 \\ I_0 = \dfrac{10^{-6}}{T} \text{ (A).} & \text{for } 0 < t < T \\ 0 & \text{for } t>0 \end{cases}$$

Answer.

The figure above has the voltage increasing, dv/dt, for time 0<t<T.
When S_T(t) is constant for t>T here dv/dt = 0 hence i_C = 0.
So only the slope is applicable because we have a voltage change per time change; dv/dt.
The other two, cases here t<0 and t>T, the current is 0. <---Check thru.

Example 6.18 (Unit Delta/Impulse - Sifting Property).

Sifting property in the Delta function this may come up in the signals coursework (will come up). Otherwise in circuit analysis I dont have any personal experience with it. The explanation on it in Schaums was brief, so better check with your text book. Overall yes the main idea behind it here. No numerical evaluation example.



<---Something like this.
Notice on the left side its dT(t) the derivative. Right side is the delta function.

Basically the previous discussion, shown above, is used in the sifting property its NOT called shifting or delay, rather sifting. *To sieve thru, sifting, thats the meaning, get smaller objects thru the sieve, sifting. Maybe not be appropriate name but I'm not the professor. Dont sound like it to me. Maybe next course more sense in signals.*

dT(t - t0) is a narrow pulse of width T and height 1/T, which starts at t = t0.
Its width is T. So its end point or coordinate is t0 +T. As shown in figure below.

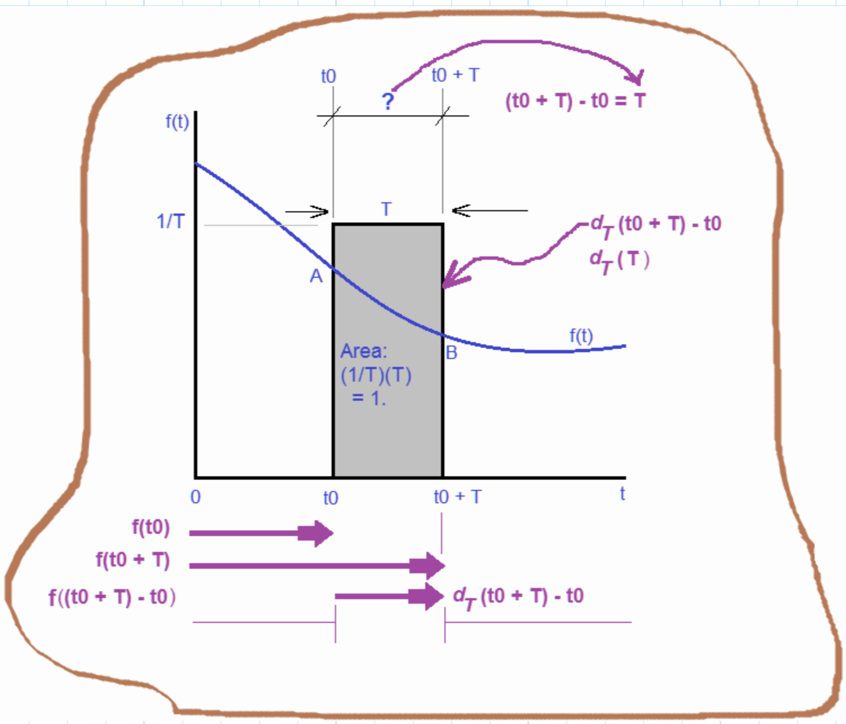
Expression below was seen in previous example. So we know there was current I involved in that pulse discussion.

$$Q = \int_0^T i_C(t)\,dt = I_0 \cdot T = 1 \cdot 10^{-6} \quad \text{Coulomb.}$$

Now can understand the expression below, resented on page 127 in Schaums.

$$I = \int_{-\infty}^{\infty} d_T(t - t_0) \cdot f(t)\,dt \qquad \text{Equation 1}$$

$$d_T(t - t_0) = \begin{cases} \dfrac{1}{T} & t0 < t < t0 + T \\ 0 & \text{Elsewhere} \end{cases} \qquad \text{Equation 2}$$



You see the area under the curve from point A to B, and can relate to its integral. This picked up in next page.

Substitute equation 2 in 1, what do we substitute? Has to be the term with a value on the RHS of equation 2.

$$I = \int_{-\infty}^{\infty} \left(\frac{1}{T}\right) \cdot f(t)\,dt$$

$$I = \left(\frac{1}{T}\right) \cdot \int_{t_0}^{t_0+T} f(t)\,dt$$

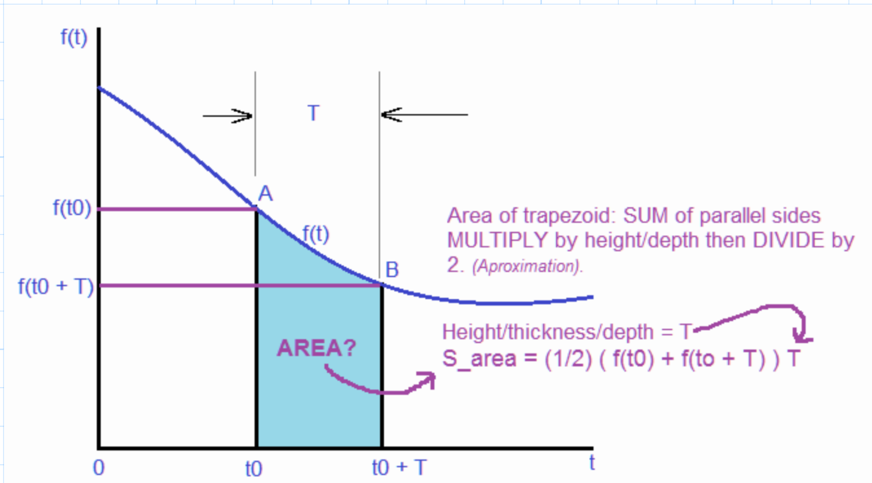
We take the intergral over the period T, t0+T - t0 = T.

$\int_{t_0}^{t_0+T} f(t)\,dt$ This term is the area under the curve, represented by an integral.

$$\int_{t_0}^{t_0+T} f(t)\,dt = S \quad ==> I = \left(\frac{S}{T}\right) \quad ==> \quad \left(\frac{1}{T}\right) \cdot \int_{t_0}^{t_0+T} f(t)\,dt$$

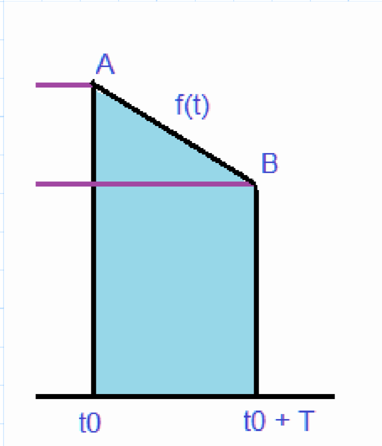
$$I = \left(\frac{S}{T}\right)$$

S is the shaded area in the figure above.
T the width of the pulse of the shaded area.



The function f(t) with the points A and B.

Assuming T to be small, the function f(t) can be made into a short straight line AB as shown below. Got the general idea. Thats all and how to get the area of the strip.



$$S = \left(\frac{1}{2}\right) \cdot \left[\left(f(t_0) + f(t_0 + T) \right) \right] \cdot T$$

$$I = \frac{S}{T}$$

$$I = \left(\frac{1}{T}\right) \cdot \left(\frac{1}{2}\right) \cdot \left[\left(f(t_0) + f(t_0 + T) \right) \right] \cdot T$$

$$I = \left(\frac{1}{2}\right) \cdot \left[\left(f(t_0) + f(t_0 + T) \right) \right]$$

Continued on next page.

$$I = \left(\frac{1}{2}\right)\cdot\left[\left(f(t_0) + f(t_0+T)\right)\right]$$

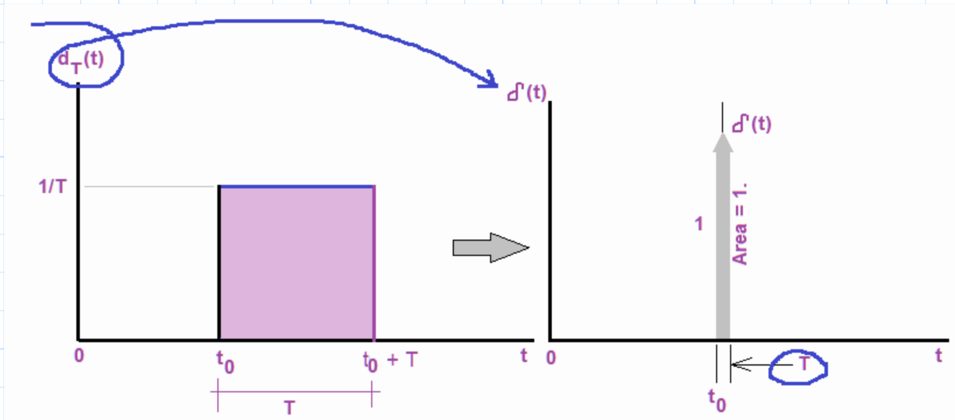
As T ---> 0, then $d_T(t-t_0)$ ---> $\delta(t-t_0)$

dT the derivative, LHS, which is same saying as the delta/impulse function RHS

then this corresponds to

function f(t) $f(t_0+T)$ ---> $f(t_0)$

now saying the function at t0+T same as at function at to BECAUSE? T is approaching zero. Same thing maybe I studied in Calculus 1 on the limit t approaching 0. Maybe. What is that saying? We need not go far and maybe solve it at t=t0.



$$\lim_{T\to 0} I = \lim_{T\to 0}\left(\frac{1}{2}\right)\cdot\left[\left(f(t_0) + f(t_0+T)\right)\right]$$ <---This T on the very right most approches 0.

$$\lim_{T\to 0} I = \lim_{T\to 0}\left(\frac{1}{2}\right)\cdot\left[\left(f(t_0) + f(t_0+0)\right)\right]$$

$$\lim_{T\to 0} I = \left(\frac{1}{2}\right)\cdot\left[\left(f(t_0) + f(t_0)\right)\right]$$ No limit on RHS now.

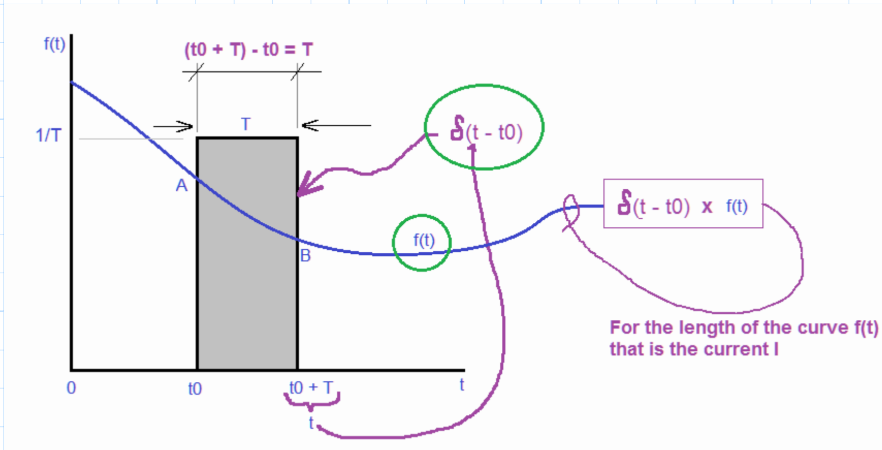
$$\lim_{T\to 0} I = \left(\frac{1}{2}\right)\cdot 2\, f(t_0)$$

$$\lim_{T\to 0} I = f(t_0)$$

Where does this take us?
LHS term eventually is going to be I.
What else can it be! Its the only variable there.

This takes us to the first integral.

$$I = \int_{-\infty}^{\infty} d_T(t-t_0)\cdot f(t)\,dt$$ Equation 1



See Equation 1 in the right side of the figure, f(t), is the current for the length of the curve. So basically its the area under the curve using the 'summation' of the delta function, which can be shown using an 'integral'.

Go back to the previous 2 pages for the expression below.

As T ---> 0, then $d_T(t-t_0)$ ---> $\delta(t-t_0)$ and $f(t_0+T)$ ---> $f(t_0)$

This term: $d_T(t-t_0)$ ---> $\delta(t-t_0)$ Same thing just the delta function is used to represent the derivative term.

$$I = \int_{-\infty}^{\infty} d_T(t-t_0)\cdot f(t)\,dt$$

Equation 1 becomes the delta equation below. Derivative to Delta function.

$$\lim_{T\to 0} I = f(t_0)$$

$$\lim_{T\to 0} I = \int_{-\infty}^{\infty} \delta(t-t_0)\cdot f(t)\,dt$$

Limit can be infinty, it is the length of the curve f(t).

$f(t_0+T)$ ---> $f(t_0)$ This is true then,

so is this $\delta(t-t_0)$ ---> $f(t_0)$

$$\lim_{T\to 0} I = \text{---}> f(t_0)$$

$$\lim_{T\to 0} I \; \text{---->} \; I = \int_{-\infty}^{\infty} \delta_T(t-t_0)\cdot f(t)\,dt = f(t_0)$$

Sifting property. Something like this. Check your textbook.

Check with your local engineer, if this is correct, the LHS = RHS, the expression in the middle is related to the LHS so it eventually should equal the RHS, shown with spacing above. Maybe like this A = C, A = B, so B = C.

Comments:

Started at f(t0) returned to f(t0).
This seems to be the usual history with differetiation and integration, taking the limit as t approaches 0.

I = the integral of the delta impulse centered at (t-t0) multiplied by f(t) basically is the area under the curve, we use the strip (1/T) x (T) unit area of 1 to carry out the summation of the area between the limits, this is the current I. If I am correct. So this results to f(t0) the function at time t = t0. So limit T --->0 then f(t0 + T) results in f(t0).

I hope not wrong, you verify. But where is the sifting? Could be the sifting was from t0+T got it back to t0 where we settle for f(t0) all this when T-->0. The period T maybe was sifted and it got back to t0. Its a theory presented by an example. Check your textbook. General idea maybe here! In signals text book there are numerical examples.

6.10 The Exponential Function *(Important).*

$j := \sqrt{-1}$ $\quad L := 1 \quad R := 1 \quad C := 1 \quad a := 1$

$f := 50$

$\omega := 2 \cdot \pi \cdot f$

$t := 1$ Let t = 1 so the variable is defined.
Its the range of time t is for a particular function.

$s := j + \omega \cdot t$ <---We focus on the 's' in another chapter in some detail.

$f(t) := e^{s \cdot t}$ <--- This is the function we are concerned with.
When s: -ve the function decays.
When s: +ve the function grows.
So we can for practical purpose make
'st' into 'at' where 'a' is a constant real number.
So we can STUDY it better by that easier.

$f(t) := e^{a \cdot t}$ same saying--> $f(t) := e^{j + \omega \cdot t}$

In RL and RC circuits we have the time constant tau. You see the explanation on this in another chpater for now just accept it for a constant. *Aim here is to appreciate the function how it moves, walk, talks, reacts,....these things, and yes the curve.*

$\tau_{RL} := \frac{L}{R}$ $\qquad \tau_{RC} := R \cdot C$

Here, the inverse of 'a' is the time constant. $\tau := \frac{1}{a}$ remember 'a' takes the place of 's'.

$f(t) := e^{s \cdot t}$ ---> $f(t) := e^{a \cdot t}$ --->

$a := \frac{1}{\tau}$ ---> $f(t) := e^{\left(\frac{1}{\tau}\right) \cdot t}$ ---> $f(t) := e^{\frac{t}{\tau}}$ <---There.

$f(t) := e^{-\left(\frac{t}{\tau}\right)}$ <--- How does the negative function behave?

When t = 0, tau = 1

$f(t) := e^{0} \quad = \quad 1$

When t = Infinity

$f(t) := e^{-\left(\frac{1 \cdot 10^{9}}{\tau}\right)} \quad = \quad 0 \qquad e^{-1} = 0.368$

$e^{-10} = 4.54 \cdot 10^{-5}$

$e^{-100} = 3.72 \cdot 10^{-44}$ <--- approaching 0.

After second,...make it t =1 second, and tau = 1:

$f(t) := e^{-\left(\frac{1}{\tau}\right)} \quad = \quad f(t) := e^{-\left(\frac{1}{1}\right)} = 0.368$

When tau equal 1 second:

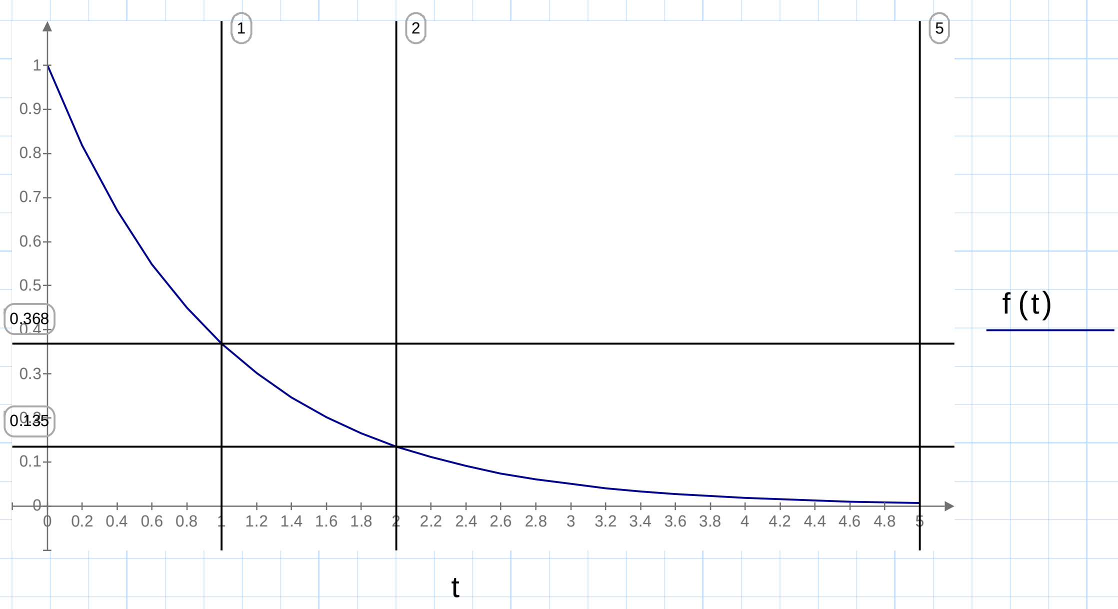
$f(t) := e^{-\left(\frac{t}{1}\right)} \quad = \quad f(t) := e^{-(t)}$ <---This function is called a 'normalised' exponential.

Lets plot a basic graph on e^-(t/tau):

clear(t)

$t := 0, 0.2 .. 5 \qquad \tau := 1$

$f(t) := e^{-\left(\frac{t}{\tau}\right)}$

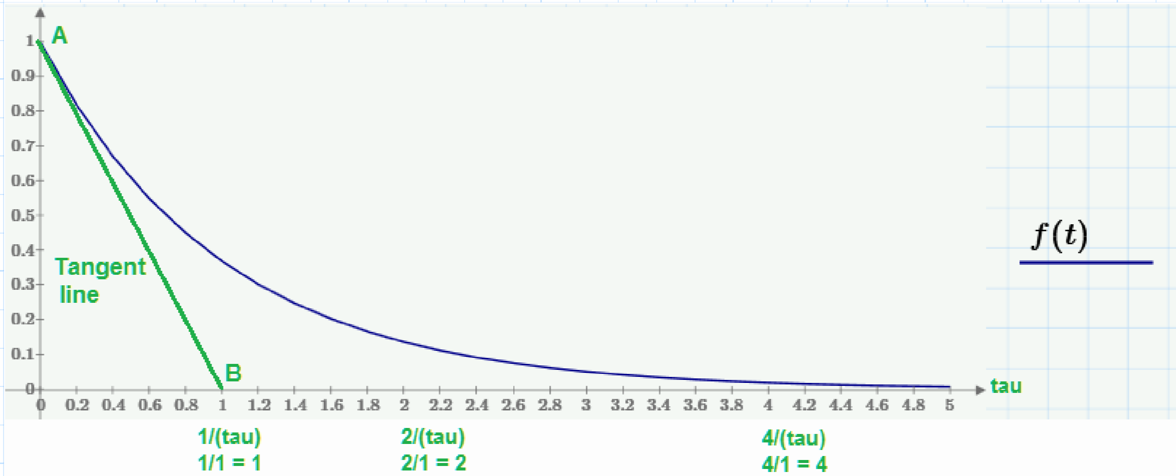


Study the graph, the curve is decaying, and dies out close to t = 5, here tau = 1.
The horizontal x-axis can be taken for t or tau, since tau = 1 in t/tau.

Example 6.19 (Exponential Function):

Show that the tangent, green line, to the graph e^-(t/tau) at t =0 intersects t axis at t = tau. As shown in the figure below, same graph as previous page.

Solution:



At t =0, f(t) = 1 we identify 1 as v.
So this is point A (0,v), the beginning of the tangent line.
Let's take the derivative of f(t) at this point:

$$\frac{d\left(e^{-\left(\frac{t}{\tau}\right)}\right)}{dt} = \left(\frac{-1}{\tau}\right)\cdot e^{-\left(\frac{t}{\tau}\right)}$$

At t = 0 at point A

$$= \left(\frac{-1}{\tau}\right)\cdot e^{-\left(\frac{0}{\tau}\right)} = -\left(\frac{1}{\tau}\right)$$ <---Slope at point A

$$= -\left(\frac{1}{\tau}\right)\cdot t$$ <---Slope at point t in general; Ax + B form.
A the slope and b intersection.

From calculus class/course the equation of the line is:
Slope + Intersection point of the line on the y-axis.
So here you have:

$$-\left(\frac{t}{\tau}\right) + 1$$ <--- Equation of line v_tan (t) = -(t/tau) + 1. Answer.

The green line intersect the t-axis (x-axis) at point 1, see graph above.
This is same as saying point t = tau (1 tau). *Later tau is the time constant and in a particular circuit there are just like here multiples of tau.*

Example 6.20 (Exponential Function):

Draw an approximate plot of v(t) = e^-(t/tau) for t>0.

Solution:

The solution uses the information in the previous example 6.19 and the notes before that to construct the graph. Typically we have a set of data from f(t) for each point t. We join the dots and pencil-in the curve! Most of the time at least. You agree?

1). Mark point A (t = 0, v = 1) similar scale we had in example 6.19.
2) Point B will intersect at t = tau = 1. t=1, tau = 1, 1/1 = 1.
At present we do not have values of R or C and R or L we are not involved with the physical electric circuit, just the math side of things with the exponential curve.
3). Draw a straight line between A and B. Thats the tangent line we had prior.
4). We need additional points? Where do we get them from? Go to the figure before the previous one. We have identified two points there on the curve.
5). Take t = 1 and t = 2, draw a vertical lines up, then from 0.368 and 0.135 on the y-axis draw horizontal lines to the right. we have points (1, 0.368) and (2, 0.135).
6). Total of 4 points A, (1, 0.368), (2, 0.135) and B. Use a flexible rule to draw in the curve e^-(t/tau). Answer.

0.135 is also got from? $0.368^2 = 0.135$

So the new point with t = 3, whats the y-axis here: $0.368^3 = 0.05$ Correct!

t = 4 $0.368^4 = 0.018$

t = 5 $0.368^5 = 0.007$ Long ways from 007, this is 0.007, correct. Getting closer to the y-axis = 0. Usually the Engineer is not going past 5(tau) because the curve has settled close to zero. *Usually maybe there are times its not the case.*

The math on why we get these accurate results, you done it in calculus course, shown below:

$e^{-\left(\frac{1}{\tau}\right)} = 0.368$ $e^{-\left(\frac{1}{\tau}\right)\cdot 2} = 0.135$ $e^{-\left(\frac{1}{\tau}\right)\cdot 3} = 0.05$ $e^{-\left(\frac{1}{\tau}\right)\cdot 4} = 0.0183$

$e^{-\left(\frac{1}{\tau}\right)\cdot 5} = 0.0067$ $e^{-\left(\frac{1}{\tau}\right)\cdot 6} = 0.0025$ $e^{-\left(\frac{1}{\tau}\right)\cdot 7} = 0.0009$

Example 6.21 Exponential function.

a). Show that the rate of change with respect to time of an exponential function
v = Ae^st is at any moment proportional to the value of that moment.

b). Show that any linear combination of an exponential function and its n derivatives is proportional to the function itself.
Find the coefficient of proportionality.

Solution:

a).

$$v(t) = A \cdot e^{st}$$ We have an amplitude A.

$$\frac{d(v(t))}{dt} = s \cdot A \cdot e^{st}$$

What does $A \cdot e^{st}$ has in common with $s \cdot A \cdot e^{st}$

Both are the same except for the variable 's 'in the RHS term which is the 'cause' for that proportionality. Which can be written as shown below:

$$v(t) = A \cdot e^{st}$$

$$v \cdot s = s\left(A \cdot e^{st}\right)$$ Answer.

b).

Continuing with part a.

$$\frac{d^2(v(t))}{dt^2} = s \cdot s \cdot A \cdot e^{st}$$

$$= s^2 \cdot A \cdot e^{st}$$

<---2nd derivative with the similar expression with the addition of the order of 2 on 's'.
Now as we know the general behaviour of the derivative of the exponential function its going to carry on this way.

$$\frac{d^n(v(t))}{dt^n} = s^n \cdot A \cdot e^{st} = v \cdot s^n$$

When we add all the terms starting from itself the first thru the higher order derivatives:

$$v + s \cdot Ae^{st} + s^2 \cdot Ae^{st} + s^3 \cdot Ae^{st} + s^4 \cdot Ae^{st} + \ldots \quad s^n \cdot Ae^{st}$$

$$v + s \cdot Ae^{st} + s^2 \cdot Ae^{st} + s^3 \cdot Ae^{st} + s^4 \cdot Ae^{st} + \cdots + s^n \cdot Ae^{st}$$

Lets make the 'a' term a coefficient which it really is because its a real number.
We seen this before in math course, the first term get the a0 which is 1.
Remember so this term is not changed because coeffficient is made 1. 's' remains and 'a' is added on as that coefficient.

The WRONG form first:

$$a_0 \cdot v + a_1 \cdot Ae^{st} + a_2 \cdot Ae^{st} + a_3 \cdot Ae^{st} + a_4 \cdot Ae^{st} + \cdots + a_n \cdot Ae^{st}$$ <---Wrong

$A \cdot e^{st}$ is actually v, as given in the problem in the beginning.

We can now change the 'a' term expression with v instead of the exponent term.

$$a_0 \cdot v + a_1 \cdot v + a_2 \cdot v + a_3 \cdot v + a_4 \cdot v + \cdots + a_n \cdot v$$

We can factor the 'v' out, we are getting somewhere, where the result will be applied in other courses or chapter.

$(a_0 + a_1 + a_2 + a_3 + a_4 + \cdots a_n) \cdot v$ We next set the 'a' terms be represented by H.

$$H = (a_0 + a_1 + a_2 + a_3 + a_4 + \cdots a_n)$$

$(a_0 + a_1 + a_2 + a_3 + a_4 + \cdots a_n) \cdot v = H \cdot v$ Looks reasonably accurate.
Whats wrong?

I left the 's' term out it CANNOT be substituted by 'a'.
'a' is not a term, its the coefficient and also represents the 's' term.
When we have equations, they are like 3x^3 + 5x^2 + 2x + 6 = 0. The x is the 's' the coefficient we are speaking about here are 3, 5, and 2. Thats the mistake, we do NOT have those equations here now, but in the future there will be.

The RIGHT way:

$$a_0 \cdot s^0 \cdot v + a_1 \cdot s^1 \cdot v + a_2 \cdot s^2 \cdot v + a_3 \cdot s^3 \cdot v + a_4 \cdot s^4 \cdot v + \cdots + a_n \cdot s^n \cdot v$$

$(a_0 \cdot s^0 + a_1 \cdot s^1 + a_2 \cdot s^2 + a_3 \cdot s^3 + a_4 \cdot s^4 + \cdots a_n \cdot s^n) \cdot v$ Equation 1

$H = (a_0 + a_1 + a_2 + a_3 + a_4 + \cdots a_n)$ Answer. Pulling out the coefficients 'a' in the LHS expression. We set them equal to H.
H does not have the 's' terms ONLY the coefficients.

So, we see the 's' terms remain in equation 1, but not in the H. We cant really separate 's' terms out and then multiply it in such a way to get equation 1.

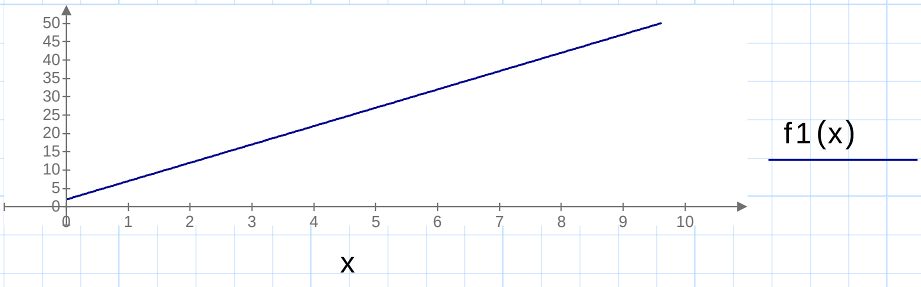
Exponential function: Specifying and Plotting f(t) = Ae^-at + B.

What if the expression was better as Ae^st + B, instead of only Ae^st?
We know from our math courses, the term Ax + B is a straight line.

clear (x)

$A := 5 \qquad B := 2$

$f1(x) := A \cdot x + B$



I belief it now, I was not sure so I graphed it. May be funny for you thats okay. You probably knew. It was a gamble. So we know the input to a circuit or the outout of a circuit may have reaction like the graph above. Similarly, we make the exponential function to Ax + B form.

$A \cdot e^{-st} + B$ <--- This function. What can it do for electric circuits?

$A \cdot e^{-at} + B$ <--- Here we set s = a. As before.

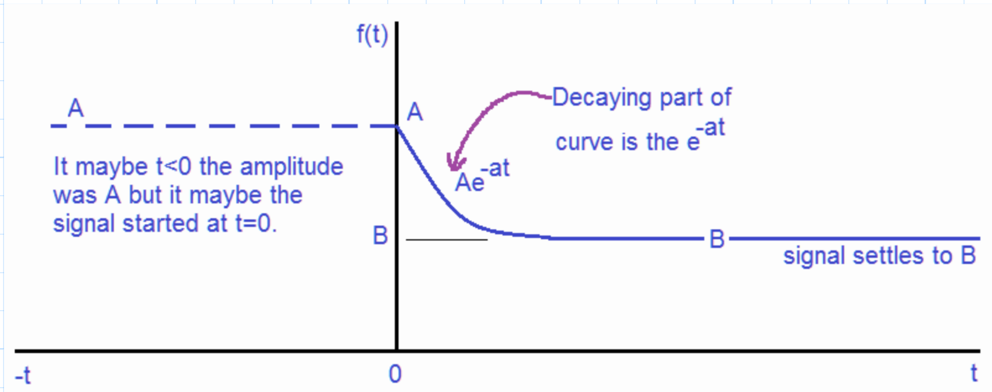


Figure above is an attempt to describe Ae^-at + B.
The text in Schaums is focussing on B being the latter part in the curve which is the final outcome, so B is called final value. A is the initial value.

Initial value: $f(0) = A + B$ Final value: $f(\infty) = B$
Time constant: 1/a. *As before. Thats how the plot above was.*

With that brief introduction we work an example, better appreciation.
Example 6.22 (Exponential function Ae^-at + B).

Find a function v(t) which decays exponentially from 5V at t=0 to 1 V at t = infinity, with a time constant of 3 seconds.
Plot v(t) using the manual plot example worked in 6.20 OR Excel/Matlab/Mathcad/......

Solution:

Lets set v(t) to an appropriate form of function.

$v(t) := A \cdot e^{-\left(\frac{t}{\tau}\right)} + B$ e^-(t/tau) as in the previous examples and notes.

$t := 0$ $v(0) = A + B = 5$ At t=0, A term = A, so A+B. This must equal 5V since the amplitude at t = 0 should reflect A, which we have equal to 5 as the starting voltage.

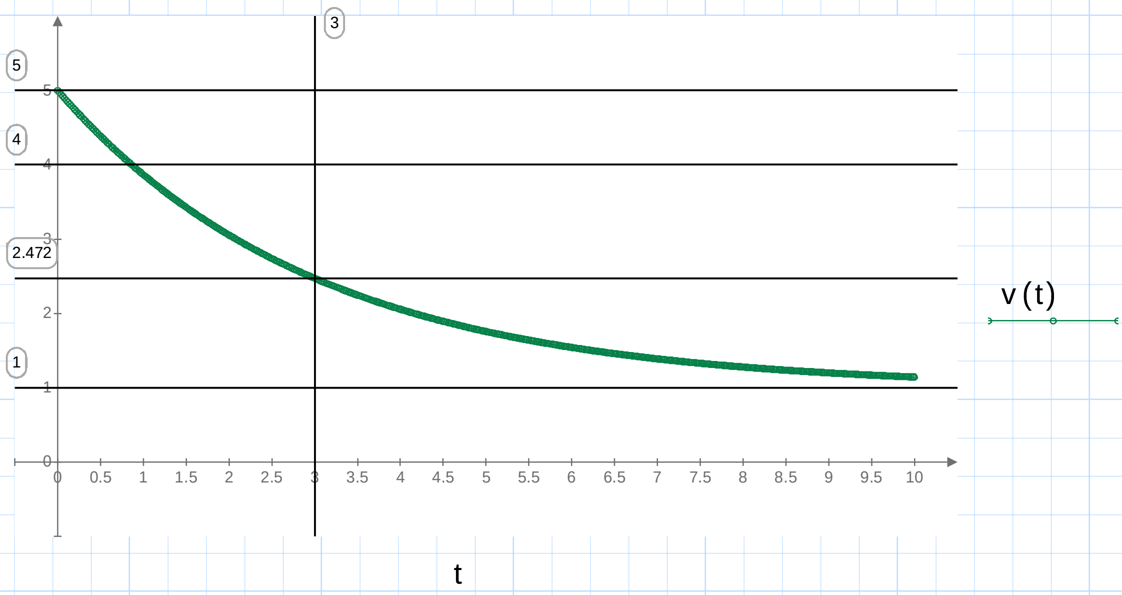
t = $\propto$ infinity $v(\propto) = 0 + B = 1$ Problem says at infinity the voltage is 1V. So this is after the decay has settled and will be the final value so this is B = 1.

$A + 1 = 5$ Substitute for B = 1 solving for A the initial value.

$A = 4$ Next substitute in for v(t), and tau = 3 second as required in problem.

clear (t)

$v(t) := 4 \cdot e^{-\left(\frac{t}{3}\right)} + 1$ Same as saying (Initial - Final value)e^-(t/tau) + (Final value)
Plot starts at 5 then drops to 1.



0.368? Remember from the e^-t/tau plot its significance. At t = 3 because here tau = 3, so 3/3 = 1, and that was why 0.368. Yes! The decay runs from 5 to 1, so 0.368 X (5 - 1) = 1.472. On the plot this will be at 1 + 1.472 i.e. (final value add 1.472) = 2.472 at t=3.

Example 6.23 (Exponential functions) <---Good example to work.

The voltage v = Vo e^-|t|/tau, where tau>0, is connected to a capacitor.
Find the current i in the capacitor?
Sketch v and i for Vo = 10V, C = 1uF, and tau (time constant) = 1ms.

Solution:

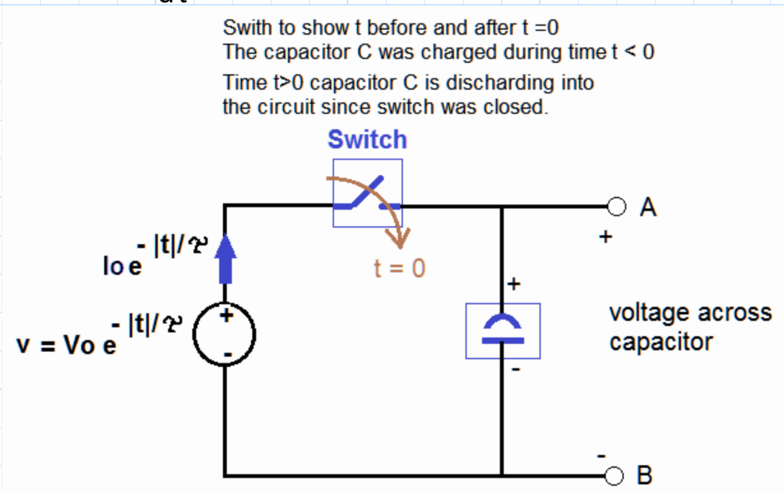
clear (t)

$V_0 := 10\ V \qquad C := 1 \cdot 10^{-6}\ uF \qquad \tau := 1 \cdot 10^{-3}$

The problem had t in the exponent term of v shown with the absolute | | symbol, |t|.
Do they mean time t is positive value only or was that an error? The negative sign is out side -|t|/tau so they may have meant both sides of 0, time is t<0 and t>0.

$$v(t) := V_0 \cdot e^{\frac{-|t|}{\tau}}$$

$$i_C(t) := C \cdot \frac{d}{dt}(v(t))$$ The usual capacitor current.



Schaums had no circuit and the solution has 2 conditions t<0 and t>0.
So, a circuit was shown here. *Sometimes, I may miss the Engineer-Author's full understanding of the question.*

NOT an eXpert here!
Not a mind reader.
With practice, you may be able to grasp the problem or work with it for a while.

What Schaums does is make 2 cases for the circuit, t<0, and t>0.
The graphs they show has the capacitor charging for t<0, and discharging for time t>0. You can conclude the capacitor was charged during the time t<0, then from t = 0 to t >0 the capacitor discharges.
These are the plots we need to show.

$\frac{-|t|}{\tau}$ ---> $\frac{-(-t)}{\tau}$ and $\frac{-('+t)}{\tau}$

---> $\frac{(t)}{\tau}$ and $\frac{(-t)}{\tau}$

Discussion: Notice the sign changes, +t becomes -ve i.e. discharging, and -t became +ve charging. That needs to be verified in the solution see the calculations and plots, looks that way.

Our voltage form is exponential, we are not given a phase angle, so we do not need to worry about who is leading or lagging.

Can we conclude the current I will have the same form? Why not, if its dc we do the usual the voltage dc so the current dc. Yes, same form.

Case when t<0:

$$v(t) = V_0 \cdot e^{\frac{-|-t|}{\tau}} = V_0 \cdot e^{\frac{t}{\tau}} \qquad \frac{d}{dt}(v(t)) = \left(\frac{1}{\tau}\right) \cdot V_0 \cdot e^{\frac{t}{\tau}}$$

$$i_C(t) := C \cdot \frac{d}{dt}(v(t)) \qquad C = 1 \cdot 10^{-6} \qquad \tau = 0.001$$

$$C \cdot \left(\frac{1}{\tau}\right) \cdot (V_0) = 0.01$$

$$i_C(t) = 0.01 \cdot e^{\frac{t}{\tau}}$$

Case when t>0:

$$v(t) = V_0 \cdot e^{\frac{-|t|}{\tau}} = V_0 \cdot e^{\frac{-t}{\tau}} \qquad \frac{d}{dt}(v(t)) = -\left(\frac{1}{\tau}\right) \cdot V_0 \cdot e^{\frac{-t}{\tau}}$$

$$\frac{d}{dt}(v(t)) = -\left(\frac{1}{\tau}\right) \cdot V_0 \cdot e^{\frac{-|t|}{\tau}}$$

$$i_C(t) := C \cdot \frac{d}{dt}(v(t)) \qquad C = 1 \cdot 10^{-6} \qquad \tau = 0.001$$

$$C \cdot \left(\frac{-1}{\tau}\right) \cdot (V_0) = -0.01$$

$$i_C(t) = -0.01 \cdot e^{\frac{-t}{\tau}}$$

Next step multiply the power of the exponent by 1000. This reason for x1000 was given in example 7.6. It makes things easier to read compared to the small decimal values.

$$e^{-\left(\frac{1 \cdot 10^3}{1 \cdot 10^3}\right) \cdot \left(\frac{-t}{1 \cdot 10^{-3}}\right)} = e^{\left(1 \cdot 10^3\right) \cdot t} \qquad \text{The other case} \qquad e^{-\left(1 \cdot 10^3\right) \cdot t}$$

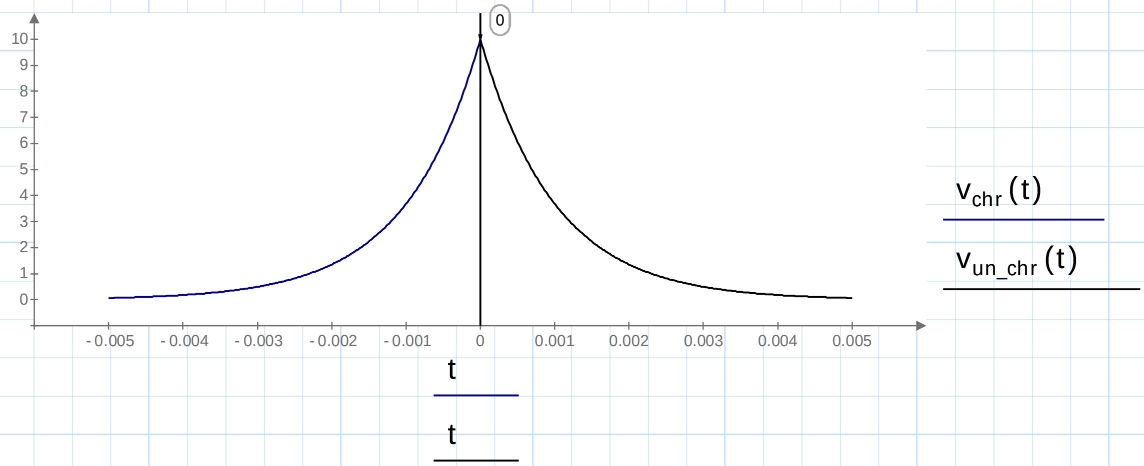
So now our voltage and current expressions look like this:

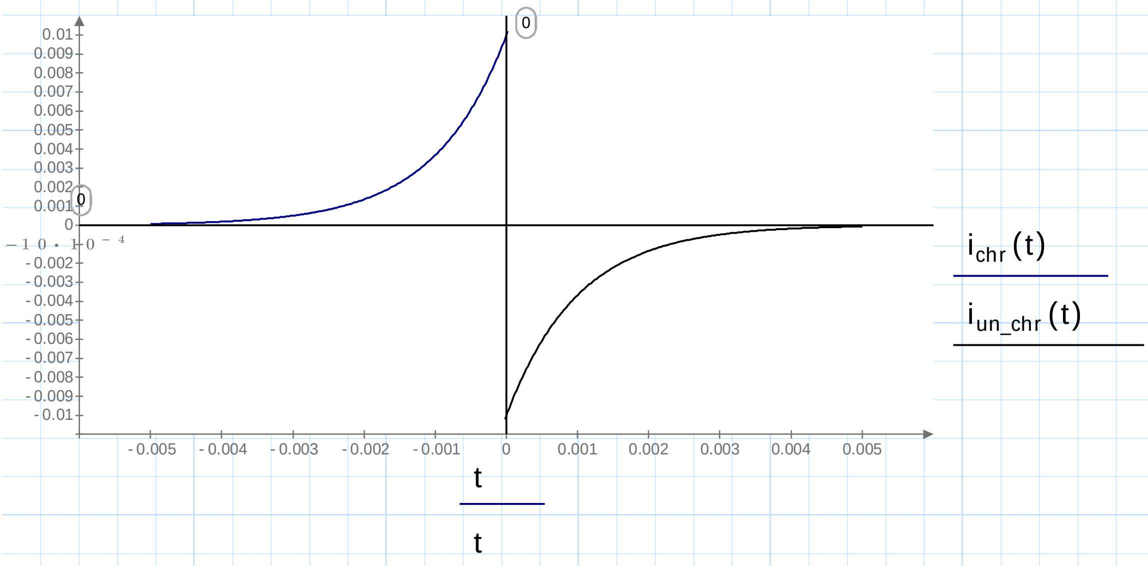
t<0 :

$$v_{chr}(t) := V_0 \cdot e^{(1 \cdot 10^3) \cdot t} \qquad i_{chr}(t) := 0.01 \cdot e^{(1 \cdot 10^3) \cdot t}$$

t>0 :

$$v_{un\_chr}(t) := V_0 \cdot e^{-(1 \cdot 10^3) \cdot t} \qquad i_{un\_chr}(t) := -0.01 \cdot e^{-(1 \cdot 10^3) \cdot t}$$





We see the same results with Schaums solution. Capacitor voltage charging, from 0 to a maximim 10V, then drop to 0. The current is different, the current increases as its charging time increases from t<0 to t =0. At time t=0 and t>0 it starts from -0.01A, so the current's magnitude sign changes, it has a negative sign current, and then decreases to 0. Answer.

6.11 Damped Sinusoids.

We know what a sinusoid is, its the sine curve or the cosine curve. *There is no tangent curve, you can try to make one and maybe find out why. Maybe there is. I never seen one.* We say sinusoidal we mean both sine and cosine.

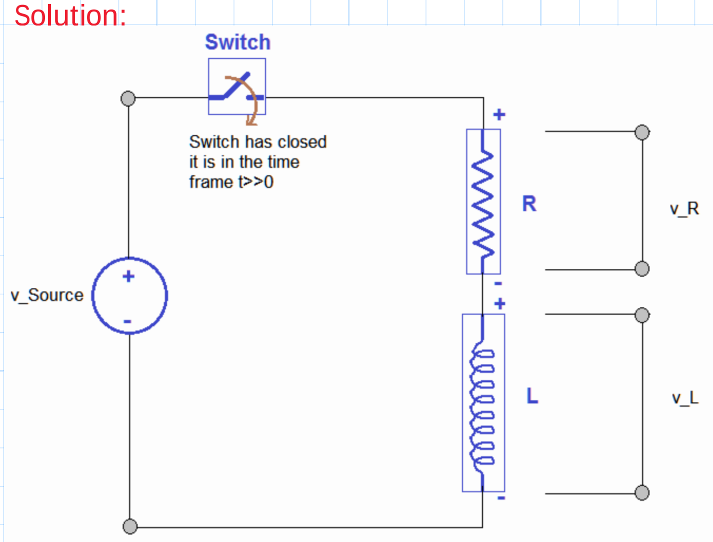
How does a sine/cosine curve become damped, i.e. become lower in amplitude with time? Multiply the sine/cosine term with a decaying exponential term!

$v(t) = A \cdot e^{-a \cdot t} \cdot \cos(\omega \cdot t + \theta)$ <---Damped Sinusoid.

Example 6.24 Damped Sinusoid.

The current i = Io_e^-at_cos(wt).
This current passes through a series RL circuit.
a). Find v_RL, voltage across the resistor and inductor combination.
b). Compute v_RL for Io=3A, a=2, w=40 rad/s, R=5ohm, and L=0.1H.
Sketch i as a function of time.

Solution:



Series RL circuit.
Asumption here the switch was closed and the circuit is in analysed in t>0. The voltage source is not removed from the circuit so that tells me the analysis is in time t>0, and also no initial conditions were given.

$i = I_0 \cdot e^{-a \cdot t} \cdot \cos(\omega \cdot t)$

Again we use the same form of expression as voltage, A e^-at (cos wt + theta), but no phase angle theta. Usually the voltage has the phase angle included, we leave it out in the current. We only need it in one or the other, so it don't further add on to it during calculations, its actually the difference between the voltage and current's angle. So if we start multiplying or adding or subracting the phase angle we would be making a change to that angle, which was supposed to remain the same for the particular circuit. *<---Show/Proof discussion was wrong?*

a).

$$vR = R \cdot i = R \cdot I_0 \cdot e^{-a \cdot t} \cdot \cos(\omega \cdot t)$$

$$vL = L \cdot \left(\frac{di}{dt}\right)$$

$$\left(\frac{di}{dt}\right) = \left(-a \cdot I_0 \cdot e^{-a \cdot t} \cdot \cos(\omega \cdot t)\right) + \left(I_0 \cdot e^{-a \cdot t} \cdot \omega \cdot -\sin(\omega \cdot t)\right)$$

$$= -I_0 \cdot e^{-a \cdot t} \cdot (a \cdot \cos(\omega \cdot t) + \omega \cdot \sin(\omega \cdot t))$$

$$vL = -L \cdot I_0 \cdot e^{-a \cdot t} \cdot (a \cdot \cos(\omega \cdot t) + \omega \cdot \sin(\omega \cdot t))$$

Voltage across R and L combined ?

$$vRL = vR + vL$$

$$= R \cdot I_0 \cdot e^{-a \cdot t} \cdot \cos(\omega \cdot t) - L \cdot I_0 \cdot e^{-a \cdot t} \cdot (a \cdot \cos(\omega \cdot t) + \omega \cdot \sin(\omega \cdot t))$$

$$vRL = I_0 \cdot e^{-a \cdot t} \cdot (R - L \cdot a) \cdot \cos(\omega \cdot t) - L \cdot \omega \cdot \sin(\omega \cdot t)$$ Answer.

$I_0 := 3$ $a := 2$ $\omega := 40$ $R := 5$ $L := 0.1$

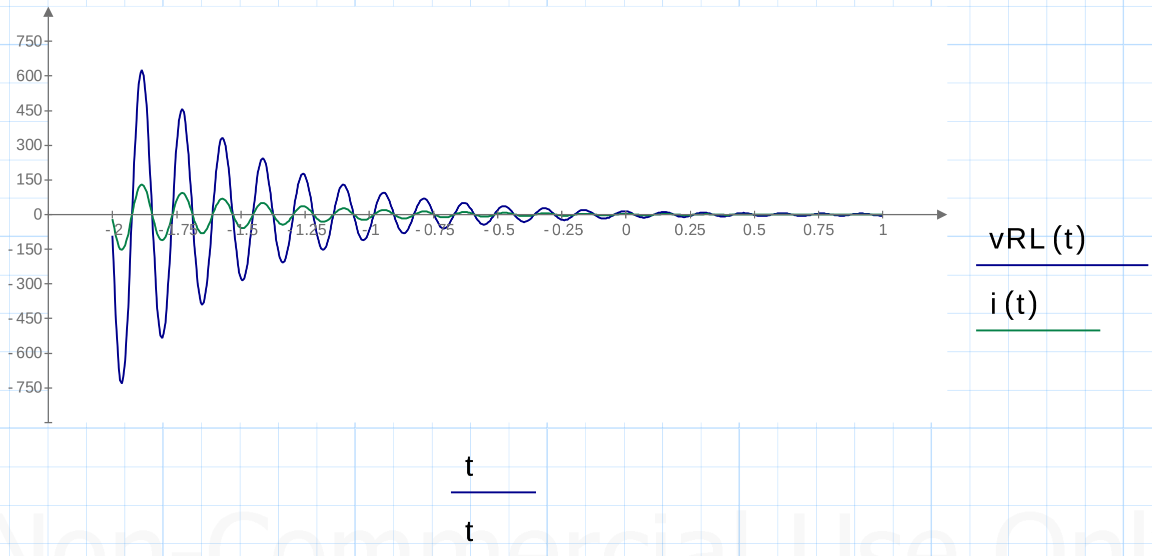
clear (t)

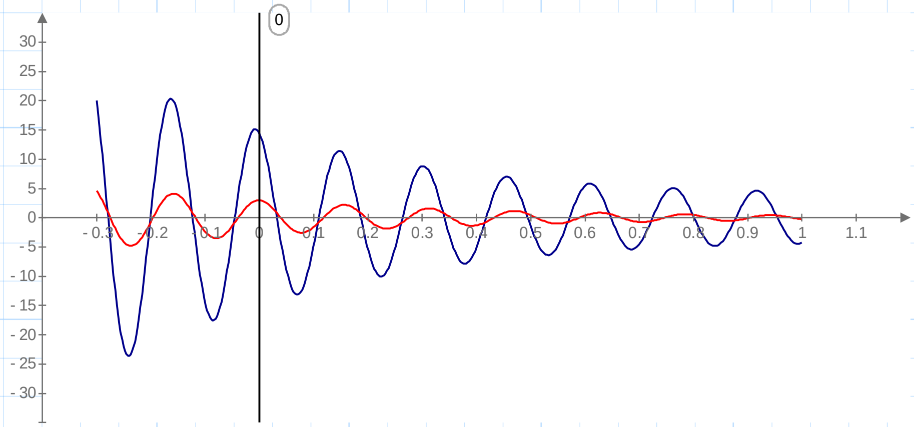
Substitute in for vRL(t) and i(t):

$$vRL(t) := I_0 \cdot e^{-a \cdot t} \cdot (R - L \cdot a) \cdot \cos(\omega \cdot t) - L \cdot \omega \cdot \sin(\omega \cdot t)$$

$i(t) := I_0 \cdot e^{-a \cdot t} \cdot \cos(\omega \cdot t)$ <--- Expression for current we set earlier. Answer.

The general plots for voltage and current applying the values given, followed by a zoomed in plot.





b).

$I_0 := 3$ $a := 2$ $\omega := 40$ $R := 5$ $L := 0.1$

clear(t)

Substitute in for vRL(t) and i(t):

$$vRL(t) := 3 \cdot e^{-2 \cdot t} \cdot (5 - (0.1) \cdot 2) \cdot \cos(40 \cdot t) - 0.1 \cdot 40 \cdot \sin(40 \cdot t)$$

$$vRL(t) := 14.4 \cdot e^{-2 \cdot t} \cdot \cos(40 \cdot t) - 4 \cdot \sin(40 \cdot t)$$

$V_0 := I_0 \cdot \sqrt{(R - L \cdot a)^2 + L^2 \cdot \omega^2}$ Equation 1 $V_0 = 18.745$ V Answer.

$\theta := \operatorname{atan}\left(\frac{L \cdot \omega}{(R - L \cdot a)}\right)$ Equation 2 $\theta = 39.806$ **deg** phase angle Answer.

$$v_{RL}(t) := 18.75 \cdot e^{-2 \cdot t} \cdot \cos(40 \cdot t + 39.8\ \mathbf{deg})$$

$i_{values}(t) := 3 \cdot e^{-2 \cdot t} \cdot \cos(40 \cdot t)$ <--- Expression for current with values.

Equation 1 and 2 maybe gotten from the expression below.

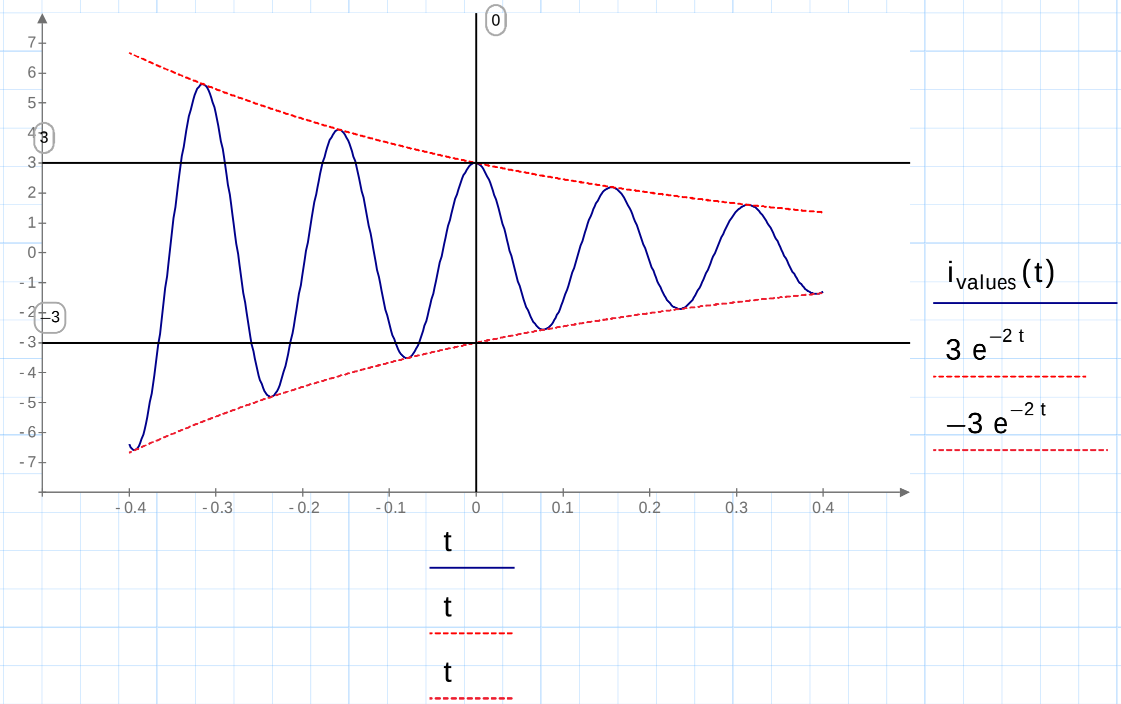
v_Source $= V_0 \cdot e^{-a \cdot t} \cdot \cos(\omega \cdot t + \theta)$ *Same form as Io with the?... Phase angle theta.*

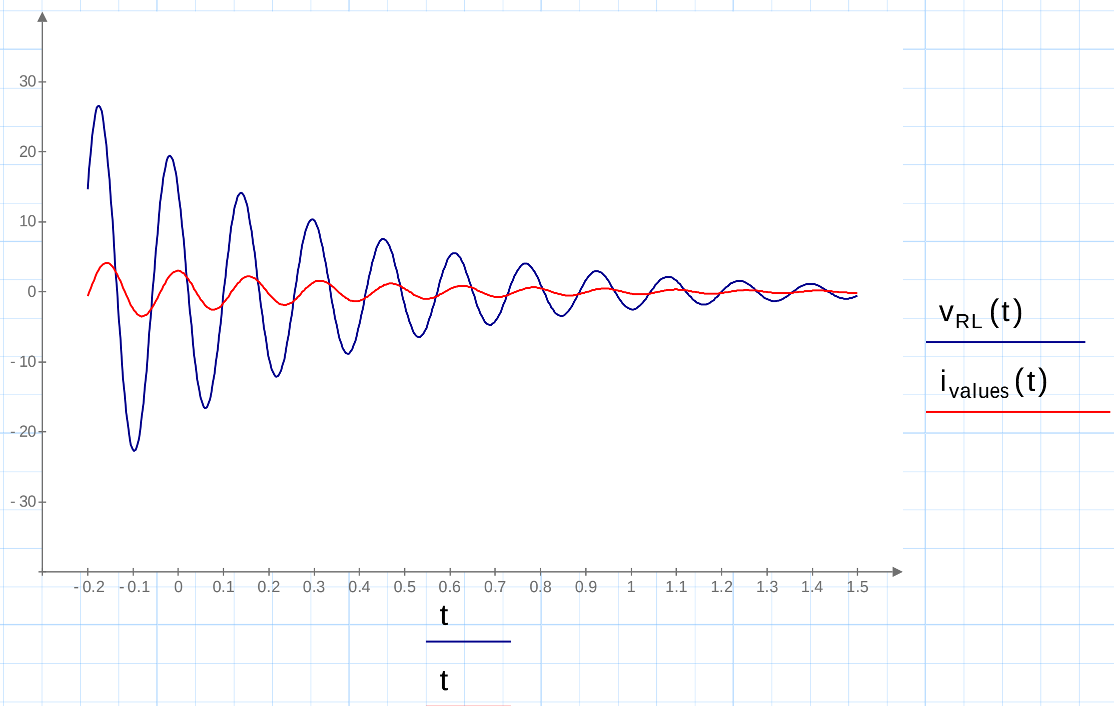
$$V_0 \cdot e^{-a \cdot t} \cdot \cos(\omega \cdot t + \theta) = I_0 \cdot e^{-a \cdot t} \cdot (R - L \cdot a) \cdot \cos(\omega \cdot t) - L \cdot \omega \cdot \sin(\omega \cdot t)$$

You may try, its not easy, divide by e^-at, next divide by maybe cos(wt)... rearrange,.....this will NOT do it. Not worth dwelling on at this time. Application of trig identities, ...etc. Some local Engineer or Professor may help you. If you need to know.

Next, AGAIN, Sinusoidal Function in detail, some direction to the equation 1 and 2.

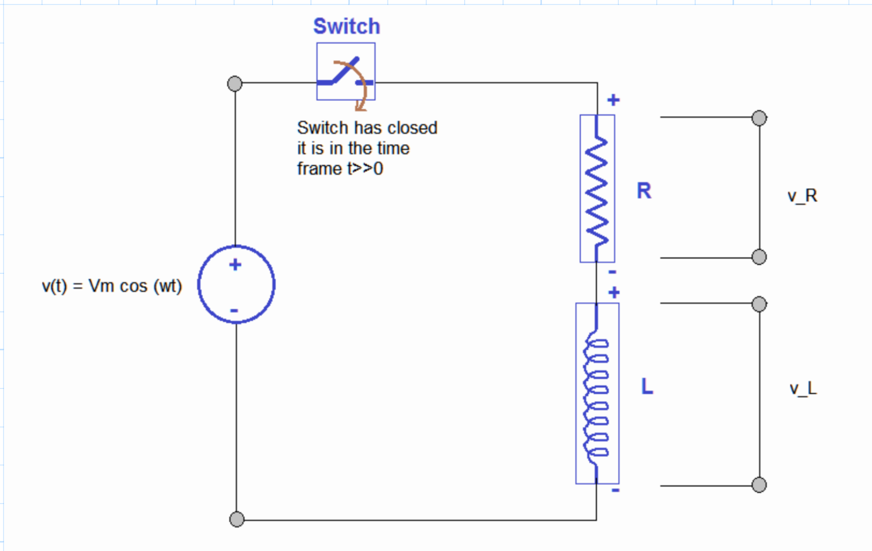
Plot below of current i as a function of time. With envelope 3e^-2t in dashed red.
The lower plot has voltage and current on a larger time range.





Sinusoidal Forcing Function *(IMPORTANT)*.

Forcing function now is the sinusoidal function, its applied as input to the circuit, we see a forced response, the output, that is also sinusoidal.



Series RL circuit.

The complete solution this equation is composed of 2 parts, the complementary (called the natural response) and the particular solution (also called the forced response).

Natural response depends on the circuit type, element values, and the initial conditions. By reducing the equation to a linear homogeneous differential equation.

The forced response has the mathematical form of the forcing function, plus all its derivatives and its first integral.

Important mathematical property of the sinusoidal function: Its derivatives and integrals are also all sinusoids. Since the forced response takes on the form of the forcing function, its integral and its derivatives, the sinusoidal forcing function will produce a sinusoidal forced response throughout a linear circuit. The sinusoidal forcing function thus allows a much easier mathematical analysis than does almost every other forcing function. It is an easy function to generate and is the waveform used predominantly through out the electric industry. ...*Engineering Circuit Analysis, Hayt and Kimmerly.*

Sinusoidal is the sine function, Cosinusoidal is the cosine function. Though sinusoidal function is used to refer to both.

RL series circuit.

$$V_M \cdot \cos(\omega t) = L \cdot \left(\frac{di}{dt}\right) + Ri \qquad \text{Equation 1}$$

Lets start with i(t) it can have only 2 forms Isin(wt) and Icos(wt), so the forced response the current must have the general form with both.

$$i(t) = I_1\cos(\omega t) + I_2\sin(\omega t)$$

I1 and I2 are constants whose values depend on Vm, R, L, and *w*.

$$\frac{di(t)}{dt} = -I_1\omega\cdot\sin(\omega t) + I_2\omega\cdot\cos(\omega t)$$

$$V_M\cdot\cos(\omega t) = L\cdot\left(\frac{di}{dt}\right) + Ri \qquad \text{substitute in RHS}$$

$$L\cdot\left(-I_1\omega\cdot\sin(\omega t) + I_2\omega\cdot\cos(\omega t)\right) + R\left(I_1\cos(\omega t) + I_2\sin(\omega t)\right)$$

$$\left(-L\cdot I_1\omega\cdot\sin(\omega t) + R\cdot I_2\sin(\omega t)\right) + \left(L\cdot I_2\cdot\cos(\omega t) + R\cdot I_1\cos(\omega t)\right)$$

$$\left(-L\cdot I_1\omega + R\cdot I_2\right)\cdot\sin(\omega t) + \left(L\cdot I_2\omega + R\cdot I_1\right)\cdot\cos(\omega t)$$

$$V_M\cdot\cos(\omega t) = \left(-L\cdot I_1\omega + R\cdot I_2\right)\cdot\sin(\omega t) + \left(L\cdot I_2\omega + R\cdot I_1\right)\cdot\cos(\omega t) \qquad \text{Equation 1}$$

$$\left(-L\cdot I_1\omega + R\cdot I_2\right)\cdot\sin(\omega t) + \left(L\cdot I_2\omega + R\cdot I_1 - V_m\right)\cdot\cos(\omega t) = 0$$

The equation above must be true for all values of t. This can be achieved only if the factors multiplying sin(wt) and cos(wt) are each zero.
That will surely make the expression equal to zero.
If t =0 the cosine term is not zero, its 1 so the factor multiplying it must be zero.
If t =0 the sine term is zero, so the factor multiplying need not be zero other then when t is not equal to zero.

$$-L\cdot I_1\omega + R\cdot I_2 = 0$$
$$L\cdot I_2\omega + R\cdot I_1 - V_m = 0$$

Rearranging:

$$-L\cdot I_1\omega + R\cdot I_2 = 0$$
$$L\cdot I_2\omega + R\cdot I_1 = V_m$$

Solving the simultaneous equations

$$I_1 = \frac{RV_M}{R^2 + \omega^2 L^2} \qquad I_2 = \frac{\omega L V_M}{R^2 + \omega^2 L^2}$$

We were given the expression for the voltage.
We need now to find the expression of the current this is the forced response *(or loosely what I called the output). In circuit the generator has a set voltage it may be AC or DC, the current depends on the? Load which is resistive, inductive, capacitive,.......*

The current as a function of time will have the sine and cosine terms, just as we started with i(t).

$$i(t) = I_1\cos(\omega t) + I_2\sin(\omega t)$$

$$i(t) = \left(\frac{RV_M}{R^2+\omega^2 L^2}\right)\cdot\cos(\omega t) + \left(\frac{\omega L V_M}{R^2+\omega^2 L^2}\right)\cdot\sin(\omega t)$$

<---Forced response. Equation 2.

From our electric singal waveform, three phase circuits, the power triangle,...... we know the phase angle is missing above.

$$i(t) = A\cdot\cos(\omega t - \theta)$$

Current i(t) in cosine term with amplitude and phase angle!

Lets use the cosine term for its benefits, maybe beneficial, and plug in the phase angle, then set that for the current i(t) RHS of equation 2.

Simplify first using sum/difference identity.

$$A\cdot\cos(\omega t - \theta) = A\cos(\theta)\cos(\omega t) + A\sin(\theta)\sin(\omega t)$$

Now equate it:

$$A\cos(\theta)\cos(\omega t) + A\sin(\theta)\sin(\omega t) = \left(\frac{RV_M}{R^2+\omega^2 L^2}\right)\cdot\cos(\omega t) + \left(\frac{\omega L V_M}{R^2+\omega^2 L^2}\right)\cdot\sin(\omega t)$$

Equation 3.

Similarly again equating the factors/coefficients of cos wt and sin wt to zero.......in equation 3

$$A\cos(\theta)\cos(\omega t) = \left(\frac{RV_M}{R^2+\omega^2 L^2}\right)\cdot\cos(\omega t) \qquad A\sin(\theta)\sin(\omega t) = \left(\frac{\omega L V_M}{R^2+\omega^2 L^2}\right)\cdot\sin(\omega t)$$

$$A\cos(\theta) = \left(\frac{RV_M}{R^2+\omega^2 L^2}\right)$$

Equation 4

$$A\sin(\theta) = \left(\frac{\omega L V_M}{R^2+\omega^2 L^2}\right)$$

Equation 5

What variables we need to find? R, L, w, and V we know? Maybe. A? Need A and Theta.

If we divide one term by the other we can get a tangent term; sin/cos=tan.

$$\frac{A\sin(\theta)}{A\cos(\theta)} = \frac{\left(\dfrac{\omega L V_M}{R^2+\omega^2 L^2}\right)}{\left(\dfrac{RV_M}{R^2+\omega^2 L^2}\right)}$$

$$\tan(\theta) = \frac{\omega L V_M}{RV_M}$$

$$\tan(\theta) = \frac{\omega L}{R}$$

$$\theta = \operatorname{atan}\left(\frac{\omega L}{R}\right)$$ <--- Phase angle.

Lets give it another try to find the coefficient A.

Square both equation 4 and 5 then add.

$$A^2\cdot\cos^2\cdot(\theta) = \left[\frac{R^2\cdot V_M{}^2}{\left(R^2+\omega^2 L^2\right)^2}\right] \qquad A^2\cdot\sin^2\cdot(\theta) = \left[\frac{\omega^2\cdot L^2\cdot V_M{}^2}{\left(R^2+\omega^2 L^2\right)^2}\right]$$

Add equations 4 and 5.

$$A^2\cdot\cos^2\cdot(\theta)+A^2\cdot\sin^2\cdot(\theta) = \left[\frac{R^2\cdot V_M{}^2}{\left(R^2+\omega^2 L^2\right)^2}\right]+\left[\frac{\omega^2\cdot L^2\cdot V_M{}^2}{\left(R^2+\omega^2 L^2\right)^2}\right]$$

$$A^2\cdot\left(\cos^2\theta+\sin^2\theta\right) = \left[\frac{R^2\cdot V_M{}^2+\omega^2\cdot L^2\cdot V_M{}^2}{\left(R^2+\omega^2 L^2\right)^2}\right]$$ *LHS: Trig identity sin^2 (theta) + cos^2(theta) = 1*

$$A^2\cdot(1) = \left[\frac{V_M{}^2\cdot\left(R^2+\omega^2\cdot L^2\right)}{\left(R^2+\omega^2 L^2\right)^2}\right] = \left[\frac{V_M}{\left(R^2+\omega^2 L^2\right)}\right]$$ Next square-root for A.

$$A = \frac{V_M}{\sqrt{\left(R^2+\omega^2 L^2\right)}}$$ <---Amplitude.

We can substitute in A and Theta Phase Angle into the i(t) equation.

$$i(t) = A\cdot\cos(\omega t-\theta)$$ <--- Cosinusoidal form made the better and complete solution.

$$i(t) = \left[\frac{V_M}{\sqrt{\left(R^2+\omega^2 L^2\right)}}\right]\cdot\cos\left(\omega t-\operatorname{atan}\left(\frac{\omega L}{R}\right)\right)$$ <---The forced response.

Engineering Circuit Analysis 4th Edition, Hyatt and Kimmerly. McGrawHill.

Continuing Example 6.24.

May be now closer to Schaums but you check OR *start up the steps again for 6.24 for the long expressions.*

$$A = \frac{V_M}{\sqrt{(R^2 + \omega^2 L^2)}}$$

Lets say A now is the current amplitude. Lets set that to Io.

$$I_0 = \frac{V_M}{\sqrt{(R^2 + \omega^2 L^2)}}$$

$$V_M = I_0 \cdot \sqrt{(R^2 + \omega^2 L^2)}$$ Almost there?

Discussion (Maybe) - Bonus pick-up in end of next chapter section 7:14:
In our problem solving R is equal to? (R-La) So we substitute that in. <---Here?
Here the resistance is not all Ohms its got Inductance but you know there is a thing called IMPEDANCE, so maybe the impedance of the Inductor that is the resistance of the inductor in ac conditions. You will come to that in your 3 phase or ac analysis - easy in comparion. Not here, we are more interested in getting some intermediate thru advanced 'refresher or first time course work' for most, to prepare for Laplace studies. STOP. Don't get over confident with the fruits of ambition. 'Life time of studies'.......

Now we set R - La <---*Looks like an easy way out I admit.*
Current i = Io_e^-at_cos(wt) <---Chapter 8 Schaums.
e^(-st) where a=s. Some we look in next chapter.

$$R = (R - La)$$

$$V_M = I_0 \cdot \sqrt{\left((R - La)^2 + \omega^2 L^2\right)}$$ Voltage amplitude.

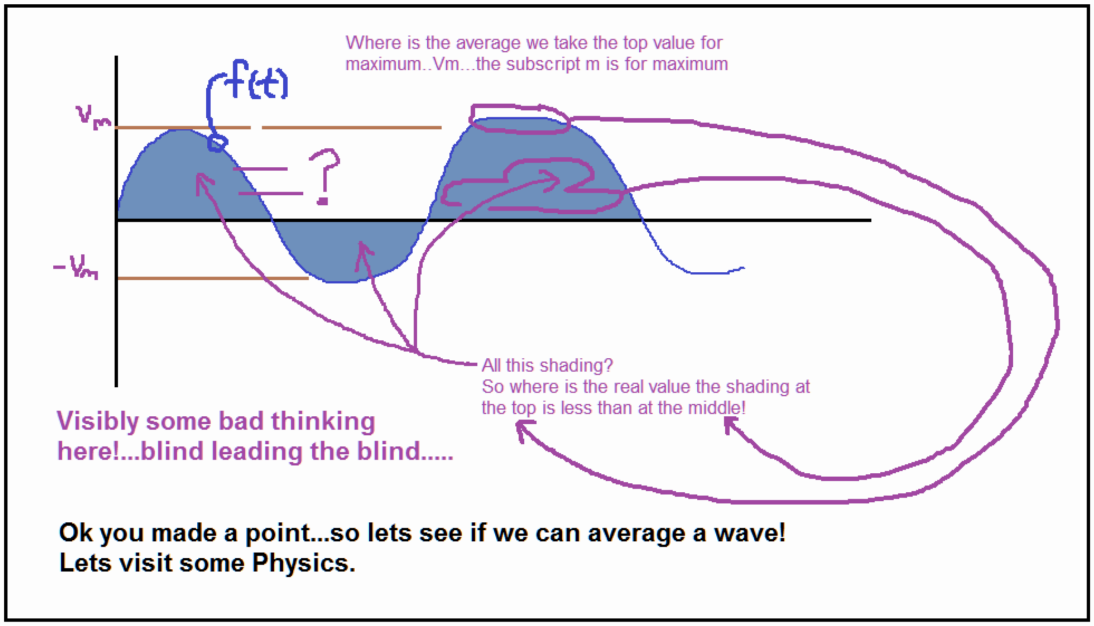
$$\theta = \text{atan}\left(\frac{\omega L}{R}\right)$$ <---Substitute (R-La) for R

$$\theta = \text{atan}\left(\frac{\omega L}{(R - La)}\right)$$ Phase angle.

That should have you completing example 6.24 because you wanted to know how those expressions equation 1 and 2 were attained/derived/.....you may have a better choice of words. You got the general idea wasnt easy.
Schaums: This function will be discussed in more detail in Chapter 8 - page 131.

6.6 Average and Effective (RMS) Values.

We been thru the sinusoidal function so a little further on how we get an average value or some definite value in a sinusoidal function.
Enjoy this journey.



Discussion:

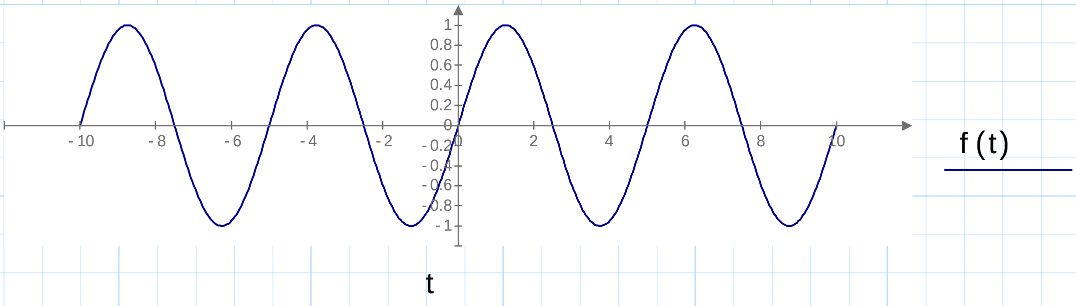
Alright so we got a little troubling matter.
So in Math there is a topic called Fourier its about applying integration a topic NOT so favourite of mine as you can see. Maybe youre the same. This topic when explained slowly and carefully like handling explosive devices, then an average student and below average can understand. Here its not a math class so we will move along applying the Fourier expression, its taught in signals course work. Its a Physics course subject as well. As far as I know the other engineering disciplin do not use it that frequently, its a strong subject matter in signals.

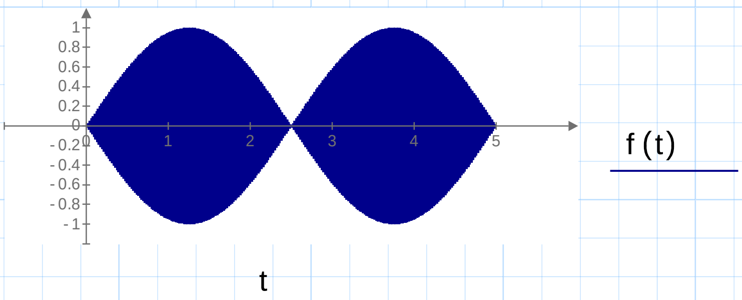
Discussion:
A periodic function is that sine wave you seen all your high school and college math life, and for that local engineer all his working life.
Same for that cosine. You know what a period T is, the frequency f, the radian frequency omega equal 2 Pi f.
These things have a place in a repeating wave, same pattern/shape running thru, periodic is another name for it, *Keep It Simple Smarty KISS.*

FIRST A DEVIATION, SKILLS HERE TOO, GOOD ONES. THEN WE COME BACK TO HERE TO CONTINUE,

$f := 50$ $\quad T := \frac{1}{f} = 0.02$ $\quad \omega := 2 \cdot \pi \cdot f$ $\quad f(t) := \sin(\omega \cdot t)$

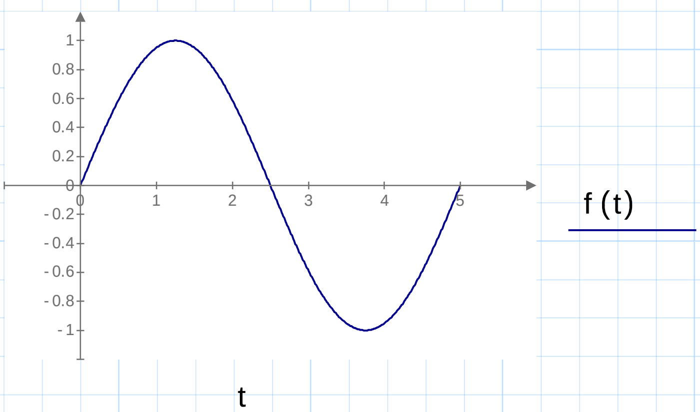
The periodic wave shown below from -t *to* t =0 *to* t+. Easily done in Excel.





For the same values this plot filled up the curve, hashed in too many time.
Solid instead of just a line. Why?

New values.
$f := 0.2$ $\quad T := \frac{1}{f} = 5$ $\quad \omega := 2 \cdot \pi \cdot f$ $\quad f(t) := \sin(\omega \cdot t)$

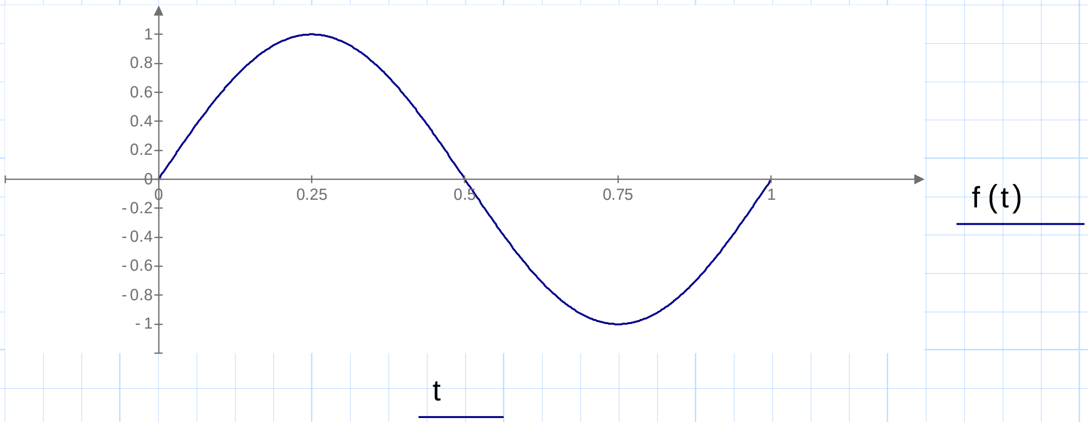


Why did I have to change the frequency to get the plot the same as the top for one cycle or period? Should have worked the same plotting from 0 to 5. Maybe the frequency? because it shaded in the whole curve, too many points at f = 50, so I changed it to 0.02.

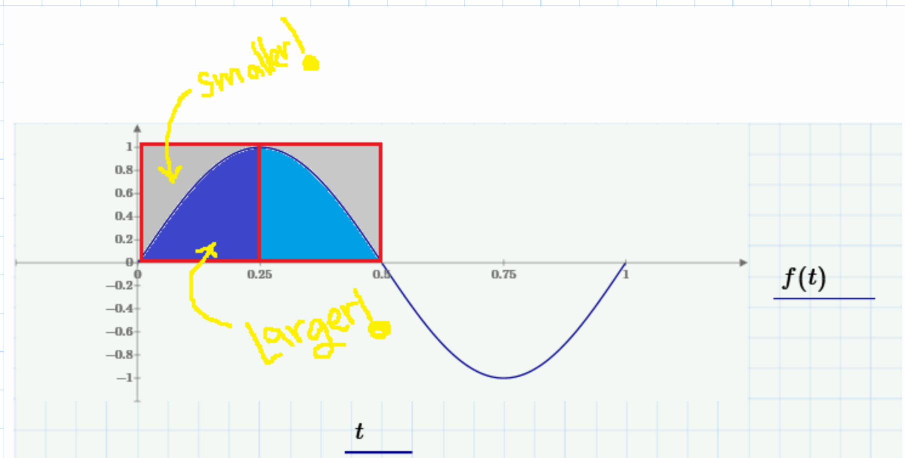
This is no different than any other plot so *dont be quick to blame some conspiracy* is involved. You may use Excel. It has solved many engineering problems of depth.

We use these new values with a different wave/curve/function.

$f := 1 \qquad T := \frac{1}{f} = 1 \qquad \omega := 2\cdot\pi\cdot f \qquad f(t) := \sin(\omega\cdot t)$



Fequency is 1 in 1 second, hence the period is 1.
Simple graph to make the shaded area more determinable next.



If we think average, surely the left side of the first half of the cycle would be same area for dark blue and grey. If this side of the curve is not agreeing neither the rest. So if area was what we were seeking then its not going to average out correctly for a definite value for the total area, which we come to next on that theory. BUT, maybe a triangle wave, a triangle wave may be equal. You may not doubt then.

$$F_{average} = f(t) = \left(\frac{1}{T}\right)\cdot\int_0^T f(t)\,dt$$

<----(1/T) x Area of the function
= Frequency x Area of Function within one period. Area can be voltage, current,.....
(1/T) is frequency. Which you know in the plot the frequency runs thru the plot time t.
Sin(wt), w = 2 Pi **f**. Function taken over 1 period.

$$F_{average} = f(t) = \left(\frac{1}{T}\right)\cdot\int_0^T f(t)\,dt$$

Agreed. Discussion wise the final result is Faverage, thru the frequency range, and in one cycle T(Lim 0-->T). The whole frequency is in there the number of oscillations (1/T), the final result is an average over the period, ie the intergral divided by T.

$$\left(\frac{1}{T}\right)\cdot\int_0^T f(t)\,dt = \left(\frac{1}{T}\right)\cdot\int_{t0}^{to + T} f(t)\,dt$$

This is the general form where instead of 0 its set to t0. So its taken any where on the time axis where you want it to be t0.

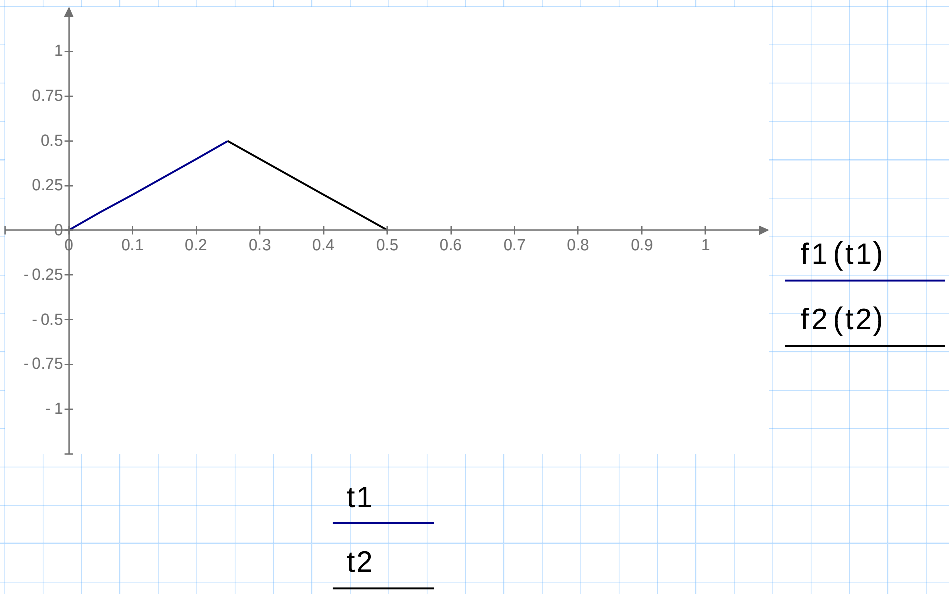
Discussion wise maybe you agree but would a triangle wave prove it? Give it a try? Most textbooks do not show it with the triangle wave, may be not as cool looking in comparison to a sine wave, there may been one or two textbooks, and they may been easier to belief. It may matter for some, especially, if you are a practicing lawyer or law student.

clear(t, t1, t2)

$t1 := 0, 0.05 .. 0.25$ $\qquad$ $t2 := 0.25, 0.30 .. 0.5$

$f1(t1) := 2 \cdot t1$ $\qquad$ $f2(t2) := 1 - (2) \cdot t2$

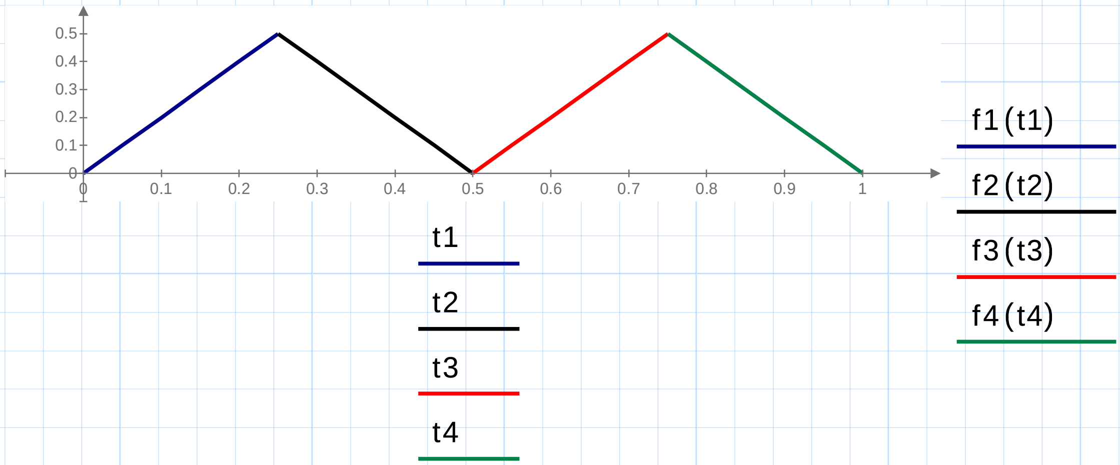
*About how we got f1(t1) and f2(t2): (1 - 2*t2) the 1? Start at -2*t2, t2 = 0.25, so -2*0.25=-0.5. The f1? at t1 = 0.25, f1(0.25) =2*0.25 = 0.5. We need to make f2(t2) = 0.5 so we place 1 in there and subtract f2(0.25) from it; f2(t2) = 1 - 2(t2). Solves it.*



Let do the whole one cycle.

$t1 := 0, 0.05 .. 0.25$ $\quad t2 := 0.25, 0.30 .. 0.5$ $\quad t3 := 0.5, 0.55 .. 0.75$ $\quad t4 := 0.75, 0.80 .. 1.0$

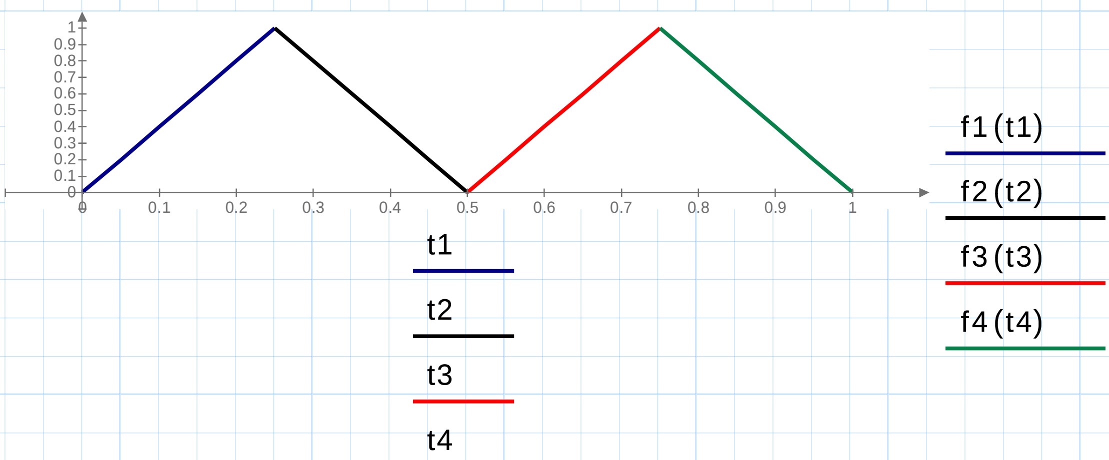
$f1(t1) := 2 \cdot t1$ $\quad f2(t2) := 1 - (2) \cdot t2$ $\quad f3(t3) := 2 \cdot t3 - 1$ $\quad f4(t4) := 2 - (2) \cdot t4$



Next we move it up to a magnitude 1 instead of the half above. All this is, YES, studies in electrical engineering circuits. So you know the range and how to graph/plot each function. *Tricks! TRICKS!* This is all on the positive side below, **how about positive and negative**? Next

$t1 := 0, 0.05 .. 0.25$ $\quad t2 := 0.25, 0.30 .. 0.5$ $\quad t3 := 0.5, 0.55 .. 0.75$ $\quad t4 := 0.75, 0.80 .. 1.0$

$f1(t1) := 4 \cdot t1$ $\quad f2(t2) := 2 - (4) \cdot t2$ $\quad f3(t3) := 4 \cdot t3 - 2$ $\quad f4(t4) := 4 - (4) \cdot t4$

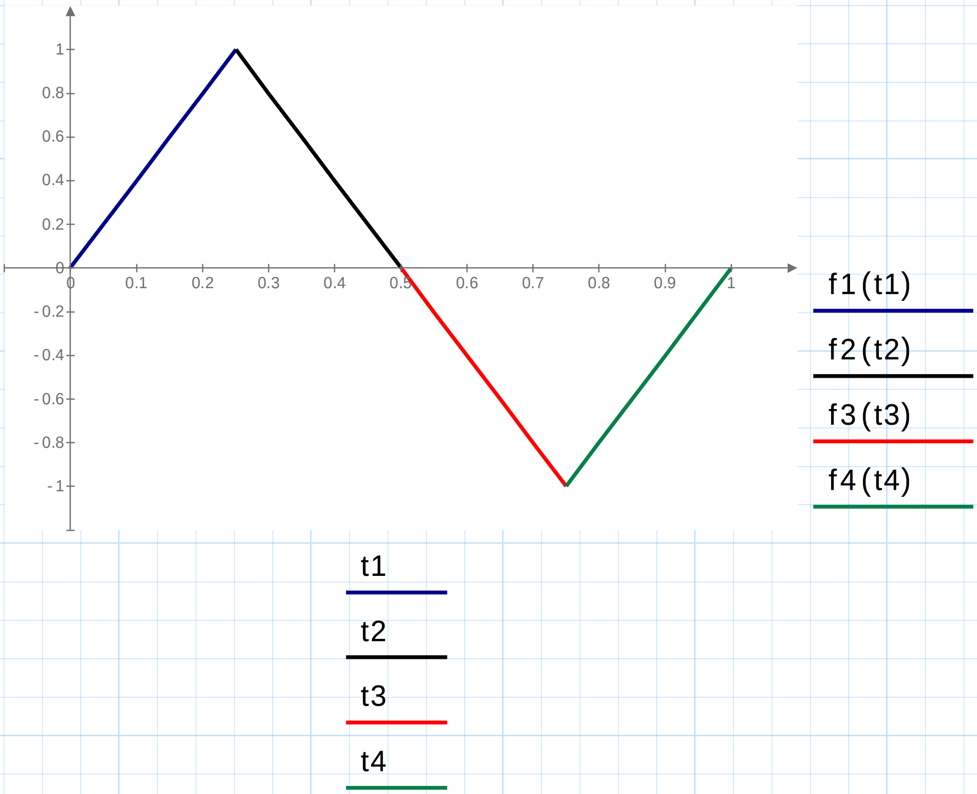


Positive & Negative triangle wave? Not easy for me, never done it. NEVER had use for it. Its typically used in electronics, various forms of triangles. My work was construction electrical engineering into sinusoidal waveforms for power systems.

clear (t , t1 , t2 , t3 , t4)

$t1 := 0, 0.05 .. 0.25$ $t2 := 0.25, 0.30 .. 0.5$ $t3 := 0.5, 0.55 .. 0.75$ $t4 := 0.75, 0.80 .. 1.00$

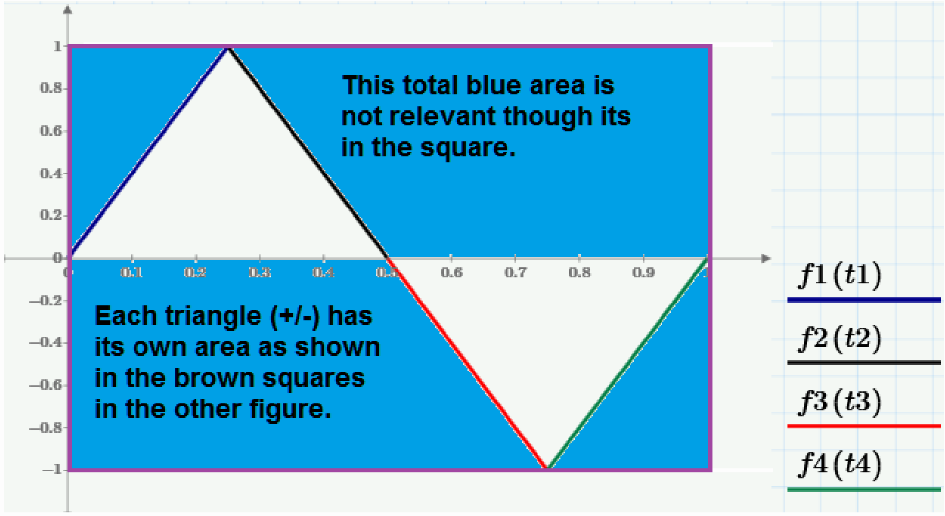
$f1(t1) := 4 \cdot t1$ $f2(t2) := 2 - (4) \cdot t2$ $f3(t3) := 2 - 4 \cdot t3$ $f4(t4) := (4) \cdot t4 - 4$



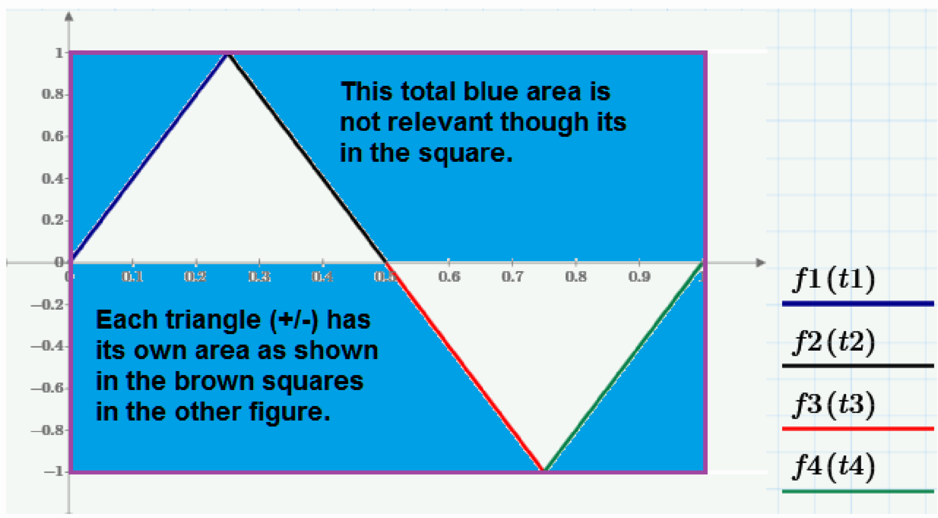
Happy!

You work through each functions, do them next on Excel. Though you have a math software or some similar software that does it, work it in Excel so you get a good feel for Excel. This comes in use more times for an Engineer then other software, had in the past,...... It will improve your programming skills too, manually. Excel is more used than any single other software.

Discussion:



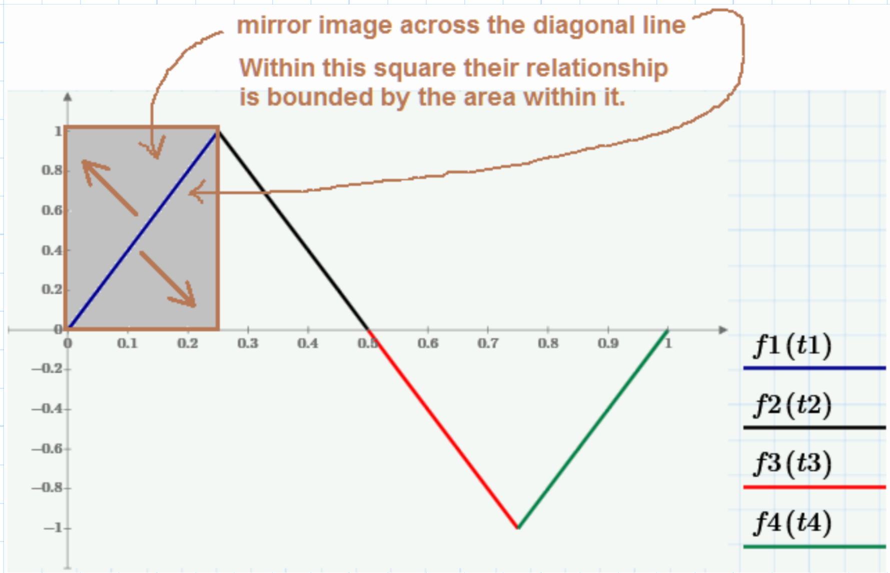
The two brown squares, shown below, with each square representing its own triangle. Each triangle is half the area of each square. AGREED. So the total of the area of the triangles are (1/2) + (1/2) = 1.



The area of the grey shaded part is equal to the dark blue shaded triangle, its half in each brown outline square. Grey area total in both top and bottom is also 1.

What ever the grey area is the dark blue is same area. So we can settle on that. Why would each be half instead of 1, area under the slope of the triangle would be ? 1/4 + 1/4 = 1/2. Not (1/2) + (1/2) = 1. True. But the area of concern from the beginning was the area under the? Function f(t). So now? Its (1/2) + (1/2) = 1. So now we take the integral over the period 0 to T = 1, the area of the triangle (dark blue shaded) equal 1.

To clarify further, so that we can proceed with the area equal 1 under the function bounded by the time axis and f(t).



Now, given we had a discussion and it was debatable, but we pushed forward.
Hey, you could had used area of triangle, (1/2) base X height. Done! No arguement, but (1/2)(0.5)(1) = 0.25. Half base and thats 0.5 for whole top triangle, the height equal 1. Now the same for the bottom. Add them that equal? 0.5. NOT ACCEPTABLE.

PUSHED forward.

The F average equation we saw earlier, we now apply to the triangle wave.

$$F_{average} = f(t) = \left(\frac{1}{T}\right)\cdot\int_0^T f(t)\,dt$$

$$T = 1$$

$$\int_0^T f(t)\,dt = 1$$ **Pushed through**, ROBUST ENGINEERING.

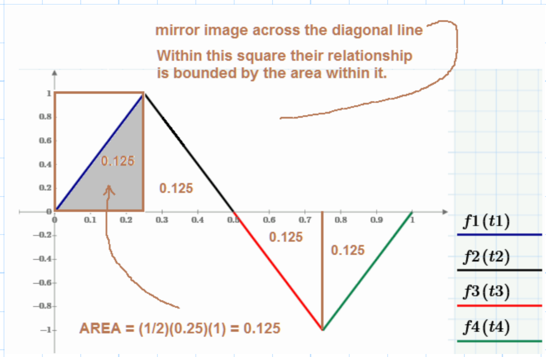
$$\left(\frac{1}{T}\right)\cdot\int_0^T f(t)\,dt = \frac{1}{T}\cdot 1 = 1$$

So, the formula does work.

Hopefully, well, forget hopefully, dont bother with it further to say its not accurate or its OFF. Clearly the area under the curve f(t) for any periodic function this formula will be applicable. This is the starting point in the signals study. **HAPPY! NOT?**

AGAIN, let see if we can correct that area of triangle case. Last defense.

Using area of triangle:



Upper LEFT SIDE triangle area: 0.125

$\left(\frac{1}{2}\right)\cdot(0.25)\cdot(1) = 0.125$

Upper RIGHT SIDE triangle area: 0.125

Lower LEFT SIDE triangle area: 0.125

Lower RIGHT SIDE triangle area: 0.125

Total: 0.500

Where I went WRONG!

Each slope is an independent equation, the formula has to be applied each time for each slope. So now we are looking for the final area of 0.5 NOT 1.
Will this work?

$$F_{upper\_LEFT\_triangle} = f(t) = \left(\frac{1}{T}\right)\cdot\int_0^T f(t)\,dt$$

$$T = 0.25$$ Each slope time interval is 0.25 seconds.

$$\int_0^T f(t)\,dt = 0.125$$ Area.

$$\left(\frac{1}{T}\right)\cdot\int_0^T f(t)\,dt = \frac{1}{T}\cdot 0.125 = \frac{1}{0.25}\cdot 0.125 = 0.5$$

If this is correct, then the upper left and right triangle halves add up to 0.5 + 0.5 = 1

Upper triangle = 1, so lower triangle = -1.
But the area will not be negative, is there such a thing as a negative area, only in relation to the graph axis, so we have the absolute value of the magnitude. Now we have the total area 1 + 1 = 2.

BUT THE FUNCTION IS f1(t1) = 4t1...the plots we did for the first upper right slope.

$f1(t1) := 4\cdot t1$ $f2(t2) := 2-(4)\cdot t2$ $f3(t3) := 2-4\cdot t3$ $f4(t4) := (4)\cdot t4-4$

Those functions shown above. Lets try the math.

Intergal of f1(t1), the Upper Left triangle:

$$f1(t1) := 4 \cdot t1$$

$$\int_0^{0.25} f1(t1)\,dt1 = \int_0^{0.25} 4\ t1\,dt1 = \left(\frac{4}{2}\right) \cdot t1^2 + C$$

Since there are no initial conditions to the area C = 0.

$$\left(\frac{4}{2}\right) \cdot t1^2 = 2 \cdot (0.25)^2 = 0.125$$ SAME AREA !

BUT we are not finished yet we have to divide by T.

$$T1 := 0.125$$

$$\left(\frac{1}{T1}\right) \cdot \int_0^{0.25} f1(t1)\,dt = \frac{1}{T1} \cdot 0.125 = \frac{1}{0.125} \cdot 0.125 = 1$$

Looking at the steepness of the slope we know all are the same, so each quarter triangle final result is 1, that totals to 1+1+|-1|+|-1| = 4. If we dont take the absolute value the top mius the bottom is 1+1 +(-1-1) = 2 + (-2) = 0.
From our understanding the triangle is half the square so the area certainly is correct by that calculation. WHERE IS THE MISTAKE?

IF YOU FIND THIS FUNNY THEN YOU GOT A GOOD GROUND TO ASK THE LOCAL ENGINEER AND THE ELECTRICAL ENGINEER PROFESSOR.

I am not sure if there is a mistake here, if there is I will get an email form you.

For this discussion and THEORY the ***robust engineering*** way is the way.
Here we managed to push through the discussion to match the theory. Elsewhere it did not proof encouraging.

We all enjoyed this short journey. You fix the mistakes in the discussion there maybe a few, maybe.

Well its just math language it has its short comings - Karl Bogha.

END OF DEVIATION, WE CONTINUE WITH AVERAGE AND EFFECTIVE (RMS ROOT MEAN SQUARE) VALUES.

$$F_{average} = f(t) = \left(\frac{1}{T}\right)\cdot\int_0^T f(t)\,dt$$ <----(1/T) x Area of the function
Frequency x Area of Function within one period. Area can be voltage, current,.....

F_average is the function's average, frequency runs thru the time duration t, 50 Hz means for 1 second its cycled 50 times, next second it does another 50 cycles, for whatever time t is under consideration.

What then is the value of the integral over the period T (time) multiplied by the frequency?

Lets say f = 50, thats 50 cycles per second. 1/f = 0.02 = T. This T is the?
Upper limit of the integral.

What is the value of the integral over a period of time T (1/f) i.e. 0.02 second?
The voltage or current for that period T (0.02 s), ...what ever f(t) represents be it power, voltage, current,...... This T here is the upper limit of the integral.

So thats all we got for time (1/f) that is T, not the full length of 50 cycles rather 1 cycle.

We got the integral solved next we multiply it with the frequency. The frequency is shown as 1/T. Frequency we know is f, period T is 1/f, and 1/T is the? frequency.

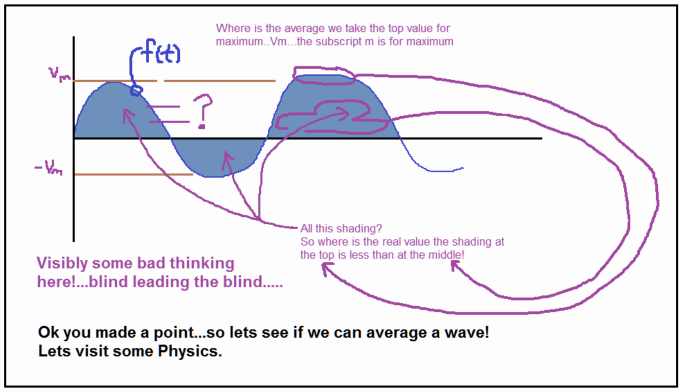
So we multiply the integral by the frequency 50 Hz which is over period of 1 second.

Does this give us the voltage, current, power,.....f(t) over 1 second? Yes.
Not roughy speaking in terms of the accuray of the calucation rather the logic first?
Yes. The logic is thats the final value for a time duration of 1 second.

Can we call this an average value? Not really when we usually average we have it over a wider range, but f(t) over one period is the same pattern for the next period, its the same value if nothing changes. True. So, its not the usual average. BUT, if we see this period the same over the other periods and for the duration of the 1 second, 50 cycls/period, can we then call this a sort of generated thru some average sort of thing? Yes.

So finally we got an average for 1 second that is over the entire frequency.

$$F_{average} = f(t) = \left(\frac{1}{T}\right)\cdot\int_0^T f(t)\,dt$$ <----Agreeable.



$$F_{average} = f(t) = \left(\frac{1}{T}\right)\cdot\int_0^T f(t)\,dt$$

<----Agreeable.
We seen this figure in the early discussion last.
BUT where is the point where we can say that is the voltage?
If we used an instrument where is the voltage we ageee to?
We got an average, so far.

Well, the theory goes a little further, with the use of the phrase 'root mean square'.
When we square a value, a negative value becomes positive.
We know this much. So if we squared the function f(t) then each value of f at time t is positive.

$$\left(\frac{1}{T}\right)\cdot\int_0^T f(t)^2\,dt$$

<--- See the square placed in for f(t).

Our values obviously have gotten BIGGER than they actually were.
How do we fix this? We wanted to get things postive, you know the sinusoidal curve has positive and negative amplitudes, we solved that, squared it, now take the?
SQUARE ROOT of the whole expression. This will be a positive number, and brought back to the same before squared. True? Almost true. It does bring it closer to the actual compared to not taking the square root, the slight difference being in the (1/T) was not squared but got square rooted. This is negligible, say its within a margin of error.
Otherwise? You got nothing!

$$\left(\left(\frac{1}{T}\right)\cdot\int_0^T f(t)^2\,dt\right)^{\frac{1}{2}}$$

<--- See the square root inserted over the entire expression.

So we read it like this from the outside going inside from the right to the left:

1. **ROOT** - the square root over the whole expression.
2. **MEAN** - 1/T, 1 divided by T isnt really the mean?
   The integral is from 0 - T, then we multiply by? 1/T so this 1/T is the idea of mean.
3. **SQUARE** - Going from left to right the function f(t) was? squared.

Now, the root-mean-square (RMS) or also known as effective value, of f(t), during the period T, is:

$$F_{RMS} = F_{EFF} = \left(\left(\frac{1}{T}\right)\cdot\int_0^T f(t)^2\,dt\right)^{\frac{1}{2}}$$

What you were looking for.

As usual, we can play around with squaring and taking the square root to reach an understanding on the desired outcome - Karl Bogha.

Now if we square F_RMS what do we get?

$$(F_{RMS})^2 = \left[\left(\left(\frac{1}{T}\right)\cdot\int_0^T f(t)^2\,dt\right)^{\frac{1}{2}}\right]^2$$

$$(F_{RMS})^2 = \left(\frac{1}{T}\right)\cdot\int_0^T f(t)^2\,dt$$

Now we see (F_RMS)^2 equal function f(t) squared over interval T and averaged by 1/T

$$(F_{RMS})^2 = \left(f(t)^2\right)$$

NOW, thru this $F_{RMS} = \left(\left(\frac{1}{T}\right)\cdot\int_0^T f(t)^2\,dt\right)^{\frac{1}{2}}$ we have a specific value for the function f(t).

When you see a voltmeter/current meter reading it says RMS, this is what it is.
Is it common sense its always RMS? No, meter wise you can get peak value also, which is Vm or Im. Its not. Most time yes. We were looking for a definite value in the periodic function f(t), we got to a close as we can get. Root_Mean Square.

Its always been that the AVERAGE and RMS/EFFECTIVE values are computed over one period.

You will find in electrical power systems the square root of 2, applied to solve for the RMS.
Peak voltage is 400V so the V_RMS? 400/ Sqrt(2).

$$\frac{400}{\sqrt{2}} = 282.843 \quad \text{V\_rms}$$

$$\frac{1}{\sqrt{2}} = 0.707 \quad \text{OR} \quad 0.707\cdot 400 = 282.8 \quad \text{V\_rms}$$

This is the end of the RMS story.
Apologies for all my errors.

Few Words of Encouragement:

The 1st edition of this book or first publication for Electric Circuits was in 1965. Later in 2014 this edition was called the 6th edition. This book has 480 pages.

Its a supplement to a textbook now. At 480 pages? A textbook today on circuits maybe 800 pages, no less than 700. The Hyat and Kemmerly 4th edition was 600 pages in 1986. Very intense and in-depth. 600 pages for engineering electric circuits. These days with the graphics quality available the number of pages increase. Some textbooks are highly over ambitious.

What I am getitng at is that its NOT possible to collect all the studies provided in the textbook in a 2 semester course. One semester is impossible these books content is for a minimum 2 semesters. The detailed knowledge in them is NOT possible to be colleted, gathered, studied in full. Not all chapter can be covered in full in 2 semesters either, whilst solving exercise problems, taking tests, and exams.

PLUS if youre a college student full or part time you got 5 or 2 courses a semester. That makes it impossible to cover the depth of the circuits subject. You are welcome to belief otherwise. I said it was NOT possible.

Some of it seems applicable for specific upper level courses. Some sections are more in tune to particular upper level or advanced courses, other sections more for other courses. And not all the chapters can be fully covered page after page. That's the few words of encouragement.

Next we work 5 or so examples on the sinusoidal function.

It is taxing, tiring, exhaustive, but the sinusoidal function subject matter you know as well is of significant importance based on the wave form's capability, its mathematical benefits, and its use in the power industry. So, we will attempt them. Some examples have to do with providing a theory thru worked examples.

Then to the last section on Random signals just a short introduction on this. Just so you got an idea on this topic like whats its about.

Example 6.9.

Find the average and effective values of the **cosine wave**
v(t) = Vm cos(wt + Theta). Using the F_average math expression. *Here place V for F.*

$$V_{average} = v(t) = \left(\frac{1}{T}\right)\cdot\int_0^T v(t)\,dt$$

Solution:

a).

$$V_{average} = \left(\frac{1}{T}\right)\cdot\int_0^T V_M\cdot\cos(\omega t+\theta)\,dt$$

You know v(t) = f(t). Hello!....Yes.

$$= \left(\frac{V_m}{\omega t}\right)\cdot\left(\frac{1}{T}\right)\cdot\sin(\omega t+\theta)$$

Lim t: T-->0

$$= \left(\frac{V_m}{T\cdot\omega t}\right)\cdot\sin(\omega t+\theta)$$

Lim T-->0

$$= \left(\frac{V_m}{T\cdot\omega T}\right)\cdot\sin(\omega T+\theta) - \left(\frac{V_m}{0\cdot\omega 0}\right)\cdot\sin(\omega 0+\theta)$$

First term T equal 0 because **wT** is one revolution, like 360 deg = 0.
Second term is 0 as sin(0) = 0.

$$= 0 - 0$$

$$V_{average} = 0$$ Answer.

b).

$$V_{RMS} = \left(\left(\frac{1}{T}\right)\cdot\int_0^T v(t)^2\,dt\right)^{\frac{1}{2}}$$

<---V eff OR V rms.
We cannot jump into it straight,
first we need to evaluate v(t)^2.

$$v^2(t) = V_M^2\cdot\cos(\omega t+\theta)^2$$

Trig identity: $\cos(\omega t+\theta)^2 = \left(\frac{1}{2}\right)(1+\cos 2(\omega t+\theta))$

$$v^2(t) = \frac{V_M^2}{2}\cdot(1+\cos 2(\omega t+\theta))\,dt$$

$$\frac{V_m^{\,2}}{2}\int_0^T (1+\cos 2(\omega t+\theta))\,dt$$

$$\int_0^T 1+\cos 2(\omega t+\theta)\,dt \quad = \quad t+\int_0^T \cos 2(\omega t+\theta)\,dt$$

$$\int_0^T \cos 2(\omega t+\theta)\,dt \quad = ?$$

$$u = (\omega t+\theta) \qquad \frac{du}{dt} = \omega \qquad du = \omega\,dt$$

$$\int_0^T \cos 2(\omega t+\theta)\,dt = \left(\frac{dt}{du}\right)\cdot\int_0^T \cos 2(u)\,du$$

<--- You check thru this integration.

$$= \frac{1}{\omega}\cdot\int \cos 2(u)\cdot\omega\,dt$$

$$= \frac{1}{\omega}\cdot\int \cos 2(\omega t+\theta)\cdot\omega\,dt$$

$$= \frac{1}{\omega}\cdot\sin 2(\omega t+\theta)$$

......(1/w)(1/w)(sin2(wt+theta)w

Lim t: T-->0, substitute T for 1/f.

$$= \left(\frac{1}{\omega}\cdot\sin 2\left(2\pi f\frac{1}{f}+\theta\right)\right)-\left(\frac{1}{\omega}\cdot\sin 2(2\pi f(0)+\theta)\right) = \left(\frac{1}{\omega}\cdot\sin 2(2\pi+\theta)\right)-\left(\frac{1}{\omega}\cdot\sin 2(0+\theta)\right)$$

$= 0$ *Sin2 (2 Pi + Theta) = (Sin 2(0) + Theta) - (Sin(0) + Theta) = 0.*
Theta is ONLY the phase angle, not the t-axis (time). Theta - Theta = 0.

$$\frac{V_m^{\,2}}{2}\int_0^T (1+\cos 2(\omega t+\theta))\,dt = \frac{V_m^{\,2}}{2}\cdot(t-0) = \frac{V_m^{\,2}}{2}\cdot(t)$$

Lim t: T-->0

$$= \frac{V_m^{\,2}}{2}\cdot(T)$$

$$V_{RMS} = \left[\left(\frac{1}{T}\right)\cdot\int_0^T f(t)^2\,dt\right]^{\frac{1}{2}} = \left[\left(\frac{1}{T}\right)\cdot\left(\frac{V_m^{\,2}}{2}\cdot(T)\right)\right]^{\frac{1}{2}} = \left(\frac{V_m^{\,2}}{2}\right)^{\frac{1}{2}}$$

$$V_{RMS} \quad \text{OR} \quad V_{EFF} = \frac{V_m}{\sqrt{2}} = 0.707\cdot V_m$$ **Answer.**

Both V_average and V_effective are independent of ? Frequency and Phase Angle. So the conclusion is the average and rms of a cosine wave are always 0.707 Vm respectively.

Example 6.9 B. Lets say we make it sine wave?

Find the average and effective values of the **sine wave**
v(t) = Vm sin(wt + Theta). Using the F_average math expression.

$$V_{average} = v(t) = \left(\frac{1}{T}\right)\cdot\int_0^T v(t)\,dt$$

Solution:

<---This was not a Schaums example, so you verify.
Merely changed the pervious example to sine.

a).

$$V_{average} = \left(\frac{1}{T}\right)\cdot\int_0^T V_M\cdot\sin(\omega t+\theta)\,dt$$

$$= -\left(\frac{V_m}{\omega t}\right)\cdot\left(\frac{1}{T}\right)\cdot\cos(\omega t+\theta)$$

Lim T-->0

$$= -\left(\frac{V_m}{T\cdot\omega t}\right)\cdot\cos(\omega t+\theta)$$

Lim T-->0

$$= \left(-\left(\frac{V_m}{T\cdot\omega T}\right)\cdot\cos(\omega T+\theta) - \left(\frac{V_m}{0\cdot\omega 0}\right)\cdot\cos(\omega 0+\theta)\right)$$

First term T equal 1 because wT is one revolution, like 360 deg = 0
Second term is 1.

$$= -(1-1)$$

$$V_{average} = 0$$ Answer.

b).

$$V_{RMS} = \left(\left(\frac{1}{T}\right)\cdot\int_0^T v(t)^2\,dt\right)^{\frac{1}{2}}$$

<---V eff OR V rms.
We cannot jump into it straight,
first we need to evaluate v(t)^2.

$$v^2(t) = V_M{}^2\cdot\sin(\omega t+\theta)^2$$ Trig identity: $\sin(\omega t+\theta)^2 = \left(\frac{1}{2}\right)(1-\cos2(\omega t+\theta))$

$$v^2(t) = \frac{V_M{}^2}{2}\cdot(1-\cos2(\omega t+\theta))\,dt$$

$$\frac{V_m^{\,2}}{2}\int_0^T (1-\cos 2(\omega t+\theta))\,dt$$

$$\int_0^T 1-\cos 2(\omega t+\theta)\,dt \quad = \quad t-\int_0^T \cos 2(\omega t+\theta)\,dt$$

$$-\int_0^T \cos 2(\omega t+\theta)\,dt \quad = ?$$

$$u = (\omega t+\theta) \qquad \frac{du}{dt} = \omega \qquad du = \omega\,dt$$

$$-\int_0^T \cos 2(\omega t+\theta)\,dt = -\left(\frac{dt}{du}\right)\cdot\int_0^T \cos 2(u)\,du$$

<--- You check thru this integration.

$$= -\frac{1}{\omega}\cdot\int \cos 2(u)\cdot\omega\,dt$$

$$= -\frac{1}{\omega}\cdot\int \cos 2(\omega t+\theta)\cdot\omega\,dt$$

$$= -\frac{1}{\omega}\cdot\sin 2(\omega t+\theta)$$

......(1/w)(1/w)(sin2(wt+theta)w

Lim t: T-->0, substitute T for 1/f.

$$= -\left(\frac{1}{\omega}\cdot\sin 2\left(2\pi f\frac{1}{f}+\theta\right)\right)+\left(\frac{1}{\omega}\cdot\sin 2(2\pi f(0)+\theta)\right) = -\left(\frac{1}{\omega}\cdot\sin 2(2\pi+\theta)\right)+\left(\frac{1}{\omega}\cdot\sin 2(0+\theta)\right)$$

$= 0$

Sin2 (2 Pi + Theta) => -(Sin 2(0) + Theta) + (Sin(0) + Theta) = 0.
Theta is ONLY the phase angle, not the t-axis (time). -Theta + Theta = 0.

$$\frac{V_m^{\,2}}{2}\int_0^T (1-\cos 2(\omega t+\theta))\,dt = \frac{V_m^{\,2}}{2}\cdot(t-0) = \frac{V_m^{\,2}}{2}\cdot(t)$$

Lim t: T-->0

$$= \frac{V_m^{\,2}}{2}\cdot(T)$$

$$V_{RMS} = \left(\left(\frac{1}{T}\right)\cdot\int_0^T f(t)^2\,dt\right)^{\frac{1}{2}} = \left(\left(\frac{1}{T}\right)\cdot\left(\frac{V_m^{\,2}}{2}\cdot(T)\right)\right)^{\frac{1}{2}} = \left(\frac{V_m^{\,2}}{2}\right)^{\frac{1}{2}}$$

$$V_{RMS} \quad \text{OR} \quad V_{EFF} = \frac{V_m}{\sqrt{2}} = 0.707\cdot V_m$$ Answer.

Both V_average and V_effective are independent of ? Frequency and Phase Angle. So the conclusion is the average and rms of a sine wave are always 0.707 Vm respectively. You verify with your local engineer. *Comment: Of course its correct. Whats the difference just their starting values of sine and cosine at 0 deg. Surprising results are the same.*

Example 6.10 A

Find the V average and V effective of the half rectified sine wave.
v(t) = Vm sin(wt + Theta). Using the F_average math expression.

$$v(t) = \begin{cases} V_m \sin(\omega t) & \textit{when } \sin(wt) > 0. \\ 0 & \textit{when } \sin(wt) < 0. \end{cases}$$

Solution:

Half rectified means? Half the wave, (T/2).

$$V_{average} = v(t) = \left(\frac{1}{T}\right)\cdot\int_0^{\frac{T}{2}} v(t)\,dt$$

a).

$$V_{average} = \left(\frac{1}{T}\right)\cdot\int_0^{\frac{T}{2}} V_M \cdot \sin(\omega t)\,dt$$ <-- Note: No phase angle here.

$$= \left(\frac{V_M}{\omega T}\right)\cdot -\cos(\omega t)$$

Lim t: T/2-->0

$$= -\left(\left(\frac{V_M}{\omega (T)}\right)\cdot\cos\left(\omega\cdot\left(\frac{T}{2}\right)\right) - \cos(\omega\cdot 0)\right)$$

$$= -\left(\left(\frac{V_M}{2\pi f\left(\frac{1}{f}\right)}\right)\cdot\cos\left(2\pi f\cdot\left(\frac{1}{2f}\right)\right) - \cos(\omega\cdot 0)\right)$$

$$= -\left(\frac{V_M}{2\pi}\right)\cdot(\cos(\pi) - \cos(0))$$

$$= -\left(\frac{V_M}{2\pi}\right)\cdot((-1)-1) = -\left(\frac{V_M}{2\pi}\right)\cdot(-2) = \left(\frac{V_M}{\pi}\right)$$

$$= \left(\frac{V_M}{\pi}\right)$$ Anwer. I made some mistakes on the 'sign', so it took me an eternity and almost another half, for a half wave!

b).

$$V_{RMS} = \left(\left(\frac{1}{T}\right) \cdot \int_0^{\frac{T}{2}} v(t)^2 \, dt \right)^{\frac{1}{2}}$$

<---V eff OR V rms.
We need to evaluate v(t)^2.

$$v^2(t) = V_m{}^2 \cdot \sin(\omega t)^2$$

Trig identity: $\sin(\omega t)^2 = \left(\frac{1}{2}\right)(1 - \cos 2(\omega t))$

$$v^2(t) = \frac{V_M{}^2}{2} \cdot \int_0^{\frac{T}{2}} (1 - \cos 2(\omega t)) \, dt$$

$$\int_0^{\frac{T}{2}} (1 - \cos 2(\omega t)) \, dt = t - \int_0^{\frac{T}{2}} \cos 2(\omega t) \, dt$$

Lim t: T/2-->0

$$-\int_0^{\frac{T}{2}} \cos 2(\omega t) \, dt = ?$$

$u = (\omega t)$ $\quad \frac{du}{dt} = \omega$ $\quad du = \omega \, dt$

$$-\int_0^{\frac{T}{2}} \cos 2(\omega t) \, dt = -\left(\frac{1}{\frac{du}{dt}}\right) \int_0^{\frac{T}{2}} \cos 2(u) \, du$$

<--- You check thru this integration.

$$= -\frac{1}{\omega} \cdot \int \cos 2(\omega t) \cdot \omega \, dt$$

$$= -\frac{1}{\omega} \cdot \sin 2(\omega t)$$

......(1/w)(1/w)(sin2(wt)w

Lim t: T/2-->0

$$= -\left(\frac{1}{\omega} \cdot \sin 2\left(\omega\left(\frac{T}{2}\right)\right)\right) + \left(\frac{1}{\omega} \cdot \sin 2(\omega 0)\right)$$

$$= -\left(\frac{1}{2\pi f} \cdot \sin 2\left((2\pi f) \cdot \frac{1}{2f}\right)\right) + \left(\frac{1}{2\pi f} \cdot \sin 2(0)\right)$$

$$= -\left(\frac{1}{2\pi f} \cdot \sin 2(\pi)\right) + \left(\frac{1}{2\pi f} \cdot 0\right) = -\left(\frac{1}{2\pi f} \cdot 0\right) + 0 = -0 + 0 = 0$$

2nd term is 0

$$v^2(t) = \frac{V_M^2}{2}\cdot(t)$$ <---We have the first term

Lim t: T/2-->0

$$= \frac{V_M^2}{2}\cdot\left(\frac{T}{2}\right) - 0$$

$$= \frac{V_M^2}{4f}$$

$$= \left(\frac{V_M^2}{4}\right)\cdot T$$ Not over yet continuing........

$$V_{RMS} = \left(\left(\frac{1}{T}\right)\cdot\int_0^T f(t)^2\,dt\right)^{\frac{1}{2}}$$

$$= \left(\left(\frac{1}{T}\right)\cdot\left(\frac{V_M^2}{4}\right)\cdot T\right)^{\frac{1}{2}}$$

$$= \left(\frac{V_M^2}{4}\right)^{\frac{1}{2}}$$

V_{RMS} OR V_{EFF} $= \left(\frac{V_M}{2}\right)$ Answer.

You verify the integration steps.
Thats the answer in Schaums.
I hesitate to do the half rectified cosine wave because it does not start at 0 when t = 0 rather cos(0) = 1, ends maybe earlier!
Yes, the integration sorts things out.
May be you give it a try to find out if its the same answer. Should be?
Give it a try next.

Example 6.10 B.

Find the V average and V effective of the half rectified cosine wave.
v(t) = Vm cos(wt + Theta). Using the F_average math expression.

$$v(t) = \begin{cases} V_m \cos(\omega t) & \text{when } \cos(wt) > 0. \\ 0 & \text{when } \cos(wt) < 0. \end{cases}$$

Solution:

Half rectified means? Half the wave, (T/2).

$$V_{average} = v(t) = \left(\frac{1}{T}\right) \cdot \int_0^{\frac{T}{2}} v(t)\, dt$$

a).

$$V_{average} = \left(\frac{1}{T}\right) \cdot \int_0^{\frac{T}{2}} V_M \cdot \cos(\omega t)\, dt$$ <-- Note: No phase angle here.

$$= \left(\frac{V_M}{\omega T}\right) \cdot \sin(\omega t)$$

Lim T/2-->0

$$= \left(\left(\frac{V_M}{\omega\left(\frac{T}{2}\right)}\right) \cdot \sin\left(\frac{\omega}{2} \cdot \left(\frac{T}{2}\right)\right) - \left(\frac{V_M}{\omega\, 0}\right) \cdot \sin(\omega\, 0)\right)$$

$$= \left(\frac{V_M}{2\pi f \frac{1}{2\,f}}\right) \cdot \sin\left(\frac{2\pi f}{2} \cdot \frac{1}{2\,f}\right) - \left(\frac{2\, V_M}{2\pi f \cdot 0}\right) \cdot \sin(2\pi f \cdot 0)$$

$$= \left(\frac{V_M}{\pi}\right) \cdot \sin\left(\frac{\pi}{2}\right) - (0) \cdot \sin(0)$$

$$= \left(\frac{V_M}{\pi}\right) \cdot (1) - 0$$

$$= \left(\frac{V_M}{\pi}\right)$$ Anwer. Same Answer! You verify. Wave may start early but it finishes late in comparison to sine wave within the same period.

b).

$$V_{RMS} = \left(\left(\frac{1}{T}\right)\cdot\int_0^{\frac{T}{2}} v(t)^2\,dt\right)^{\frac{1}{2}}$$ <---V eff OR V rms.
We need to evaluate v(t)^2.

$$v^2(t) = V_m^{\,2}\cdot\cos(\omega t)^2$$ Trig identity: $\cos(\omega t)^2 = \left(\frac{1}{2}\right)(1+\cos2(\omega t))$

$$v^2(t) = \frac{V_M^{\,2}}{2}\cdot\int_0^{\frac{T}{2}}(1+\cos2(\omega t))\,dt$$

$$\int_0^{\frac{T}{2}}(1+\cos2(\omega t))\,dt = t+\int_0^{\frac{T}{2}}\cos2(\omega t)\,dt$$

Lim T/2-->0

$$\int_0^{\frac{T}{2}}\cos2(\omega t)\,dt = ?$$

$$u = (\omega t) \qquad \frac{du}{dt} = \omega \qquad du = \omega\,dt$$

$$\int_0^{\frac{T}{2}}\cos2(\omega t)\,dt = \left(\frac{1}{\frac{du}{dt}}\right)\int_0^{\frac{T}{2}}\cos2(u)\,du$$ <--- You check thru this integration.

$$= \frac{1}{\omega}\cdot\int\cos2(\omega t)\cdot\omega\,dt$$

$$= \frac{1}{\omega}\cdot\sin2(\omega t)$$

Lim T/2-->0

$$= \frac{1}{\omega}\cdot\sin2\left(\omega\left(\frac{T}{2}\right)\right)-\frac{1}{\omega}\cdot\sin2(\omega 0)$$

$$= \frac{1}{2\pi f}\cdot\sin2\left((2\pi f)\cdot\frac{1}{2f}\right)-\frac{1}{2\pi f}\cdot\sin(0)$$

$$= \left(\frac{1}{2\pi f}\right)\cdot\sin2(\pi)-0 \qquad = \left(\frac{T}{2\pi}\right)\cdot\sin2(\pi)-0$$

$$= 0-0$$

$$= 0$$

$$v^2(t) = \frac{V_M^{\,2}}{2}\cdot(t)$$ We have first term, 2nd term resulted in 0.

Lim T/2-->0

$$= \frac{V_M^{\,2}}{2}\cdot\left(\frac{T}{2}\right)$$

$$= \frac{V_M^{\,2}}{4\ f}$$

$$= \left(\frac{V_M^{\,2}}{4}\right)\cdot T$$ Next calculate the rms or effective.

$$V_{RMS} = \left(\left(\frac{1}{T}\right)\cdot\int_0^T f(t)^2\ dt\right)^{\frac{1}{2}}$$

$$= \left(\left(\frac{1}{T}\right)\cdot\left(\frac{V_M^{\,2}}{4}\right)\cdot T\right)^{\frac{1}{2}}$$

$$= \left(\frac{V_M^{\,2}}{4}\right)^{\frac{1}{2}}$$

V_{RMS} OR V_{EFF} $= \left(\frac{V_M}{2}\right)$ Answer. Same answer with sine wave.
Check with your local engineer.

Example 6.9 and 6.10 both had so much of the same.

Key words: *SO MUCH OF THE SAME!*
....why not, makes things simple, the benefits of sinusoidal wave expression mostly the same trignometry functions.

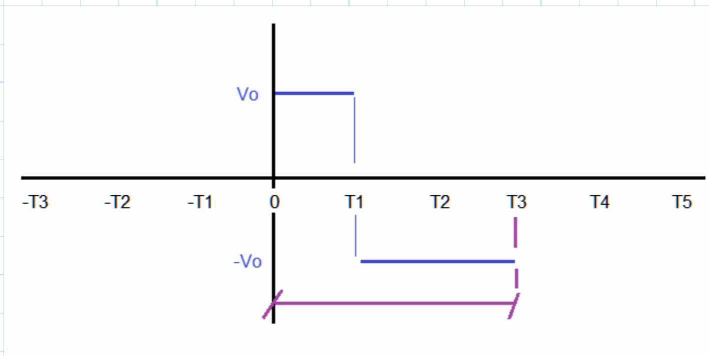
Example 6.11 (Periodic Function).

Find V_avg and V_eff of the periodic function v(t) where, for one period T,

$$v(t) = \begin{cases} V_0 & \text{for } 0 < t < T1 \\ -V_0 & \text{for } T1 < t < 3T1 \end{cases}$$

Period T = 3T1.

Solution:

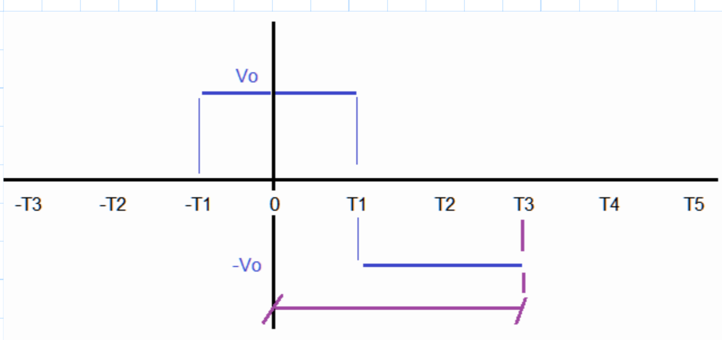


Working toward the expression for V_average below.

$$V_{average} = \left(\frac{1}{T}\right) \cdot \int_0^T v(t)\, dt$$

<---- v(t) waveform.
Not in textbook.
Sketched here.

Could the waveform be as shown below going into t<0? Reason I ask is on how to make the decision on the signal or waveform. Should we see it as below or how the engineers authors decided in textbook.

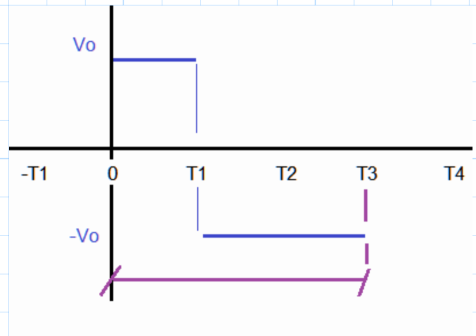


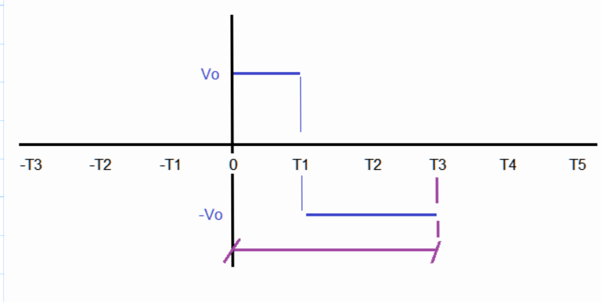
<---could it be like this?

We may go by either one but strictly speaking keep it within the scope of the problem information first. The first figure information is all we got it maybe the waveform does not go int the -t. Over the period T1-T3 the value is -Vo. This is over two intervals. The other is Vo over one interval. Vo positive is over 1 time intveral, then we have 2 time interval where its -Vo. I suggest following the solution for the gaining the insight first then that may put behind other thoughts.

The problem states.....'where , for one period T'.

We have the waveform changing from Vo to -Vo at time T1.
We say the period is from 0-T3.





<--- Late entry!
what about this?

Top = Vo x (T1-0) = VoT1

Bottom: -Vo x (T3-T2)= -Vo(2T1).
T3-T2 = Two time intervals of T1.

Add both: VoT1 + (-2VoT1)
Do we subtract or add? Since the magnitude maybe what we are concerned at this stage, otherwise there is loss of waveform if we subtract, we make the 2nd term positive.

Area = VoT1 + |-2VoT1|
= 3VoT1

The waveform or signal is over a period of T3-0 = T3 = 3(T1).
Now, lets say we average are over a period of 3T1.
Average = 3VoT1 / 3T1
= 3Vo. WRONG. *It was a GOOD attempt.*

Now we follow the engineer-author solution:

First part: VoT1
Second part: -Vo(T3-T1) = -Vo(2T1)......2T1 is two intervals of T1.
Period: T3-0 = T3 = 3(T1)...3 time intervals of T1.
Add first and second part: VoT1 + (-Vo2T1). Apply the addition per sign.
VoT1 - V02T1
Vo(T1-2T1)....This what the engineers-authors have.
Next we divide by the time period which is 3T1.
V_average = Vo(T1-2T1)/3T1

$$V_{average} = \frac{V_0}{3\ T1}\cdot(T1-2\ T1) = \frac{V_0}{3\ T1}\cdot(-T1) = -\left(\frac{V_0}{3}\right)$$ Answer.

Beautiful!

Again showing the terms:

$V_0 \cdot T1$ Square it --> $V_0^2 \cdot T1$

$-V_0 \cdot 2\ T1$ Square it --> $V_0^2 \cdot 2\ T1$

Period: $3\ T1$

The time interval is not squared? Yes, we focus on the magnitude of the voltage. Time is representative of the interval T1 we could have set it to T. Not squared. First interval is 1 second squared is 4, thats not in proportion as 1:2, its 1:4. NOT squared.

$$V_{RMS} = \left(\left(\frac{1}{T}\right) \cdot \int_0^{\frac{T}{2}} v(t)^2\, dt \right)^{\frac{1}{2}}$$ <---We squared to fit this expression.

Now what is our total of the two: $V_0^2 \cdot T1 + V_0^2 \cdot 2\ T1 = 3\ V_0^2 \cdot T1$

Now we divide it by the interval T which is 3T1: $\dfrac{3\ V_0^2 \cdot T1}{3\ T1}$

$= V_0^2$

Next we take the? SQUARE ROOT. $v_{EFF} = \sqrt{V_0^2}$

$v_{EFF} = V_0$ Answer.
Beautiful!

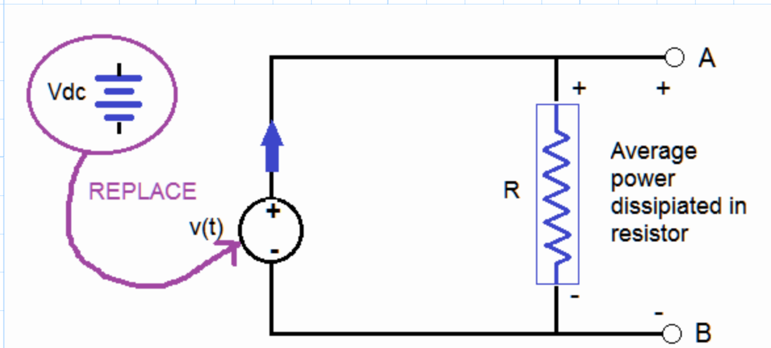
Example 6.13 (Average power in a Resistor):

Compute the average power dissipated from 0 to T in a resistor connected to a voltage v(t).

Replace v(t) by a constant voltage Vdc.

Find Vdc such that the average power during the period remains the same?

Solution:



Power = vi
i = v/R
Now power = v^2/R
Why do we want to get it in the form of V^2?
In this form v^2, the voltage will not have a negative value, its been SQUARED-OFF.
Ok. Later we want it to be readily replaced by a dc voltage, so no negative or oscillating values so the average power remains the same.

$$p = vi = \frac{v^2}{R}$$

$$P_{average} = \frac{1}{T}\cdot\int_0^T \frac{v^2}{R}(t)\,dt = \left(\frac{1}{R}\right)\cdot\frac{1}{T}\cdot\int_0^T v^2(t)\,dt$$

$$= \left(\frac{1}{R}\right)\cdot v^2(t)$$

We dont need to really integrate since we have the required voltage in v^2 form over the period T.

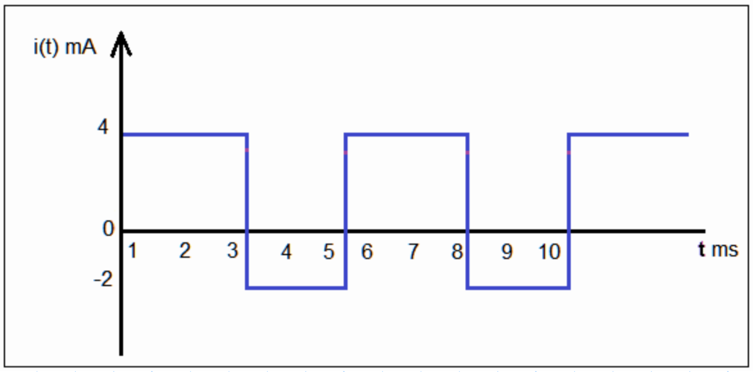
$$P_{average} = \frac{V_{dc}^2}{R}$$

Answer. We swap v(t) for Vdc a constant voltage.
Here Vdc^2 is the same as Veff^2.

Next the last example for Average and RMS (Effective) Values.

Example 6.13 (Capacitor voltage and current waveform):

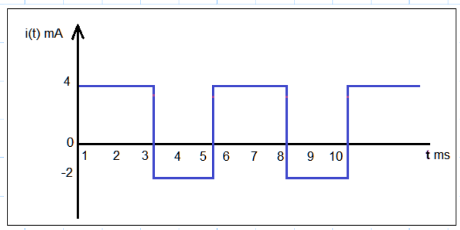
The current i(t) shown in the figure below passes through a 1 uF capacitor.
Find
(a) v_ac, the voltage across the capacitor at t = 5k ms (k = 0,1,2,3,4,5.....)?
(b) value of a constant current source Idc, which can produce the same voltage across the capacitor at t = 5 k ms when applied at t>0.
Compare Idc with <i(t)>, the average of i(t) in the figure, for a period of 5 ms after t>0.



Solution:

a).

i_c = C (dv/dt)



v_t = (1/C) integral of i(t) dt <---- This is what we will use.

$$v_{ac} = \left(\frac{1}{C}\right)\cdot\int_{0}^{5\ ms} i(t)\,dt$$

The current cycles between 4 mA and -2 mA.
So we have two integrals? Yes. First fror the +4 then -2, which will plave a negative sign between them. Firstly when I was going thru I noticed there was a concern since the period T=5ms had both pos and neg sides to the wave. *Ah huh.....so 2 expressions solves that!*

$$v_{ac} = \left(\frac{1}{C}\right)\cdot\int_{0}^{3\ ms} i(t)\,dt - \left(\frac{1}{C}\right)\cdot\int_{3\ ms}^{5\ ms} i(t)\,dt$$

1st expression on LHS has i(t) = 4 mA, and the 2nd -2 mA. With C = 1uF.

$$v_{ac} = \left(\frac{1}{1\cdot 10^{-6}}\right)\cdot\int_{0}^{3\ ms} 4\cdot 10^{-3}\,dt - \left(\frac{1}{1\cdot 10^{-6}}\right)\cdot\int_{3\ ms}^{5\ ms} 2\cdot 10^{-3}\,dt$$

$$v_{ac} = \left(\frac{10^{-3}}{1\cdot 10^{-6}}\right)\cdot\int_{0}^{3\ ms} 4\,dt - \left(\frac{10^{-3}}{1\cdot 10^{-6}}\right)\cdot\int_{3\ ms}^{5\ ms} 2\,dt$$

$$v_{ac} = \left(\frac{10^{-3}}{1\cdot 10^{-6}}\right)\cdot 4\ t \quad - \quad \left(\frac{10^{-3}}{1\cdot 10^{-6}}\right)\cdot 2\ t$$

Lim t: 0-3ms — Lim t: 3-5ms

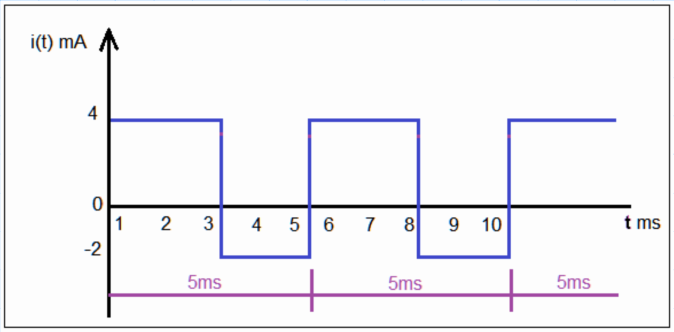
$$v_{ac} = \left(\frac{10^{-3}}{1\cdot 10^{-6}}\right)\cdot 4\ \left(3\cdot 10^{-3}-0\right) - \left(\frac{10^{-3}}{1\cdot 10^{-6}}\right)\cdot 2\ (5-3)\cdot 10^{-3}$$

$$v_{ac} = 12\cdot\left(\frac{10^{-6}}{1\cdot 10^{-6}}\right) - 4\left(\frac{10^{-6}}{1\cdot 10^{-6}}\right)$$

$$v_{ac} = (12-4)\cdot(1) \quad = 8\ \ V$$

v_{ac} = 8 This is the voltage across the capacitor at t = 5 ms.

v_{ac} = 8 V



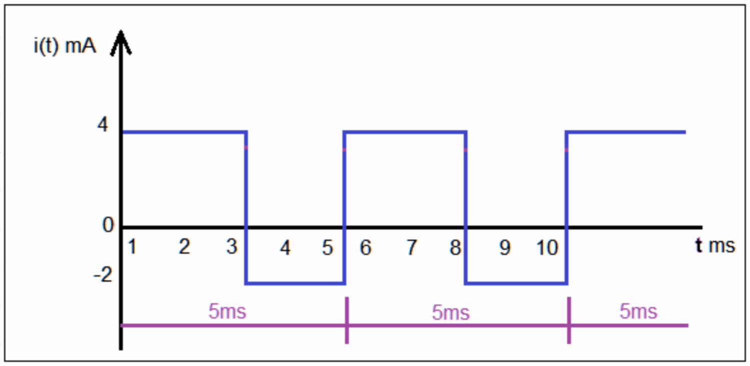
What the result is expressing is that every 5 ms, one period T, the circuit is generating a voltage of 8V across the capacitor. Correction, a net voltage of 8V.

Intepretation: For each period of 5 ms, the current i(t)'s net charging effect on the capacitor results with a net voltage of 8V. Next 5ms the same i(t) add another 8V to the capacitor voltage. So the k in the problem was NOT the multiple of 1000 to the five ms rather the k = 1,2,3,..... So it was 5k NOT 5x1000.

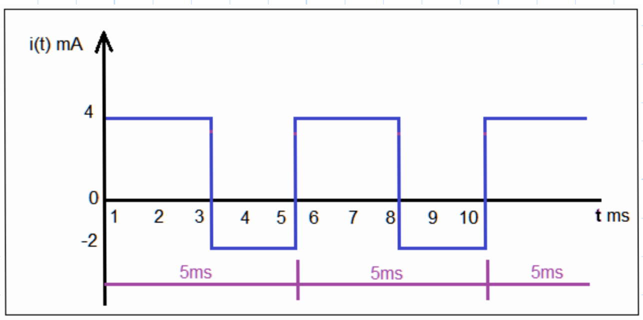
$v_{ac} = (12-4)\cdot(k) \quad = 8\cdot k \quad = 8\cdot k\ \ V$ Answer. k is not 1000, its a multiple like n.

Comment: *Personally I do NOT build capacitors, you only hear of them in such small values, they play a critical role in power systems and electrical construction (HV - LV). So if youre are planning to go in those courses, this is some indication of its importance. One of its functions is to compensate for voltage drop, as you saw working thru the example, its has ability to build up voltage.*

b).



Worry not about the voltage source? I ask myself is the capacitor able to work in ac and dc conditions? Capacitor is for storing charge, storing energy, charge increases? Potential increases. v(t)_C = (1/C) Integral of i(t).
Would the same v(t) generate I-dc? No. Schaums shows a v_dc in the solution.



The voltage source would need to be V_dc but it need to generate the same value as v_ac at end of 5 ms i.e. 8kV. So we want to solve for? I-dc. AND I_dc runs thru each ***multiple*** of five miliseconds i.e. 5**k** ms.

<--- This my understanding of what's happened, you are welcome to correct it.
v_dc = v_ac
Same average voltage.

$$v_{dc} = \left(\frac{1}{C}\right)\cdot\int_0^{5\,k\,ms} I_{dc}\,dt = 8\,k\quad V = v_{ac}$$

*Upper limit of integral is 5**k** ms*
Confusing *but what its saying is* ***continously*** *for every 5ms period; t>0.*

$$v_{dc} = \left(\frac{1}{10^{-6}}\right)\cdot I_{dc}\cdot t = \left(\frac{1}{10^{-6}}\right)\cdot I_{dc}\cdot(5\,k-0)\cdot 10^{-3} = 8\,k\quad V$$

Lim t: 5ms - 0.

$$v_{dc} = \left(10^{6}\right)\cdot I_{dc}\cdot(5\,k-0)\cdot 10^{-3} = 8\,k\quad V$$

$v_{dc} = (10^3) \cdot I_{dc} \cdot (5\ k) = 8\ k \quad V$

$(10^3) \cdot I_{dc} \cdot (5\ k) = 8\ k \quad V$

So we solve for I_dc.

$I_{dc} = \dfrac{8\ k}{(10^3) \cdot (5\ k)} \quad V$ 8k and 5k cancel off the k to 8/5

$I_{dc} = \dfrac{8}{(10^3) \cdot (5)} = 0.0016\ A$

$I_{dc} = 1.6 \quad mA$ Answer.

Compare Idc with <i(t)>, the average of i(t) in the figure, for a period of 5 ms after t>0?

If <i(t)> is the average its the average running thru every 5 ms that is 5 k ms.

This average will have to be equal to what? I_dc = 1.6 mA. Yes.
This value 1.6 mA is a dc current. No other current can build with the voltage of 8V at end of 5ms, which for 5 k ms is 8 k V.

So, for the period t>0 is the time passing thru the 1st 5ms k = 1, and the next 5ms where k=2,.....followed by k=3,..... For this time t>0, average of i(t) = <i(t)> = I_dc = 1.6mA. Answer.

Comments: Any errors here you should be able to sort things through.
Some confusion did arise in my thinking with regards to k, dc voltage,.....

Next a table on some of the wave forms we have come across and worked some examples through. Maybe?

All the waves except for sinusoidal (sine/cosine) did not have a period whereby we could detect a cycle. Seems like it if I remember correctly. The exponential wave form had a steep incline or decline then it moves in closer to to zero. The other seem to die out or settle to zero. Some had straight line at an angle which were identified as ramp, then yet a straight line a step. So, they say in the electrical enigneering business there are the periodic waveform and the non-periodic wave forms. Well lets not do a table. You can find it in your textbook.

6.12 Random Signals:

A signal is specified when its voltage is known, for that it requires the amplitude, frequency, and phase angle be known. Similarly for current, with it either leading or lagging the voltage waveform. These signals are called deterministic.

*Why deterministic the proper name for them?
Determine requires some search be made some calculation be done then we reach to the waveform's characteristics and shape. Also, if one cycle is known the next cycle would be expected to be the same. To me the name may been because it makes it easily identifiable. We can spot them. So they call them 'deterministic'.*

Random? If we seen one shape of the waveform we got the amplitude, frequency, and phase angle, but the next we dont know? Next is as good as a guessing game, or chances of winning the next number in a lottery, maybe some probability may help make sense of the next cycle. These so are called random. Really, isnt it a guess signal! Now, lets do a short brief study into it. You may study in detail and depth and make a career in the communication industry, used there.

Certain signals can be specified partly through certain statistical measures such as the mean, rms values, and frequency ranges. These are called random signals.

Are these signals important?

If youre using a device like a sensor it picks up signals in raw format to be processed. The signal picked up because of the environment in which the sensor operates may be limited to a certain range of freqeuncy, amplitude,....... So some statistics may be applied to make it predictable because we have some data on it. The input is what we are worried about in the sensor, we dont know exactly, so some guess may be applied, this taken into the design of a circuit. Simple. Agreeable, so this is a heavy duty subject, not studied here. Here, we just want to know what its about, a brief intro. May apply in some circuits in future or you study the advanced course, like Data Communications!.

Example 6.25 (Random signal):

Samples from a random signal x(t) are recorded every 1 ms and designated by x(n).

Approximate the mean, and rms values of x(t) from samples given in Table 6-2.

Table 6-2.

n:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
x(n):	2	4	11	5	7	6	9	10	3	6	8	4	1	3	5	12

Solution:

$N := 16$ count of 0 thru 15 = 16

$Sum\_x\_n := 2 + 4 + 11 + 5 + 7 + 6 + 9 + 10 + 3 + 6 + 8 + 4 + 1 + 3 + 5 + 12$

$Sum\_x\_n = 96$

$Average\_x\_n := \frac{Sum\_x\_n}{N} = 6$ Answer.

$Sum\_x\_n\_SQR1 := 2^2 + 4^2 + 11^2 + 5^2 + 7^2 + 6^2 + 9^2 + 10^2 + 3^2 + 6^2 + 8^2 + 4^2 + 1^2 + 3^2$

$Sum\_x\_n\_SQR2 := 5^2 + 12^2$ The expression was long so it was **split up into 2.**

$Sum\_x\_n\_SQR := Sum\_x\_n\_SQR1 + Sum\_x\_n\_SQR2 = 736$ <---X_rms^2. SQUARE - **S**

$\frac{Sum\_x\_n\_SQR}{N} = 46$ MEAN - **M**

Whats the square root of the above? RMS or Effective.

$x\_n\_RMS := \sqrt{\frac{Sum\_x\_n\_SQR}{N}} = 6.782$ ROOT - **R**

$X_{rms} := 6.782$ Answer. *Above we done all three R, M, and S. Yes, without the application of integration.*

One random signal example next that brings this topic here to end.

Example 6.25 (Random Signals):

A binary signal v(t) is either at 0.5 or -0.5V. *Binary meaning two states, here +.5 Or -.5.*
It can change its sign at 1 ms interval.

The sign change is not known at priori, not known before hand, but it has an equal chance for positive or negative values.

Therefore, if measured for a long time, it spends an equal amount of time at the 0.5V and -0.5V levels.

Determine its average and effective values over a period of 10s?

Solution:

Its a little difficult to visualise the problem because its not a circuit problem.
We have period which is an interval of 1 ms. We want to evaluate over 10s. In this range of 10 seconds, for the complete 10 s, what is the average value, will it be +0.5 or -0.5 or something else. Then for the same we also want to know the effective value which is the RMS.

How many times can the sign change in 10s?

$t\_duration := 10 \qquad interval := 1 \cdot 10^{-3}$

$Nos := \frac{t\_duration}{interval} = 10000$ the number of times the value can change.

We know how statistical thought on this is, 50% of the times heads 50% of the times tail. Same here we got 50-50 for 0.5 and -0.5. Obviously if each side is same chance as the other, percentage wise or probability wise, their average would be 50%. How? (50%+50%)/2 = 50%. If we only had two chances, divide by 2. Right? WRONG. But if you placed your bets on both sides? Yes. This solution is NOT looking at it this way, rather from the voltage perspective.

$v\_average := (0.5 \cdot 5000 + (-0.5) \cdot 5000) = 0$ Answer. <--- Mean **M**
10,000 intervals, and 50% for each thats 5,000.

$v\_effective\_sqrd := \frac{\left((0.5)^2 \cdot 5000\right) + \left((-0.5)^2 \cdot 5000\right)}{Nos} = 0.25$ <--- Squared **S**

$v\_effective := \sqrt{(v\_effective\_sqrd)} = 0.5$ Answer. <--- Root **R**.
The rms or eff.

Comment: Because we squared it (-0.5) then square root it, seems in this case as expected the result is 0.5V. But if the values were different then!

2 short bonus example problems.
These can be tricky.
Math more than EE.

6.5 Combinations of Periodic Functions.

TWO sinusoidal waves. Can we add them or subtract them?
Each has a different period T.

Example 6.5

Find the period of v(t) = cos (5t) + 3 sin (3t + 45 deg)?

Cos 5t:

We know the cosine wave has a complete turn in radian.
That is 2 Pi. Hence, the period of cos 5t is 2 Pi.
Whats the period of cos 5t?

$$v1(t) = \cos(5\ t)$$

$$T1 := \frac{2 \cdot \pi}{5} = 1.257$$

3 sin (3t + 45 deg):

We know the sine wave has a complete turn in radian.
That is 2 Pi. Hence, the period of sin 3t is 2 Pi.
45 deg is the phase angle which is not the sine term. This is the same exact thing, phase angle, described in our circuits textbook.
What is the period of sin 3t?

$$v2(t) = \sin(3\ t)$$

$$T1 := \frac{2 \cdot \pi}{3} = 2.094$$

How do we proceed next? Well this for me is one case the theory/explantion, ie the horse, was best behind the cart the solution.
Both have one revolution in? 2 Pi. for sine and cosine 1 revolution is in 2 Pi.

Let T = 2Pi $T := 2 \cdot \pi$

cos(5t) = sin(3t) = 2Pi = T <--- Not exactly but the next line down.

5T1 = 3T2 = 2Pi = T

$$5\left(\frac{2\pi}{5}\right) = 3\left(\frac{2\pi}{3}\right) = 2\pi = T$$

$(2\pi) = (2\pi) = 2\pi$ Answer. *Now Horse before the cart!*

So, we see the common denominator now is 2 Pi.

We know for a given function when we add the period T it does not change it:

$v(t) = v(t+T)$ The T is just cycling thru again and again leaving the value attached to 't' the wave shaper.

Schaums take it further to show:

v(t + T) = cos (5t+T) + 3 sin ((3t +T)+ 45 deg) = cos (5t) + 3 sin (3t + 45 deg) = v(t)

The theory NOW:

cos(5t) 5 is the n1, T1 we found was: $T1 := \frac{2\cdot\pi}{5} = 1.257$ $n1 := 5$

sin(3t) 3 is the n2, T2 we found was: $T2 := \frac{2\cdot\pi}{3} = 2.094$ $n2 := 3$

$n1\cdot T1 = n2\cdot T2 = T = 2\pi$

Requires T1/T2 = n2/n1......reasonable from this example done earlier, looks ok.

$\frac{T1}{T2} = 0.6$ $\frac{n2}{n1} = 0.6$ Same? WOW!

The result has to be a rational number otherwise, otherwise the sum is not a periodic function. It does NOT mean the waves hit all the points at the exact same locations on the graph, NO, just that its has the same periodicity, here it is 2 Pi, it could be PI, PI/2,.......

Example 6.8

Express v(t) = cos 5(t) sin(3(t) + 45 deg) as the sum of 2 cosine functions and find its period.

Solution:

Trig identity we use:

sin(a) cos(b) = (1/2) [sin(a+b) + sin(a-b)].

Then the LHS turned to cosine, using the sin(theta) = cos(-theta).

$$\cos 5t \cdot \sin(3\,t + 45\text{ deg}) = \left(\frac{1}{2}\right)\cdot(\sin(5\,t + 3\,t) + 45\text{ deg}) + (\sin(5\,t - 3\,t) - 45\text{ deg})$$

$$= \left(\frac{1}{2}\right)\cdot(\sin(8\,t) + 45\text{ deg}) + (\sin(2\,t) - 45\text{ deg})$$

Next turn it to cosine.

$$= \left(\frac{1}{2}\right)\cdot(\cos(8\,t) - 45\text{ deg}) + (\cos(2\,t) + 45\text{ deg})$$

Answer.

Next a worked example problem.
Worthy one relevant to this topic.
After which it will require lots less time and effort to go through the worked example problems and you completing any of the unsolved problems. Should make life easier.

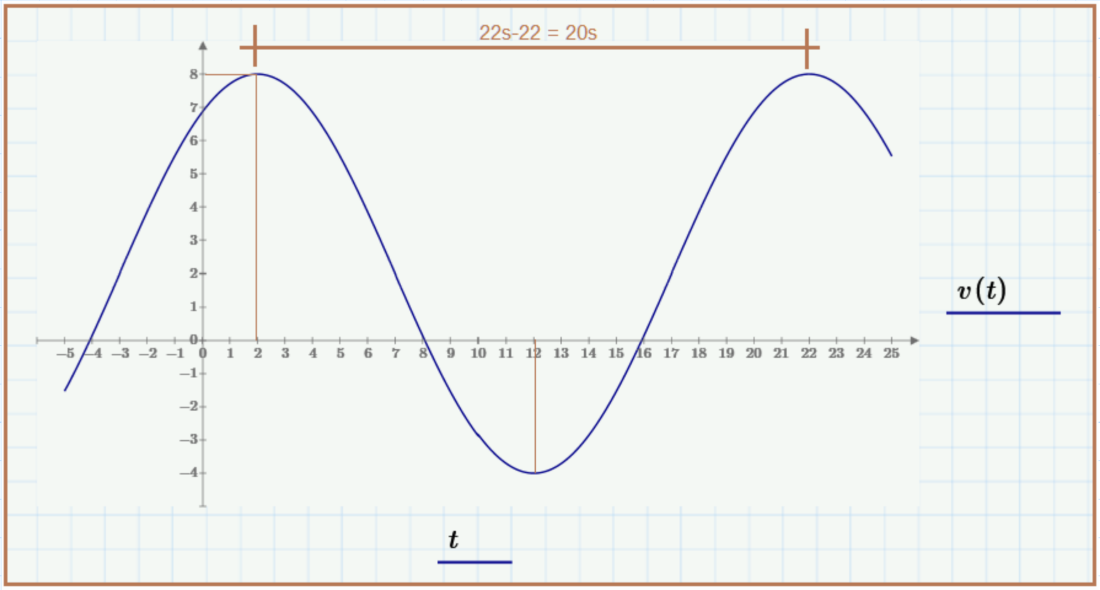
Continued on next page.

Example 6.7 Period and Frequency of a Signal:

The signal v(t) in figure below is sinusoidal.

Find its period and frequency.

Express it in the form v(t) = A + B cos(wt + theta)
and find its average and rms value?



Solution:

Period:

Obvious, time t between the 2 positive peaks is 20 s.
shown in figure. The brown writing was added on the figure.

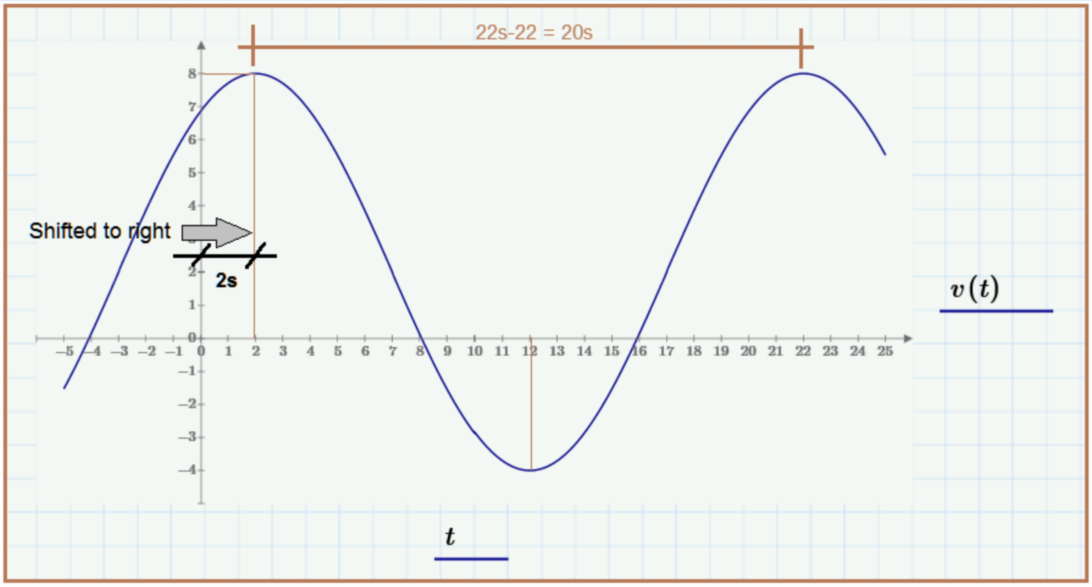
$T := 20$ s $\quad f := \frac{1}{T} = 0.05$ Hz

Our signal must be of the form: A + B cos(wt + theta).

We know from our studying, this type of function has A fixed,
while B is dependent on the cosine term. First solve for B.

$V_{max} := 8 \quad V_{min} := -4 \quad V\_B := \left(\frac{V_{max} - V_{min}}{2}\right) = 6$ <---B

$V\_A := V_{max} - V\_B = 2$ OR $V\_B := V_{min} + V\_B = 2$ <---A Same!



Look in the figure above, the wave is shifted to the right by 2 s.

The cosine wave usually starts at t = 0 with the peak voltage at t = 0.
so, that is why we know its shifted to the right by 2 s.

What is this shift? PHASE ANGLE.

Lets find this phase angle.

20 seconds is 2 Pi that is 360 degs.
Obviously we can proportion the 2s into an angle.

$T := 20 \qquad T_{shift} := 2$

$$Ph\_angle := \left(\frac{T_{shift}}{T}\right) \cdot 360\ \mathbf{deg} = 36\ \mathbf{deg}$$ Phase angle calculated in degrees.

$$\frac{2 \cdot \pi}{10} = 36\ \mathbf{deg}$$ Now in Pi form.

Trick situation, is the theta positive angle or negative angle?

Since it was supposed to peak at t=0, but did it at t = 2s, it had moved forward in the plot, to bring it back to t=0, we move it back? 36 degrees. ONE way to look at it, you may have yours.

That is - 36 degs.

We have found A, B, the period t, and the phase angle, can we form the function v(t)? A + B(cos(wt) + theta).

Discussion: Wait what's w?

w = 2 Pi f = 2 Pi(1/T)

2 Pi is one revolution or cycle in a cosine or sine wave.
2 Pi = T, because for cosine one revolution is 2 Pi.
Now 2 Pi/T = 2 Pi / 2 Pi = 1. If you agree.
I am just working it thru.
The wave has to start at 0 with the peak.
cos(0) = 1, maximum.
Does this mean we have to make it 0; wt+theta = 0?
Otherwise the function is not properly fixed? No? No.
"Maybe a term paper or a research paper here?" !

w = 2 Pi f <----This is omega, its the definition of radian frequency.
Lots of math can make you think that way!!!!

$A := 2$ $B := 6$ $\theta := -36\ \mathbf{deg}$ $f = 0.05$

$\omega := 2 \cdot \pi \cdot f = 0.3142$

In Pi terms:

$\pi = 3.142$

$\omega_{pi} := \frac{\omega}{\pi} = 0.1$ $\quad 2\ \pi \cdot f_{pi} = \frac{2\ \pi \cdot f}{\pi} = 0.1$

$\omega_{pi} = \frac{\omega}{\pi} = \frac{1}{10} = 0.1$ *(1/10) to get a simple whole number form for 0.1.*

$\omega_{pi} = \omega = \frac{\pi}{10} = 0.1$ <---Right? Pi/10. Yes.

$v(t) := A + B \cdot (\cos(\omega \cdot t) + \theta)$

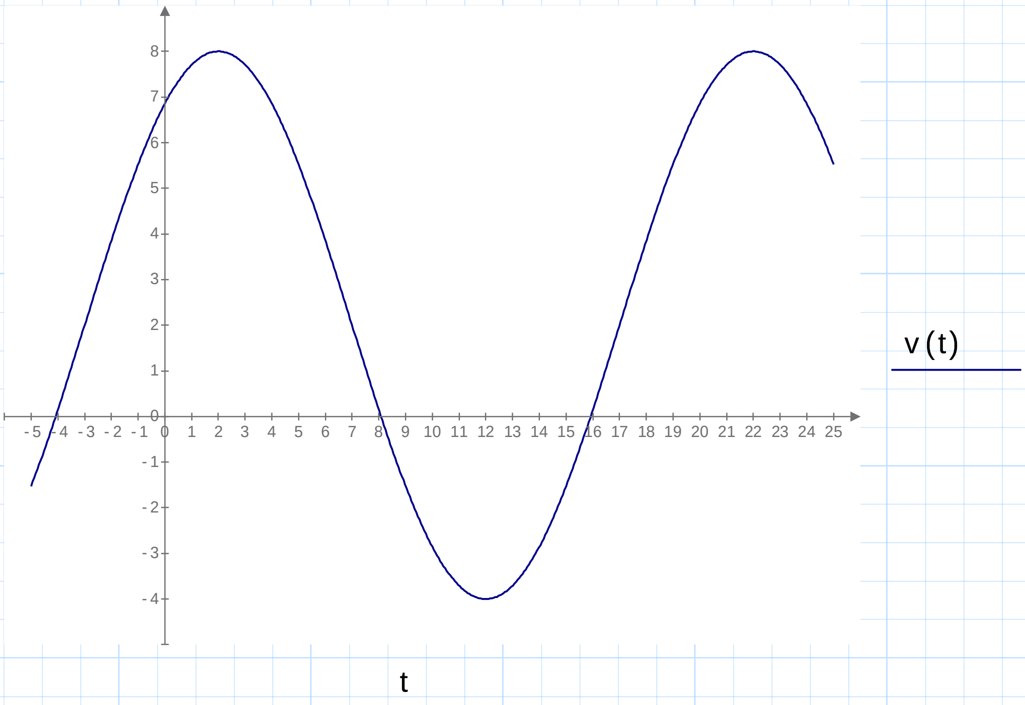
$v(t) := 2 + 6 \cdot \left(\cos\left(\frac{\pi}{10}\right) t - 36\ \mathbf{deg}\right)$ Answer. Same as Schaums.

Next page the plot of this function v(t).

clear (t)

$$v(t) := 2 + 6 \cdot \cos\left(\frac{\pi}{10} \cdot t - 36\ \mathbf{deg}\right)$$

Same plot the example provided matched here through the v(t) equation.



We have come to the end of this exercise. Could do more examples, but we have a purpose here a first time and refresher study, use your textbook, just too many to solve, some stopping point need be reached. STOP. Purchase Schaums Outline on this subject should you seek more study.

THANKS to Schaums Outline and its former name Schaums Series book on Electric Circuits.
Some thanks to my college days circuits textbook by Hyat and Kemerly.

Better position than when I, or we, started - 30 March 2020.
Little improvement. One small step!

Apologies in advance for any errors and omissions.