Rough DRAFT

DISCUSSION:

Whats all this talk about a Schrodinger Equation?

This work is on why we got to using Schrodinger Equation (SE) to solve some quantum mechanics problems.

Its not on SE rather why we needed SE.

Its a rough draft probably has errors you can spot.

The topic is on wavefunction and wave packets.

An updated version with corrections where found may be generated later, if needed, but its primarily a tool for self study and assisting in course work study too.

At present this rough draft is fit for use with your corrections as needed.

Rev: 0.

Rough DRAFT Rough DRAFT Rough DRAFT

Constants:
$\lambda := 1$ $v := 1$ $\omega := 1$ $i := \sqrt{-1}$ $A := 1$ $k_0 := 1$ $k := 1$ $\Delta k := 1$ $\omega_0 := 1$ $dk := 1$ $d\omega := 1$

DISCUSSION:

Whats all this talk about a Schrodinger Equation?

A particle like electron proton.....exhibits a wave behaviour. The sea waves has some form of a wave, its shape maybe similar its amplitude changes depending on sea conditions. So the sea has a wave like nature, similarly then the particle has a wave like nature.

We know momentum p: $p := \frac{h}{\lambda} = h'k$

$h' := \frac{h}{2\pi}$ $k := \frac{2\pi}{\lambda}$

k is the wave number

$$p := \frac{h}{\lambda} = \left(\frac{h}{2\pi}\right)\left(\frac{2\pi}{\lambda}\right) = \frac{h}{\lambda}$$

We seen all the expressions above in the previous chapters.
The frequency v of the particle and its energy E are related by Planck-Einstein relation:

$$E := h \cdot v$$

Angular frequency $w := 2\pi \cdot v$ *in math and engineering we usually see f for frequency, no matter, its troubling so get used to v.*

$v := \frac{w}{2\pi}$ Note: $\frac{1}{v} = \frac{2\pi}{\omega}$ Period $T := \frac{1}{v}$ $T := \frac{2\pi}{\omega}$

For a period T of a signal or function, the measured distance of the signal/wave travelled is the wavelength (Lamda). For a sinusoidal wave this would be the distance between two adjacent peaks or crests (ie a period).

which is $\lambda := \frac{2\pi}{w}$ so $\omega := \frac{2\pi}{\lambda}$

$h' := \frac{h}{2\pi}$ $h := h' \cdot 2 \cdot \pi$

$$E := (h' \cdot 2 \cdot \pi) \cdot \left(\frac{w}{2\pi}\right)$$

$$E := h' \cdot \omega$$

Mahesh in QM For UG textbook got these terms out of the way, next he wants to device the wave function associated to a particle.
He starts with the use of a plane monochromatic wave: $\Psi(x,t) := A \cdot e^{i(k \cdot x - \omega \cdot t)}$

Lets say for the purpose of plotting a graph on the expression: $\Psi(x,t) := A \cdot e^{i\,(k \cdot x - \omega \cdot t)}$

$\omega := 2 \cdot \pi \cdot v$ $\quad \frac{1}{v} = \frac{2\,\pi}{\omega}$ $\quad T := \frac{1}{v}$ $\quad T := \frac{2\,\pi}{\omega}$ $\quad k := \left(\frac{2\,\pi}{\lambda}\right)$

$Velocity := v \cdot \lambda$ Number of cycles of wavelength per second multiplied by wavelength, gives distance of cycles travelled per second, this is velocity (vector) or speed (scalar).

For this period T, we measure the distance the signal/ wave travelled, that would be the wavelength (Lamda).

This wavelength would have its units in meters or some other length, so is this not the 'x' in the function PSI(x,t)? Yes it is.
Maybe its NOT, but for now we see its logical and the reasoning is acceptable. It is!

So 'x' may be set to a constant equal to the wavelength in PSI(x,t).
Provided we plot for one wavelength which is the time for one period T.

Lets select a known wavelength, in the EM spectrum range, radio wavelength of 100 meters with frequency 10^6 Hz (cycles per second).

$\lambda := 100$ m $\quad x := \lambda$ m $\quad v := 10^{6}$ Hz $\quad T := \frac{1}{v} = 1 \cdot 10^{-6}$ second

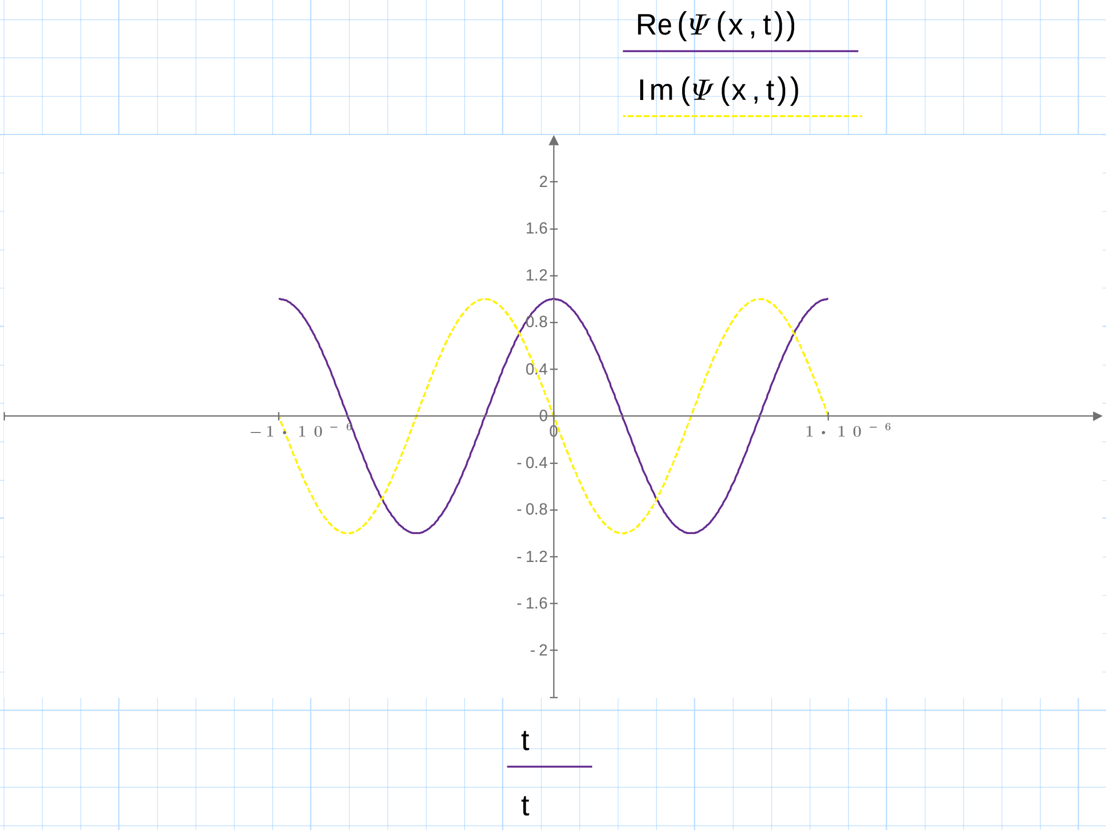
$k := \left(\frac{2\,\pi}{\lambda}\right) = 0.063$ m^-1 (per meter) $\quad \omega := 2 \cdot \pi \cdot v \quad \omega = 6.283 \cdot 10^{6}$

A to keep it simple we set it equal to 1.
$A := 1$
A is amplitude, its NOT a coefficient.

$\Psi(x,t) := A \cdot e^{i \cdot (k \cdot x - \omega \cdot t)}$ <---Plot this function.

See next page for plot.

This function chosen does plot like a sinusoidal what we were looking for.
You verify the graph.



Plot of function above. You verify using Excel.
This is the same function the textbook uses for the remaining explanation which you understand mathematically, common in electrical engineering (signals, communications,....), mechanical engineering (vibrations, acoustics,....).
If youre interested in the complex sinusoidal graph plotting you do the research, its tough for me, may not be for you.
Here, in the graph, Re part leading Im part, at zero real has a value. You verify.

Plane Monochromatic Wave? This is hard, you look it up, simple brief explanation provided below. There are those explanations with Maxwell's equation so obviously not going to that subject here.

Plane wave: $f(x,t) := A \cdot \cos(\omega \cdot t - k \cdot x + \alpha)$ plane wave, expression seen many a time.

A wave of amplitude A, wavenumber k, angular frequency omega, and phase angle +alpha, propagating in the positive x-direction, is represented by the wavefunction above. Monochromatic? Single wavelength for the wave. Which this now may explain the plane monochromatic wave.

$f(x,t) := A \cdot \cos(\omega \cdot t - k \cdot x - \alpha)$ -ve alpha may mean wave is lagging behind or opposite in direction, toward -x axis direction, you verify.

Continuing from where left-off before the graph attempt.

Page 71-75 Qm For UG (Mahesh Jain):

As a first step towards constructing a wave function to be associated with a particle, let us consider a plane, monochromatic wave:

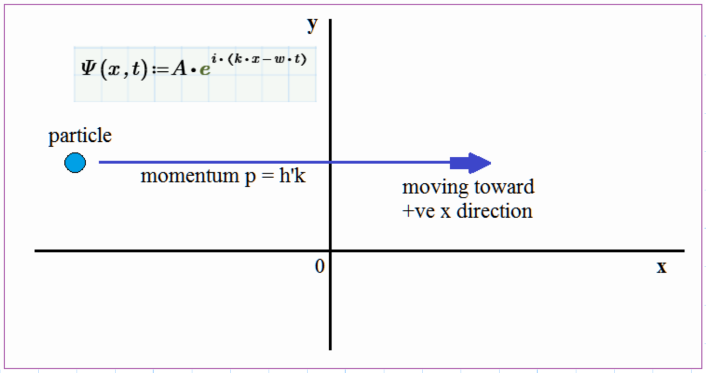
$$\Psi(x,t) := A \cdot e^{i \cdot (k \cdot x - w \cdot t)}$$

Which represents a simple harmonic disturbance of wavelength: $\lambda := \frac{2\pi}{k}$

and frequency $v := \frac{\omega}{2\pi}$

travelling towards the positive x-direction with velocity $V_{ph} := \frac{\omega}{k} \quad = \; v \cdot \lambda$

the subscript 'ph' indicates that this velocity is called a phase velocity.
The plane wave Psi(x,t) represents a particle having a definite momentum of p =h'k.



The A in the plane wave function represents the amplitude, and its constant.

As the particle moves, A remains the same,. so its dififcult to find a location where the particle could be identified in space. Constant amplitude corresponds to a lack of localisation of the particle in space.

See the figure on the next page on the functions and their localisations.

From our past experience on this expression or equation:

$$P(x)\,dx \quad = \int_{-\infty}^{\infty} |\Psi(x,t)|^2\,dx$$

P(x) dx   is the probability of finding the particle within an 'element dx' about the point x at time t.

This equation or expression is actually composed of:

$$P(x)\,dx = \int_{-\infty}^{\infty} |\Psi(x,t)|^2\,dx \quad = \int_{-\infty}^{\infty} \Psi_{conj}(x,t) \cdot \Psi(x,t)\,dx$$

Logically the probability of finding the particle in the limits -infinity to infinity is 1.
Givens the ends of the x-axis, ends of the world, surely you find the particle, so it equal 1.
So, we say the wave function should be normalised to unity, ie equal to 1.

$$\int_{-\infty}^{\infty} \Psi_{conj}(x,t) \cdot \Psi(x,t)\,dx \; = \; 1$$

Meaning the wave function is square integrable.

Our function was a plane monochromatic wave $\Psi(x,t) := A \cdot e^{i(k \cdot x - \omega \cdot t)}$

Its conjugate: $\Psi_{conj}(x,t) := A \cdot e^{-i(k \cdot x - \omega \cdot t)}$ negative on the i; *-i*

$$P(x)\,dx = \int_{-\infty}^{\infty} \Psi_{conj}(x,t) \cdot \Psi(x,t)\,dx$$

$$= \int_{-\infty}^{\infty} A \cdot e^{-i(k \cdot x - \omega \cdot t)} \cdot A \cdot e^{i(k \cdot x - \omega \cdot t)}\,dx$$

$$= \int_{-\infty}^{\infty} A^2 \cdot e^{-i(k \cdot x - \omega \cdot t) + i(k \cdot x - \omega \cdot t)}\,dx$$

$$= A^2 \int_{-\infty}^{\infty} e^0\,dx \quad = A^2 \int_{-\infty}^{\infty} 1\,dx \quad = A^2\ (x)$$

Limit infinity to -infinity

= A^2 [Infinity - (- Infinity)]
= A^2 (Infinity)
= A^2 .....when Infinity is set to constant, it results with A squared a constant.
I sense yo ucan improof on the math on this A x Infinity!

A squared results in a flat line, seefigure next page. What I am saying is the answer does not lead to a location where the particle can be found, here its somewhere in infinity, so you are back to step one ......may remind you of a song '*No Where Man' a Beatles song*.

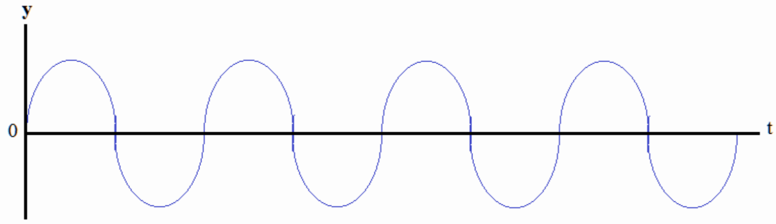
Whats the problem?

The function $\Psi(x,t) := A \cdot e^{i(k \cdot x - \omega \cdot t)}$ was NOT a suitable function to represent the wavefunction of the particle.

How do you propose to solve this? Not me.
Fourier Series....Fourier Transform? yes, the Fourier transform of a function.
Pull out math book on Fourier Series and Transforms.
If I remember correctly, Series is something about constructing a signal's value or amplitude over time, while the Transform is on generating a function from the signal's value or behaviour. You verify this.

Comments:

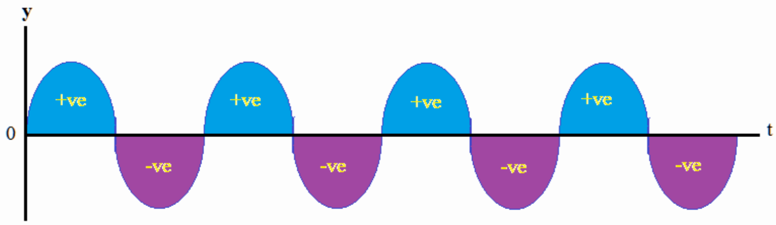
Pull out your college Math book and correct me.

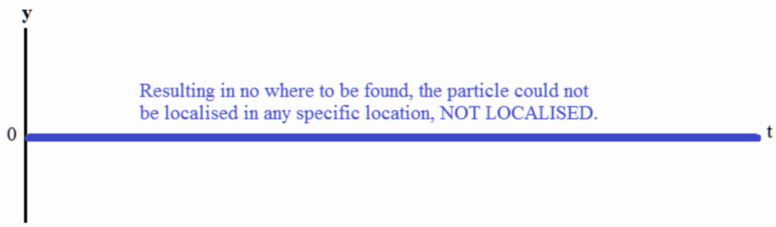


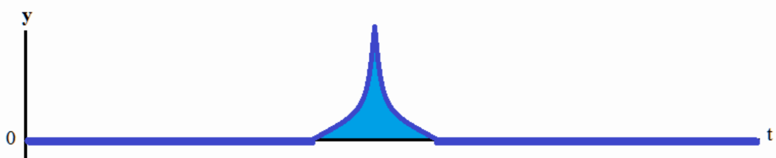
Here is a wave, it a described by a function, you can see its sinusoidal in shape. The function y = f(t) can be normalised. It can have x-axis as s instead of t, then we have y = f(x), whatever best describes the function.

Can we look at it and say over a period where the particle would be? Firstly no. Is the particle travelling on the path of the wave, that is the outline of the wave, which at each point we may say is the amplitude? Yes. Lets say for now thats right. But if we were to integrate the function we are saying we got the area under the curve for the limits of integration. Is that area where the particle is located? Yes.

Two yes! Seems the comfortable one is the area thru integration. Which leads to normalisation or some kind of probability. This we can now appreciate. Obviously for this curve the area is zero since the curve as a positive and negative side, results in zero. So in this case we cannot for this wave for the limits provided determine with certainty where the particle is. This the physcists identify as as a non-localised wavefunction. Not localised. Cannot determine the location of the particle. Key word they used is 'localised'.





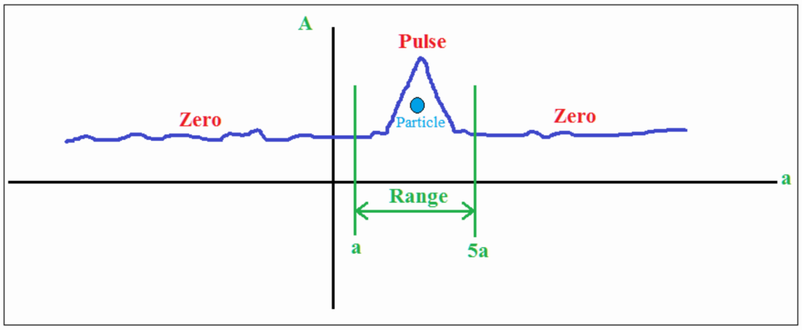


This plot above is like a pulse suddenly emerging after an absence in time, its value is zero, until we see a surge or pulse, which then turns back down to zero. Integration over the period of the pulse will give an area, some value, which may mean something, maybe where the particle is.

So, we say here the wavefunction is localised, were able to point where the particle is located.

Figure above an attempt to provide a maybe meaningful explantion on 'localised'. *I think you will agree.*

So for now what we are searching for in the wavefunction should result with an AMPLITUDE visible or present around a range of the particle, and outside the range the amplitude is negligible. The amplitude be NOT same everywhere and negligible or zero. The figure below is an attempt to describe that. You may say sure it is right but its QMs.

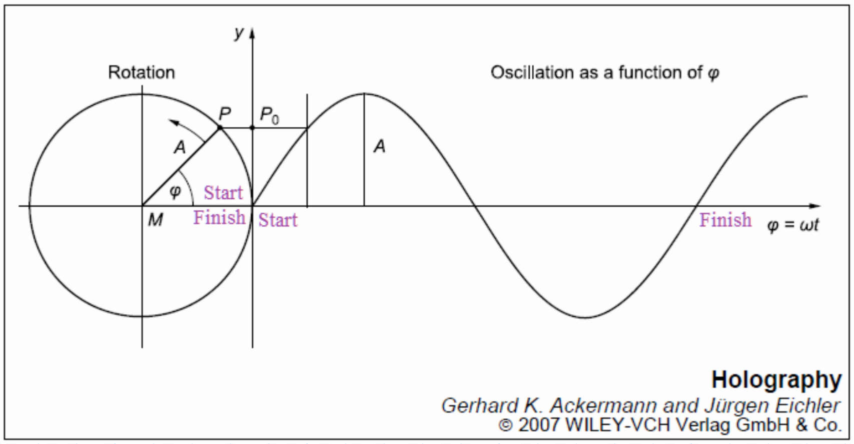


Page 72 (M Jain):
This suggest that a particle can be represented by a wavepacket.
A wavepacket can be formed by superposing plane waves of different wave numbers in such a way that they intefere with each other destructively outside of a given region of space.

The math technique for doing this is Fourier integral and transforms.

$k := \frac{2\pi}{\lambda}$ <---we see here k the wave number can differ only by the wavelength which is each wave has its own wavelength, so all the waves are different the 'signal or wave shape' is different because k has to be different.

Question: In some physics textbook they show for one revolution, 2 pi = 360 degs, the wave travels a distance of one wavelength lamda. Common in engineering textbooks. So how do we get different k values for the same wave function? We dont, each wave has one wave number. A(k1) A(k2)....each k is for a different wave. Its NOT that for some 'k' the wavelenghth travels more OR less than one revolution. For now the only way I see possible is each wave has its own wavelength, and there are many wave fucntions each with their own k. See Figure below.



Is 'k' an interger? Not likely, its a real number.
k = 2 pi / lamda will not result in an integer. Thats not a problem for Fourier Integrals.

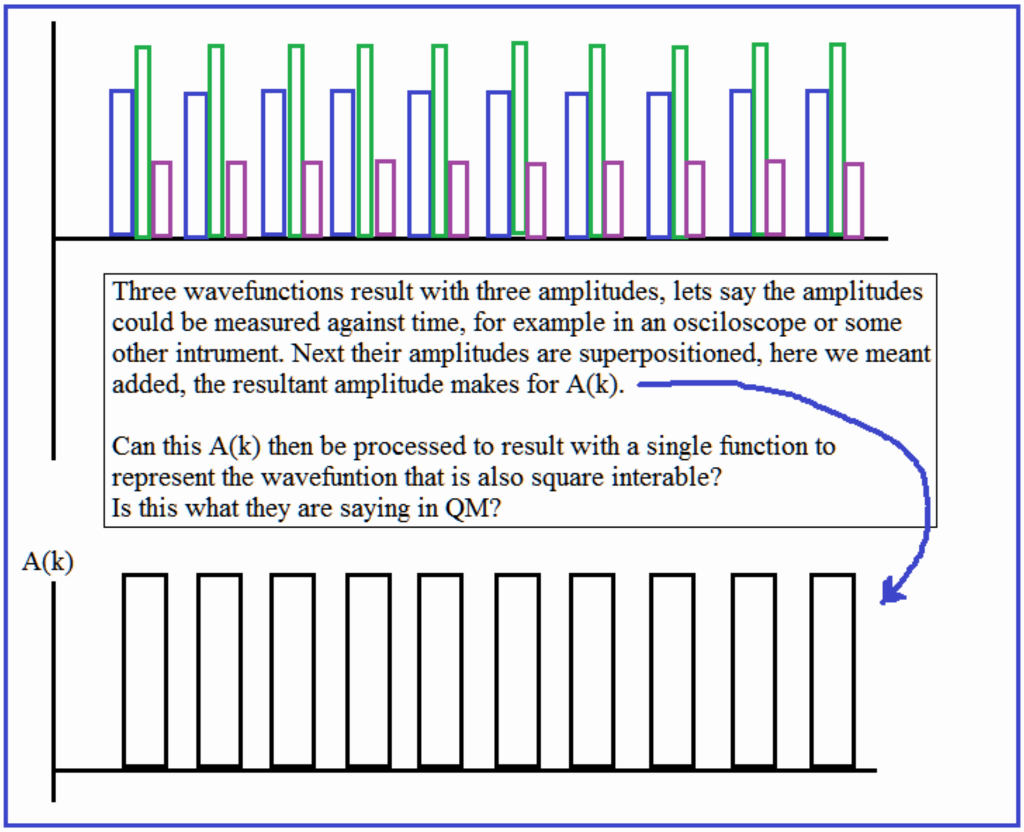


Figure above ttempts to show 3 amplitudes of 3 wave functions, then superpositioned to form the resultant amplitude function A(k). Constructively.

We need the amplitude function A(K) of the wavefunction.

Here we do not show outside the desired region, where the amplitudes work against each other, destructively, resulting with negligible amplitude.

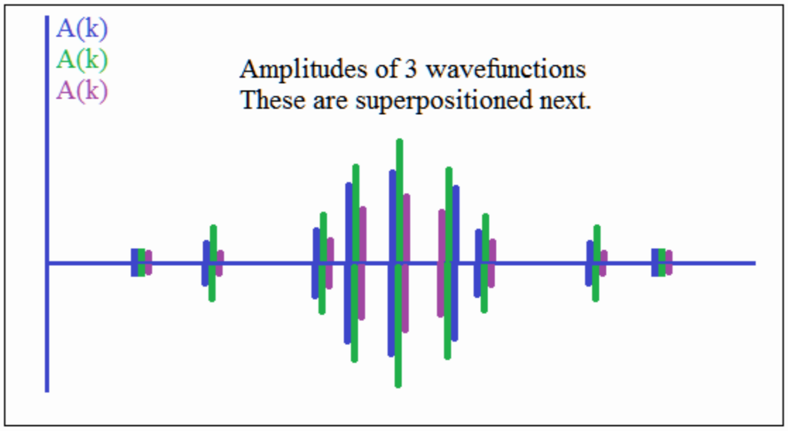
Now what is wrong with the first figure, on top, of page?

The amplitude is constant !
Yes, thats wrong.

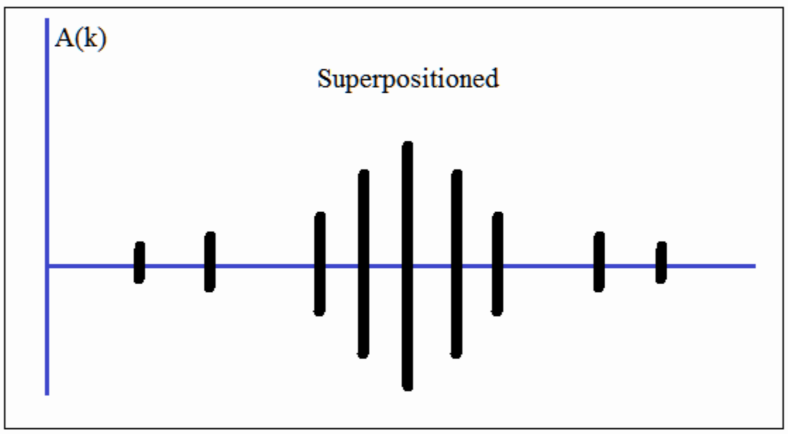
WRONG !

Lets try again on the next page.

Mahesh Jain the physicst for 'QM For UG', uses the function A(k) in his later explanation using Fourier Transform, so its a matter of getting it straight here before getting there.

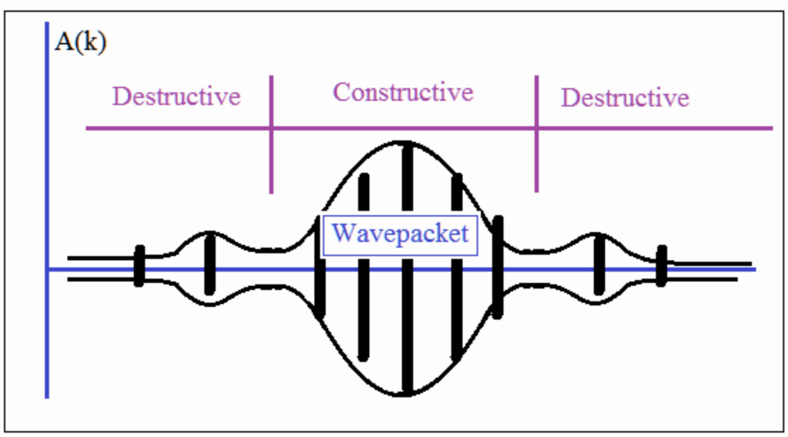


This is an amplitude plot, A(k), wrt time.
k = 2 pi/lamda, here k is wrt to each function. So we have 3 functions.
Its almost the same as x(t) with the curve, instead we select or extract the amplitude.



Here the amplitude of the 3 functions have been superpositioned.

So this is still with respect to time t but we have 1 function instead of 3.



This shows the constructive section creating the wave packet, the destructive sections cancelling out each other or negligible.
So here in the wave packet should be localised, where the particle should be found.
Figure shows the envelope outline, curve, of the wavepacket.

The figures above again attempt to depict the construction of the wavepacket.
Hope this is better but again you correct it for errors.

Comments: Lets review superposition, we know from curcuit theory there is a technique called superposition but is it the same? No. Dictionary definition maybe the same but not in practice between this physics case and electrical engineering circuit theory.
Surprise Me!. But if you think about it, the technique used in electrical may be applicable for the sum of amplitudes, though there is no circuit in QM case. You solve it.

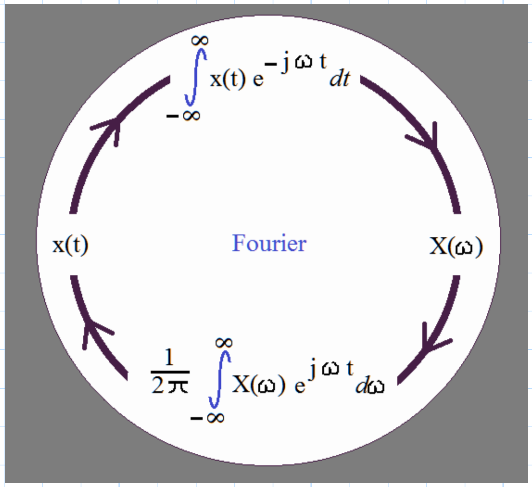
Recap: *I am saying* the wavefunction should be composed of amplitude values along points of time in the plot. In real world sense we may be analysing a signal or related function which must and can be measured that results with some form of amplitude values.

The function A(k), applying the Fourier Integral on it gives the 'Inverse Fourier Transform' f(x). Positive exp sign.

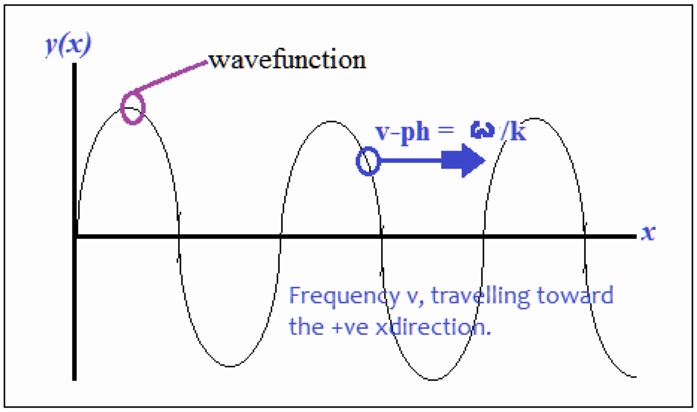
Applying the Fourier Intergal on f(x) results with the 'Fourier Transform' A(k). Negative exp sign.

$$f(x) := \left(\frac{1}{\sqrt{2\pi}}\right) \cdot \int A(k) \cdot e^{ikx}\, dk$$ <--- 'Inverse Fourier Transform' of A(k).

$$A(k) := \left(\frac{1}{\sqrt{2\pi}}\right) \cdot \int f(x) \cdot e^{-ikx}\, dx$$ <--- 'Fourier Transform' of f(x).



The figure here helps.
This from a signals course textbook.
In our application SQRT(2) was applied for convenience.

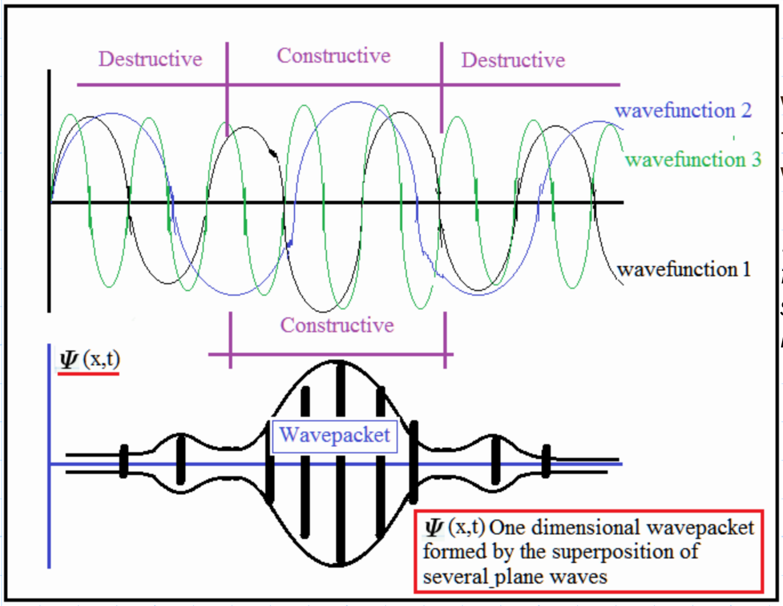


v-ph, phase velocity, is the frequency wave function travelling toward the +ve x-direction.

$w := 2\pi \cdot v \qquad k := \frac{2\pi}{\lambda}$

$$\frac{w}{k} = (2\pi \cdot v) \cdot \left(\frac{\lambda}{2\pi}\right)$$

$$\frac{w}{k} = v \cdot \lambda$$ <--This is velocity.



<--- Rough sketch, shows 3 wave functions, superpositioned to form one dimensional wavepacket.

Its not a good sketch, using 2 functions may not hit the right spots...so I used 3 functions..... rough sketch you got the idea.

Let $\Psi(x,t)$ be a one dimensional wave packet formed by the superposition of plane waves:

$$\Psi(x,t) := \left(\frac{1}{\sqrt{2\pi}}\right) \cdot \int A(k) \cdot e^{i(kx - \omega t)} \, dk$$

With the condition the amplitude A and angular frequency *w* depend on k.

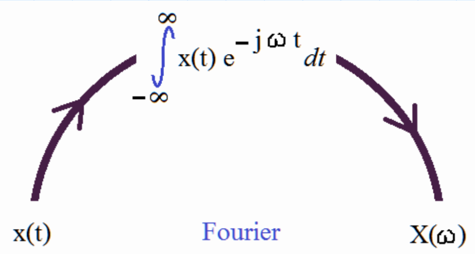
$\left(\frac{1}{\sqrt{2\pi}}\right)$ *was chosen for later convenience, you may have come across SQRT(2) often in applied science and engineering. Also similar to maths textbook on SQRT(2) for signals and systems, and other fields.*

It is clear that in order to represent a free particle by a wave packet, we must give up the requirement that the particle should have a precisely defined momentum - M Jain page 72.

Comment: *Its not so clear for me.*
Why?
Lets give it a try. I am not confident but you may correct it.
In a wavepacket there were several wave functions which have been superpositioned. We assume each superpositoned point was based on position x at a time t. Momentum is mass x velocity. For each wavefunction we need an accurate velocity which may not be easy to obtain at point x at time t. The superposition may provide the amplitude at each point which can represent the probability of finding a particle but not the velocity,makes it difficult to find a precise momentum. Velocity would need a 'space divided by time' over a period, where else here its at point x at time t on that point the superposition of wavefunction's amplitude.*You give it a try.*

$$\Psi(x,t) := \left(\frac{1}{\sqrt{2\pi}}\right)\cdot\int A(k)\cdot e^{i(kx-\omega t)}\,dk$$ <---- We have this.

We want A(k) the amplitude function by taking the Inverse Fourier Transform of PSI(x,t)



We want a -ve exponent RHS, this is from the electrical signals textbook instead of i its j, because they reserve i for current. You may excuse them for it. *Personally I prefer j for current, and leave i as it is for imaginary. Physics came first or Electrical? They say more money in electrica$.*

$$\Psi(x,t) := \left(\frac{1}{\sqrt{2\pi}}\right)\cdot\int A(k)\cdot\frac{e^{i\cdot k\cdot x}}{e^{i\cdot w\cdot t}}\,dk$$ <---WRONG!

$$\Psi(x,t)\, e^{i\cdot w\cdot t} = \left(\frac{1}{\sqrt{2\pi}}\right)\cdot\int A(k)\cdot e^{i\cdot k\cdot x}\,dk$$ <---WRONG!

$$A(k)\cdot e^{-i\cdot w\cdot t} = \left(\frac{1}{\sqrt{2\pi}}\right)\cdot\int \Psi(x,t)\cdot e^{-i\cdot k\cdot x}\,dx$$ <---WRONG!..maybe you can fix this.

Another try, multiply by e^iwt both sides:

$$\Psi(x,t)\cdot e^{i\cdot w\cdot t} = \left(\frac{1}{\sqrt{2\pi}}\right)\cdot\int A(k)\cdot e^{i\cdot\omega\cdot t}\cdot e^{i(kx-\omega t)}\,dk$$

$$\Psi(x,t)\cdot e^{i\cdot w\cdot t} = \left(\frac{1}{\sqrt{2\pi}}\right)\cdot\int A(k)\cdot e^{i(kx)}\,dk$$

Next move the functions across

Can I pull the -ve power out?
Not the same variable/function. Exp is not a variable/function like A(k) so the inverse A(k) is seen non-inverse rather the exponent is made inverse, which its multiplied too. You think?
Change the intergral wrt x RHS.

$$(A(k))^{-1}\cdot e^{i\cdot w\cdot t} = \left(\frac{1}{\sqrt{2\pi}}\right)\cdot\int (\Psi(x,t))^{-1}\cdot e^{i(kx)}\,dx$$

$$(A(k))\cdot e^{-i\cdot w\cdot t} = \left(\frac{1}{\sqrt{2\pi}}\right)\cdot\int (\Psi(x,t))\cdot e^{-i(kx)}\,dx$$

Did it here, made -ve. Next leave LHS with A(k) term only.

$$(A(k)) = \left(\frac{1}{\sqrt{2\pi}}\right)\cdot\int (\Psi(x,t))\cdot e^{i\cdot w\cdot t}\cdot e^{-i(kx)}\,dx$$

$$e^{i\cdot w\cdot t}\cdot e^{-i(kx)} = e^{-i(k\cdot x-\omega\cdot t)}$$

<---We got our exponent straight here.
At least one person thinks so.
Life is not all about +/- signs (inv/transform).

$$(A(k)) = \left(\frac{1}{\sqrt{2\pi}}\right)\cdot\int(\Psi(x,t))\cdot e^{-i(kx-\omega\cdot t)}\,dx$$ Looks good enough.

OR you may just use youre advanced engineering or plain maths book see how its setup there...

$$\Psi(x,t) := \left(\frac{1}{\sqrt{2\pi}}\right)\cdot\int A(k)\cdot e^{i(kx-\omega t)}\,dk$$ <---- We have this.

$$A(k)\cdot e^{-i\cdot w\cdot t} = \left(\frac{1}{\sqrt{2\pi}}\right)\cdot\int \Psi(x,t)\cdot e^{-i\cdot k\cdot x}\,dx$$ <---Set this, this step not necessary textbook showed it

$$A(k) = \left(\frac{1}{\sqrt{2\pi}}\right)\cdot\int \Psi(x,t)\cdot e^{-i\cdot(k\cdot x-w\cdot t)}\,dx$$ <---This is correct.

Remember, there was this idea about where is the particle?
It may look awkward but we are still looking for the particle.
Where is it?
We have the intergral so where is it?
THE LIMITS OF THE INTERGRAL?
For now it looks reasonable its the limits, do we say +inf to -inf? Where ever?

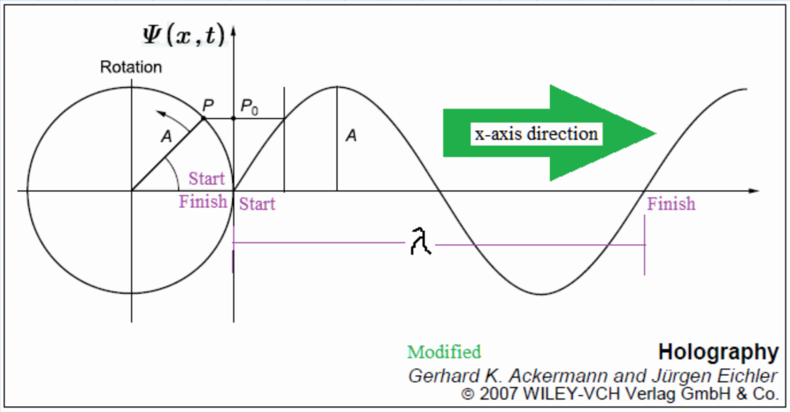
We cant go infinity now, we maybe close since we have an integral.

Let's make an ASSUMPTION. Lets say the function A(k) is centred around a value k = ko. The value ko is where the function A(k) has a significant value to point to a particle. Next we set a range or interval for where the particle maybe around the centre k.

$$k := \frac{2\pi}{\lambda}$$ k has its unit in meters because lamda is a length.

$$A(k) = \left(\frac{1}{\sqrt{2\pi}}\right)\cdot\int \Psi(x,t)\cdot e^{-i\cdot(k\cdot x-w\cdot t)}\,dx$$

But why did k get chosen instead of x?
Because x merely shows the direction where the particle is moving in the x-axis direction, its one-dimensional. Good enough for me. What if the integral was in three dimension? Still k? Looks like there is more to this.
k = 2 pi/lamda,.....one whole angular cycle per one linear wavelength.
Yes, maybe its the lamda in the x direction. So, take the limit to k because lamda is in there and its direction is in the x-axis direction. Maybe. Good enough for me. Will do! You got a better explanation pass it on to your local engineer I'm staying with this. See figure below. It must be Lamda is in x-direction. It is. Anyway check with your local engineer.

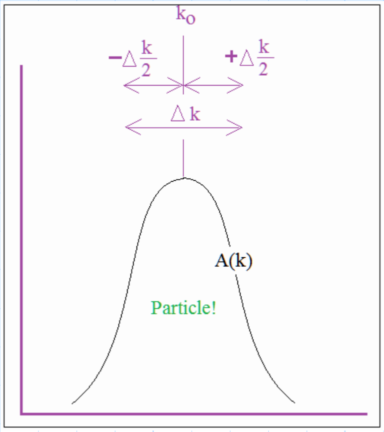


Lamda is travelling in the x-axis direction.

Continuing with the range of k.

Taking the limit to either side of k but how far of each side?

This you have done before in Maths course, delta k and +/- (delta k/2).



Maybe something like that, reason I say maybe is because the function is plotted, it says the paritcle is in this region of the graph, where there is a significant amplitude.

It maybe wrong but I took a chance since we have been searching for this particle for too long a time.

We assume that A(k) is centred about some particular value k = ko,

falling rapidly to zero outside the interval of $\left(k_0 - \frac{\Delta k}{2}\right)$ and $\left(k_0 + \frac{\Delta k}{2}\right)$ where Δk is small.
M Jain page 72-73.

Now returning to the original function PSI(x,t):

$$\Psi(x,t) := \left(\frac{1}{\sqrt{2\pi}}\right) \cdot \int_{\left(k_0 - \frac{\Delta k}{2}\right)}^{\left(k_0 + \frac{\Delta k}{2}\right)} A(k) \cdot e^{i(k \cdot x - \omega \cdot t)}\, dk$$

<---Right where we started after an improvement in the limits.
Looks like a change of plans.

Lets say a change of plans. Mahesh Jain picks up a new technique using Taylor Series.

Next an assumption, assume w (omega) varies slowly with k, so that we may expand it in a Taylor Series about ko.

$k := \frac{2\pi}{\lambda}$ $\omega := 2\pi \cdot v$

$v := 10^5$ $\lambda := 10^2$ <---Radio wave region

$\omega = 6.283 \cdot 10^6$ $k = 0.063$

cls_v $c\ l\ s\ \_\ \omega$

$v := 10^{21}$ $\lambda := 10^{-13}$ <---Atomic wave region

$k := \frac{2\pi}{\lambda}$ $\omega := 2\pi \cdot v$

$\omega = 6.283 \cdot 10^{21}$ $k = 6.283 \cdot 10^{13}$

Why would we make this assumption? *.....a few calculations above showed k << omega w.*

But what said was that it varied slowly with k. Series below has the term (dw/dk) which means change of w (omega) with respect to k. Could this be the concern of the assumption. Omega is huge in comparison to k above example calculations. We want omega slowed down when? When going from one wavefuntion to the next maybe? Phase Velocity = freq x lamda. Omega has v (frequency - 2 pi v) so by slowing down omega we slow down the phase velocity. This makes for what? If the change is slow or very small its closer to a constant. dw/dk then is the derivative of a constant, which maybe k or some real value times k, so dw/dk closer to k. Maybe yes. Maybe its a mathematical requirement for the Taylor Series. You would expect the series to be more appreciate of constants/coefficients than actual derivatives. A group of waves with velocity not far from each other, makes for the wave packet more realisable. So for now that maybe the case. You got a better explanation forward it OR discuss with your local engineer.

Taylor series about ko:

$$\omega(k) = w(k_0) + (k - k_0)\left(\frac{d}{dk}\omega\right)_{k := k_0} + \left(\frac{1}{2}\right)(k - k_0)^2 \left(\frac{d^2}{dk^2}\omega\right)_{k := k_0} + \ldots$$

Since, our problem lies in the first order values of k and ko, we can ignore the 2nd and higher order derivative terms.

I add 'also PSI(x,t) is concerned in the first order of x, because x is a position coordinate'. Agree? You cant have a position of the 2nd order (X^2)? You correct it if this is wrong. Its WRONG. Because the function w(k) above does not have x, but it may look similar for PSI(x,t). BUT, it's omega, 2 pi v, so v is dependent on x (distance) where t comes to play for time, that gives v (velocity). *Maybe.* You verify.

Neglecting 2nd and higher order terms, as we consider values of k close to ko, and letting w(ko) = wo, we obtain:

$k \sim k_0$ $\qquad \omega(k_o) := \omega_0$

$$\omega(k) = w(k_0) + (k - k_0)\left(\frac{d}{dk}\omega\right)$$

$k := k_0$

The exponential term is where we need to apply Taylor's series: $e^{i(k \cdot x - \omega \cdot t)}$

$$-\omega\, t\ : \qquad w(t_0) + (k - k_0)\left(\frac{d}{dk}\omega\right)$$

Similarly:

$$x(k) = x(k_0) + (k - k_0)\left(\frac{d}{dk}x\right)$$

Since $\left(\frac{d}{dk}x\right)$ k is not changing wrt x, its constant, so the derivative becomes x.

$$xk = x(k_0) + (k - k_0)\ x$$

Plug the terms in the exponential term:

$$e^{i(k \cdot x - \omega \cdot t)} = e^{i\left(((x(k_0) + (k - k_0)\ x)) - \left(w(t_0) + (k - k_0)\left(\frac{d}{dk}\omega\right)\right)\right)}$$

$$\Psi(x,t) := \left(\frac{1}{\sqrt{2\pi}}\right) \cdot \int A(k) \cdot e^{i\left(((x(k_0) + (k - k_0)\ x)) - \left(w(t_0) + (k - k_0)\left(\frac{d}{dk}\omega\right)\right)\right)}\, dk$$

Lets make it simpler thru assigning a function f(x,t) to some of the terms.

$$f(x,t) = \left(\frac{1}{\sqrt{2\pi}}\right) \cdot \int A(k) \cdot e^{i\left(\left(x - \frac{d\,\omega}{dk}t\right)(k - k_0)\right)}\, dk$$

Upper and lower limits?

The upper and lower limits can be reduced to:

$$\left(k_0 + \frac{\Delta k}{2}\right) - \left(k_0 - \frac{\Delta k}{2}\right) = \Delta k$$

Quantum Mechanics A Textbook for Undergraduates. Mahesh C. Jain. PHI. Department of Physics, Hindu College University of Delhi.
To Support Studies In: Modern Physics by S.N. Ghosal.
Purpose: Quantum Mechanics for 'Power Plant Engineering' Studies.
Exercise by: K S Bogha. Wavefunction and Wavepackets leading to the 'Why The Schrodinger Equation'.

$$f(x,t) = \left(\frac{1}{\sqrt{2\pi}}\right)\cdot \int_{-\Delta k}^{\Delta k} A(k)\cdot e^{i\left(\left(x-\frac{d\omega}{dk}t\right)(k-k_0)\right)}\,dk$$ Limits in.

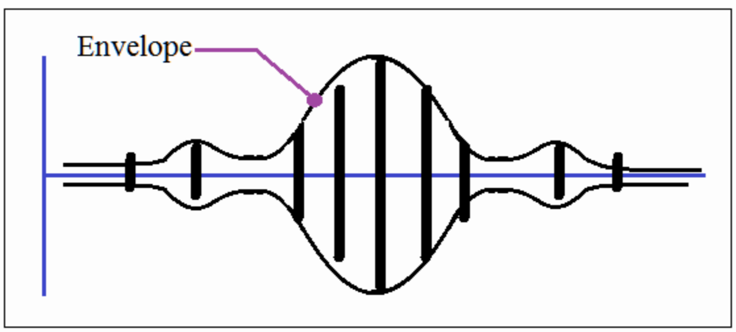
Substituting:

$\Psi(x,t) = f(x,t)\, e^{i(k_0x-\omega_0 t)}$ <---This is what M Jain has in QM For UGs, page 73.

The equation above shows the wave function $\Psi(x,t)$ is a wave of wavelength (2 PI / k0) and frequency (w0 / 2 PI) modulated by the envelope f(x,t)

$e^{i(k_0x-\omega_0 t)}$: $\lambda := \frac{2\pi}{k_0}$ and $v := \frac{\omega_0}{2\pi}$

k0x is related to wavelength because its the varaible x for length, and w0t is the frequency because its the variable t (time) for frequency.



<--- An attempt to show the envelope f(x,t) i.e. the modulator.

The envelope depends upon x and t only through the combination 'x - (dw/dk)t'.

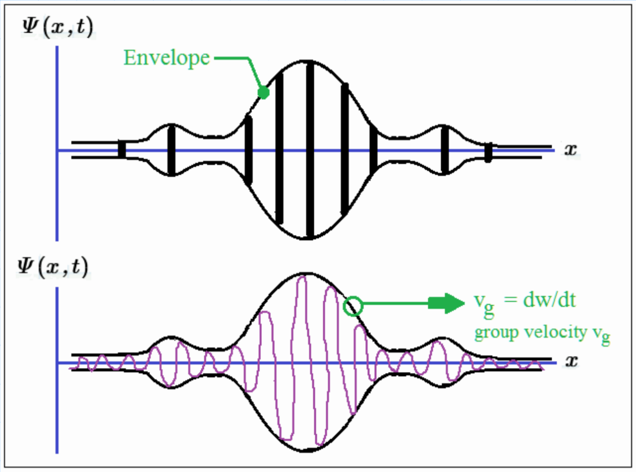
$$f(x,t) = \left(\frac{1}{\sqrt{2\pi}}\right)\cdot \int_{-\Delta k}^{\Delta k} A(k)\cdot e^{i\left(\left(x-\frac{d\omega}{dk}t\right)(k-k_0)\right)}\,dk$$

Equation from previous page shown here, so you see the (x-dw/dk)t.

This term (x -(dw/dk)t) represents the wavepacket which moves withthe group velocity v_g

$$v_g := \frac{d\omega}{dk}$$

Is M Jain and others trying to show the wavepacket can mathematically exist as a mix of wavefunctions, superpositioned? Thats what it looks like to me. M Jain in his textbook QM For UGs does it using the exponent, Fourier, and Taylor Series method which may not be found in most Modern/QM textbooks. You can check that yourself in your textbook.



<----So here we show the group velocity.

Figure above is a wave packet propagating or travelling along the x-axis.

Lastly here in this file, we have one point to make on the property of a wave packet(s).

Suppose delta(x) is the spatial extent (space extent) of a wave packet, and
delta(k) is its wave number range, then it always holds that

$$(\Delta x)\ (\Delta k) \geq 1$$

delta(x) multiplied by delta(k) must be at least equal to or greater than 1.

$$k := \frac{2\ \pi}{\lambda}$$

λ the wavelength is dependent on length, some value of x.

The smaller x gets the larger k gets because the wavelength would get smaller.
The delta(x) tells you that the width of x is small, and this results in a large k - page 73.
But when delta(k) is small, its because lamda is large, so when lamda is large it means x is large.

Is that right they have an inverse relationship?
No, I dont know about inverse relationship here, because it means
delta(x) = 1/delta(k)....this may not be true but if its equal to 1 as in the above >= expression, then maybe. I wouldnt take it that far to be proven wrong without actually conducting an experiment.

So the smaller the spatial extent (delta x) of a wave packet, the larger the range of wave numbers (delta k) in its Fourier decomposition, and vice versa. This general feature of wave packets has very deep implications in quantum mechanics in the form of Hisenberg's uncertainty principle - page 73.

What little accomplished here?
You probably got the right steps and understanding of the subject matter compared to me.
Lets give it a try on where this is truly concerned:

1) some fundamental of communication theory
2) engineering fundamentals stuff that made the radio communication and television
3) again stuff that made satellite communication
4) the internet maybe? yes and 5G......ready for 6G....holograms already there....
5) ......
99)....
100) Lots more.

Lots started with this thinking then put into hardware (production).
Check with your local engineer before you start on any experiments in wavepackets.

Comments:

We started with some accusations towards Schrodinger Equation (SE), whats this all about, never seen one here but got some idea why we needed SE.

The way I see this method is too cumbersome, not clunky too involved, and may be raise suspicion because of the too many Fourier exponents ... integrals....limits .... sorts of things advanced mathematics involved.

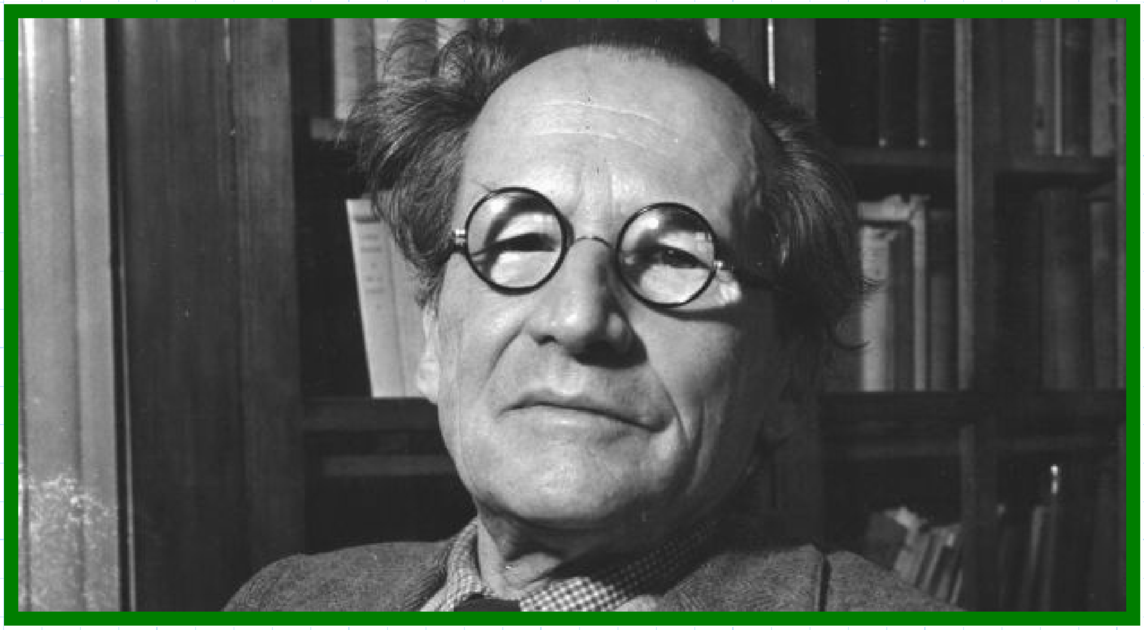
So when you read SE subject material you may see why you got there.
From my little experience in SE example problems it certainly is appreciated compared to if these techniques were required to solve the problem in physics or for that matter any other discipline. Not me!

Thanks.

Any errors and omissions apologies in advance.

Short history on Erwin Schrodinger next page, followed by a figure sketch of a person looking at the earth-world maybe implying these types of subject matter, be it intriguing or surprising.

We accept the sea performing its waves non-stop, on this sea waves some have put forward their theories. We are not discomforted by the sea waves. Similarly science has researched into many of these things and come up with models represented by mathematics to perform something useful for humanity. At times there may been seen a dark side to these things, naturally magic maybe one conclusion, but then go back to the sea waves, is that magic, its an accepted condition. Again, similary for science and its inventions through engineering or other means.



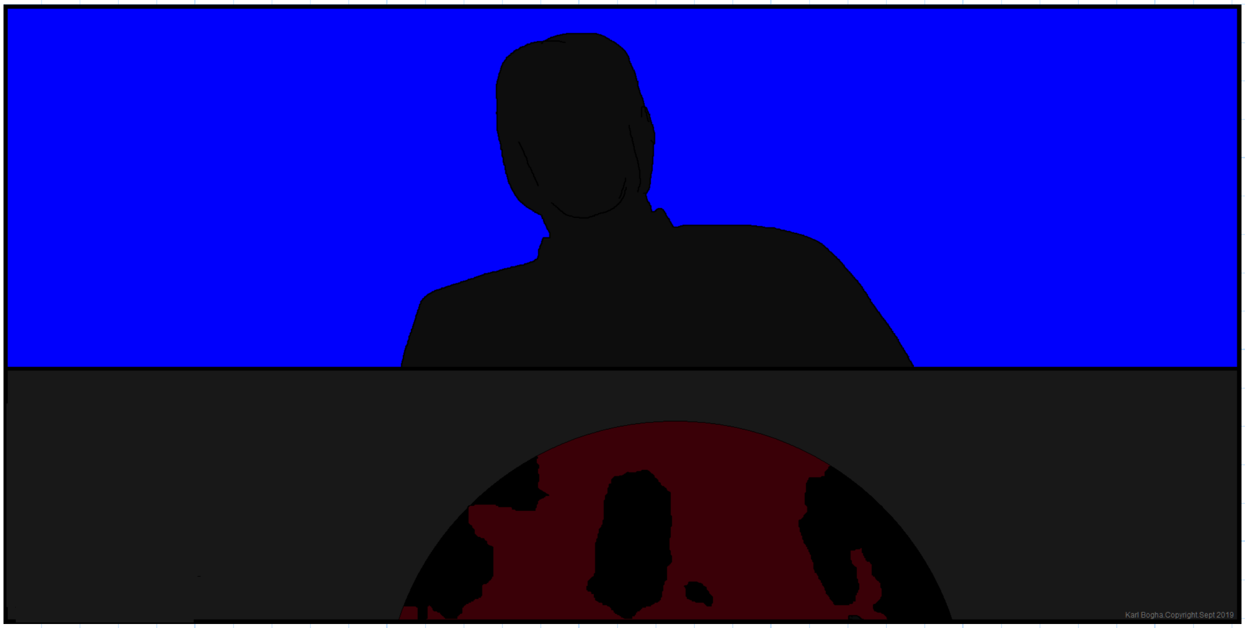
Erwin Schrodinger.

The title of the lecture “What is Life?”, was disarming in its simplicity but it was to prove to be profound in its impact. Austrian physicist Erwin Schrödinger, who had fled from Nazi Germany, was about to turn the world of biological research on its head.

His paper, delivered in the Physics Theatre of Trinity College Dublin – 75 years ago on Monday 5th Feb 2018 – is regarded as Ireland’s greatest contribution to modern science.

A newspaper link below on Schrodinger in the Ireland Times 5th Feb 2018.

https://www.irishtimes.com/news/science/dublin-institute-marks-75th-anniversary-of-schr%C3%B6dinger-lecture-1.3379859



Picture above a man looking at a hi-tech world; abstract perspective abstract art.