Study 23

How to calculate a ship

Or

How you name a boat, is how it will sail

This Study discusses the calculation of the sustainable navigation of ships in fresh and seawater, based on the consideration of forces acting on the ship. A laboratory work covering physics, mathematics, and informatics is proposed. Engineering aspects of an old shipwreck are being discussed.

Mathematics: solution of algebraic equations.

Physics: density, law of Archimedes, center of mass, center of gravity, center of the application of forces, stability of the ship, displacement.

IT: Cloud Computing.

Art: Museums, Ships decoration

Study website: https://community.ptc.com/t5/PTC-Mathcad-Questions/Study-23/td-p/544951

The author of this book work at the International Association for the Properties of Water and Steam[1]. Its members meet every year in various cities of the world closely associated with water[2, 3]. In 2015, such a meeting was in Stockholm, and the final banquet took place in the

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[1] www.iapws.org

[2] Almost all settlements of the world arose on the banks of rivers, lakes, seas, oceans. These water areas do not divide people, but, on the contrary, give them convenient ways of communication.

[3] In 2014, for example, such a meeting took place in Moscow. (In the name of the Russian capital, according to one of the legends, there is water: Moskva – akva, water). The final banquet, without which there is not a single scientific event, took place on board a motor ship cruising under the walls of the Kremlin. In 2017 IAPWS meeting was held in Japan in Kyoto, with the final banquet on the Lake Biwa, which is Japan's largest lake. And in 2018, the IAPWS conference took place in the Czech Republic capital Praha, where the Vltava river is running. The name of the river probably originate from the old Germanic words “wilt ahwa”, wild water

museum of the legendary Swedish sailing ship Vasa[1] which was launched in 1628 and immediately... sank. In 1961, the ship was raised and restored after that. She is currently exhibited in a museum specially built for her (Figure 23.1).

Fig. 23.1. Restored ship Vasa

But meetings within the framework of the International Association on the Properties of Water and Water Steam, of course, consist not only of banquets (see Figure 24.1 in Study 24). It is necessary to work — to review and approve documents describing the procedure for calculating the properties of this important substance. These *formulas* become international standards, reference books are issued and programs for computers are created. It is noteworthy that the similar Russian reference is supplemented by Internet sites with an open interactive network calculations of liquid water and water vapor properties, made using the Mathcad Calculation Server technology and Elsevier/Knovel Interactive Calculation. Figure 23.2 displays one of these

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(https://en.wikipedia.org/wiki/Vltava). One of the associated members of IAPWS is Egypt. So IAPWS members are looking forward to IAPWS meeting in Egypt, sometimes in the future. It is also often mentioned in the history of mathematics the role of the Nile river. First geometrical attempts to calculate the area of figures was associate to Nile's flooding in the Egypt land. One can find a lot of information about Egyptian mathematics in the Rhind Papyrus (1650 B.C.).

[1] https://en.wikipedia.org/wiki/Vasa_(ship)

online calculations, through which it is possible to determine the density of seawater, depending on its temperature, pressure and salinity.

http://twt.mpei.ru/MCS/Worksheets/WSP/SeaWater.xmcd

Thermodynamic Properties of Seawater (IAPWS **formulation**)

S [g/kg] := 13   T [°C] := 17   p [MPa] := 1   Recalculate

$T_0 := 273.15\ \mathrm{K}$ $T^* := 40\ \mathrm{K}$ $\tau := (T - T_0)\ /\ T^* = 0.425$

$p_0 := 101325\ \mathrm{Pa}$ $p^* := 100\ \mathrm{MPa}$ $\pi := (p - p_0)\ /\ p^* = 8.986749999999999 \times 10^{-3}$

$S_n := 35.16504\ \mathrm{g/kg}$ $S^* := S_n \cdot (40\ /\ 35) = 40.18861714285715\ \mathrm{g/kg}$

$\xi := \sqrt{S\ /\ S^*} = 0.5687483435228617$

Specific Gibbs energy water part (IAPWS-IF97)

$gw(T, p) := wspHPT(p, T) - T \cdot wspSPT(p, T)$ $gw(T, p) = -1.176\ \mathrm{kJ/kg}$

Specific Gibbs energy saline part (IAPWS 2008)

$$gs(S,T,p) := \left|\begin{array}{l} \text{return } 0\ \mathrm{J/kg} \text{ if } S = 0 \\ \pi \leftarrow (p - p_0)\ /\ p^* \\ \tau \leftarrow (T - T_0)\ /\ T^* \\ \zeta \leftarrow \sqrt{S\ /\ S^*} \\ \dfrac{J}{kg} \cdot \displaystyle\sum_{k=0}^{5} \left[\pi^k \cdot \sum_{j=0}^{6} \left[\left[(g_1)_{j,k} \cdot \zeta^2 \cdot \ln\zeta + \sum_{i=2}^{7} \left[(g_i)_{j,k}\zeta^i\right]\right] \cdot \tau^j\right]\right] \end{array}\right.$$

See $g_{i,j,k}$ >>>

$gs(S,T,p) = -0.9180517\ \mathrm{kJ/kg}$

Specific Gibbs energy saline water $g(S,T,p) := gw(T,p) + gs(S,T,p)$

$g(S,T,p) = -2.0943151602481898\ \mathrm{kJ/kg}$

Density $\rho := \left(\dfrac{\partial}{\partial p} g(S,T,p)\right)^{-1} = 1009.0649201841026\ \mathrm{kg/m^3}$

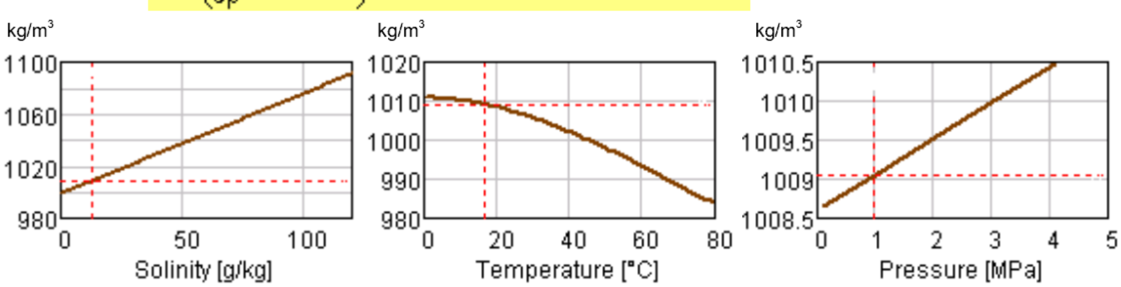


Fig. 23.2. A site where you can estimate the density of seawater

If we are at the Black Sea, where one kilogram of water contains 13 grams of salt, and we dive to a depth of 150 meters, where the pressure is 1.5 MPa (about 15 atmospheres), and the temperature is, let’s say, 17 degrees Celsius, then the density of water there will be equal to 1009.29 kilograms per cubic meter. But we can say quite calmly that the density of water there will be of the order of 1010 kg/m^3, given the measurement error and density calculation technique[1]. In addition, it is difficult to determine the salinity of water, if we take into account the fact that the composition of water in various seas, oceans and salt lakes is different. The site in Figure 23.2 not only gives the desired value of water density but also draws three isolines showing how the density of water will change if one parameter (temperature, for example) is changed and two others (pressure and salinity of water) remain constant. For pure (fresh) water, whose thermodynamic properties are determined only by two parameters — pressure and temperature, it is possible to construct a surface or a family of curves. But seawater requires three separate Cartesian graphics. We already used this technique when in study 14 we decomposed a function of four arguments into four separate planar graphs.

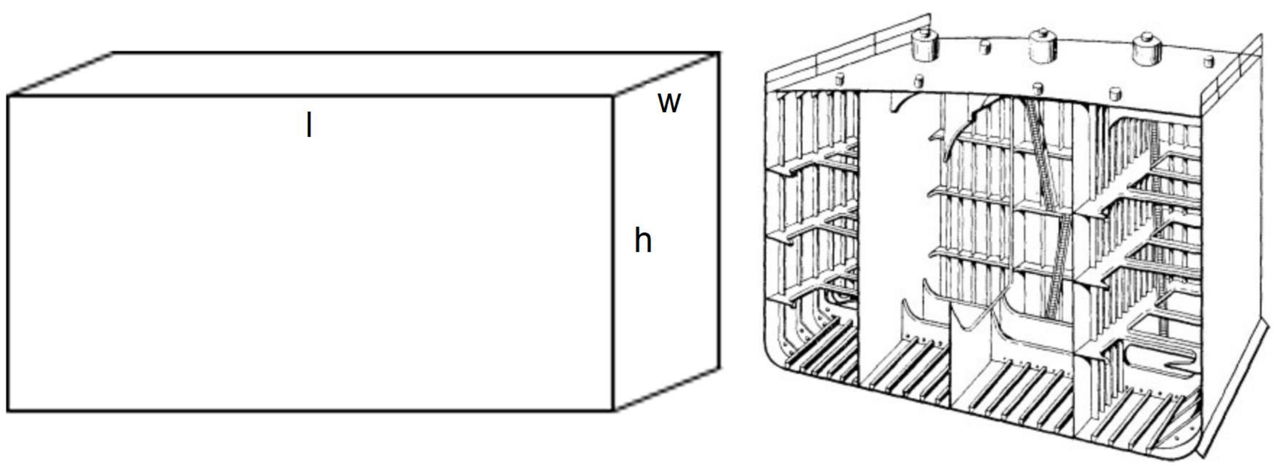
The Internet calculation, shown in Figure 23.2, came to the mind of the author when he was sitting at a banquet in the Stockholm museum at the side of that unlucky ship (Figure 23.1). And that's why.

According to one version, the toppling of the Vasa ship was due to the fact that it was not taken in account the change in the density of water when the ship left the freshwater gulf where the shipyard was located. More dense water lifted the ship, a gust of wind healed, and water poured through the open cannon hatches into the holds ...

So it was or otherwise, you can argue, but the fact that there was some engineering mistake associated with incorrect calculations – this is an undeniable fact[2].

Let's see how we can apply the Mathcad package to evaluate our hypothesis.

As a model of the ship, we take a wooden block in the form of a parallelepiped with sides of 10 mm (the width of the "deck" is variable w), 30 mm (ship's height-h) and 250 mm (length-l: Fig.2.3, left). This simplification is not so far from life: real modern merchant ships have almost rectangular central cross sections (see Figure 23.3, right), and only bow and stern are made pointed to reduce the resistance of water during movement. "Rectangular" ship is easier and cheaper to build, and then to fill it with cargo — containers for example, which also have the form of a parallelepiped.



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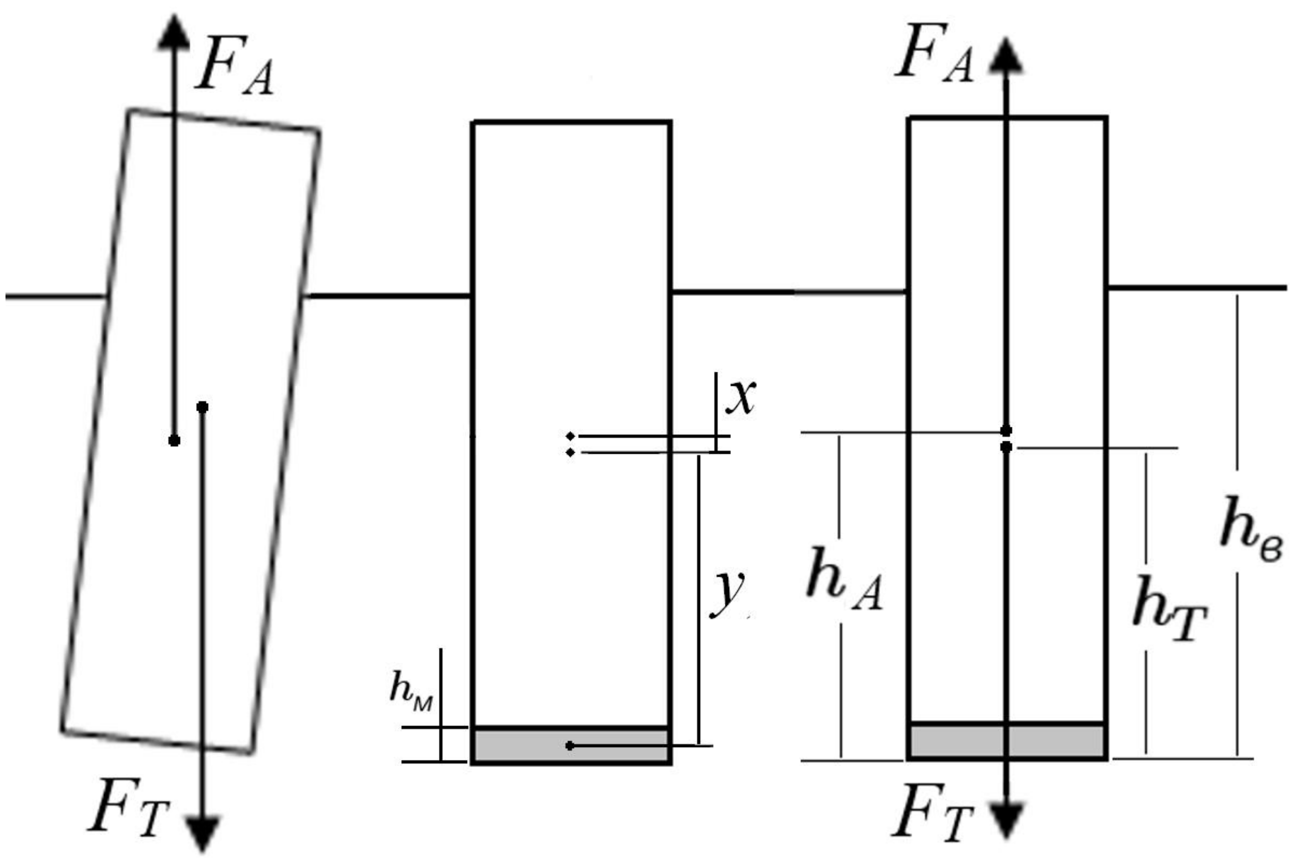
[1] In the tram hangs a sign: "Places for standing 229". A question immediately arises about how accurately the number of these places was determined. You could easily round 229 to 230. What do you say if the electronic thermometer shows that your temperature: 36.589, instead of 36.6?!

[2] Three wise men of Gotham,
They went to sea in a bowl,
And if the bowl had been stronger
My song would have been longer.
https://en.wikipedia.org/wiki/Wise_Men_of_Gotham

Fig. 23.3. The ship model and the central section of a real ship

Put our ship model in water so that the sides of 10 mm (w) and 250 mm (l) form the deck and the bottom of our "ship", and the side of 30 mm (h) — the height from the bottom to the deck. Life experience suggests that such a position of the wooden bar in the water will not be stable (see Figure 23.4, left). The force F_T (the force of gravity applied to the center of mass[1]) and the force F_A (the buoyancy force of Archimedes, applied to the center of displacement, to the center of mass of the volume of fluid displaced by the floating body) will topple our "ship", and it will lie on one side.



[1] Here it is more correct to talk not about the center of mass, but about the center of gravity. But in a homogeneous gravitational field, the center of mass (the geometrical point characterizing the motion of the body or the system of particles as a whole) and the center of gravity (the geometric point through which the resultant of all gravity acts on the particles of the body at any position of the latter in space) coincide. In study 9 we worked with a non-uniform

Fig. 23.4. Forces acting on the "ship"(left — no ballast, center and right — with ballast)

This is explained by the fact that the center of mass was above the center of displacement, and for any small deviation of such a "ship" from the equilibrium position, it will turn over under the action of a pair of forces. But this condition is necessary, but not sufficient, and we will note it below.

To lower the location of the center of mass, let's slightly work the bottom of our "ship" with a plane and attach a metal strip $h_м$ thick there. This will be the same *ballast* that should keep the "ship" from tipping over (see Figure 23.4, center and right). Sailing warships were built high, so that the cannons fired on and it was more convenient to take on board enemy ships. Without a rather massive ballast, such a ship would not have kept afloat even without raised sails [1].

Let's analyze stability of our "ship" using Mathcad (Figure 23.5).

$$\left\| \begin{array}{l} \text{“Weight of the displaced water”} \\ \left\| \begin{array}{l}\text{“Volume of the submersed part”}\\ h_d \cdot w \cdot l\end{array}\right| \cdot \rho_d \cdot g \end{array}\right| = \left\| \begin{array}{l}\text{“Ship weight”}\\ m\cdot g\end{array}\right| \xrightarrow{solve,\, h_d} \frac{m}{\rho_d \cdot l \cdot w}$$

$$\begin{bmatrix} x \cdot m_d = y \cdot m_m \\ h_T = \dfrac{h_m}{2} + y \\ h_m + \dfrac{h - h_m}{2} = \dfrac{h_m}{2} + x + y \end{bmatrix} \xrightarrow{solve,\, \begin{bmatrix} x \\ y \\ h_T \end{bmatrix}} \begin{bmatrix} \dfrac{m_m \cdot h}{2 \cdot m_d + 2 \cdot m_m} & \dfrac{m_d \cdot h}{2 \cdot m_d + 2 \cdot m_m} & \dfrac{m_d \cdot h_m + h_m \cdot m_m + m_d \cdot h}{2 \cdot m_d + 2 \cdot m_m} \end{bmatrix}$$

Fig. 23.5. The derivation of the formulas for calculating the height of the displacement center (h_d) and the center of mass (h_T)

The height of the displacement center (the center of the volume of fluid displaced by the floating

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gravitational field when we considered the motion of satellites around the planets. When we approached the earth, we began to admit that this field is homogeneous.

[1] A popular Russian book about sailing: “Scarlet sails “by Alexander Grin.

body) can be easily determined through the Archimedes law-see the upper operator in Figure 23.5, where the equation (the weight of the displaced water is equal to the weight of the ship) is solved with respect to the variable h_d. To determine the center of mass, it is necessary to solve not one equation, but three more — see the lower operator in Figure 23.5. These equations are:

- The product of the mass of the wooden part of the ship m_w from its center to the center of mass of the entire ship (x) is equal to the product of the mass of the metal part of the ship $m_м$ by the distance from its center to the center of mass of the entire ship (y) — see Figure 23.4, center.

- The height of the center of mass of the ship h_T is equal to half the ballast height $h_м$ plus the value of y.
- The height of the center of mass of the wooden part of the ship h_w is calculated in two ways:

h_m + (h – h_m)/2 and h_m/2 + x + y.

We could derive the formula for h_T manually, but having the Mathcad package at hand, we can automate this work by making the balance of torque (the first equation) and the distances (the two second equations).

Figure 23.6 shows the calculation of the stability of a ship with ballast in fresh water (ρ = 1 g / cm3) using the formula for h_T, derived in Figure 23.5. The height h_A (13.55 mm) was higher than the height h_T (13.313 mm). Our "ship" with ballast will keep upright — a pair of forces will return it to its original position with small deviations from the vertical. But this equilibrium is unstable (local potential well) — with a strong deviation of the ship from the vertical, it still lies to one side.

$$w := 10\ mm \qquad h := 30\ mm \qquad l := 250\ mm \qquad h_m := 0.5\ mm$$

$$\rho_d := 1.0\ \frac{gm}{cm^3} \qquad \text{Water density}$$

$$\rho_w := 0.8\ \frac{gm}{cm^3} \qquad \text{Wood density}$$

$$\rho_m := 7\ \frac{gm}{cm^3} \qquad \text{Metal density}$$

$$m_m := w \cdot l \cdot h_m \cdot \rho_m = 8.75\ gm \qquad m_w := w \cdot l \cdot (h - h_m) \cdot \rho_w = 59\ gm$$

$$m := m_m + m_w = 67.75\ gm$$

$$h_d := \frac{m}{w \cdot l \cdot \rho_d} = 27.1\ mm \qquad h_A := \frac{h_d}{2} = 13.55\ mm$$

Fig. 23.6. Calculation of the stability of a ship with ballast in fresh water

Let's gradually increase the density of water (take our "ship" from the freshwater harbor to the open sea) and see what happens to it. For this, let us introduce a function with the name Δ (the difference h_A and h_T) with argument $\rho_в$, construct its graph and calculate the value of the water density, where this function will change sign — where the forces h_A and h_T will start to turn our "ship" (Figure 23.7).

$$\rho_d := 1\,\frac{gm}{cm^3}, 1.001\,\frac{gm}{cm^3} .. 1.05\,\frac{gm}{cm^3}$$

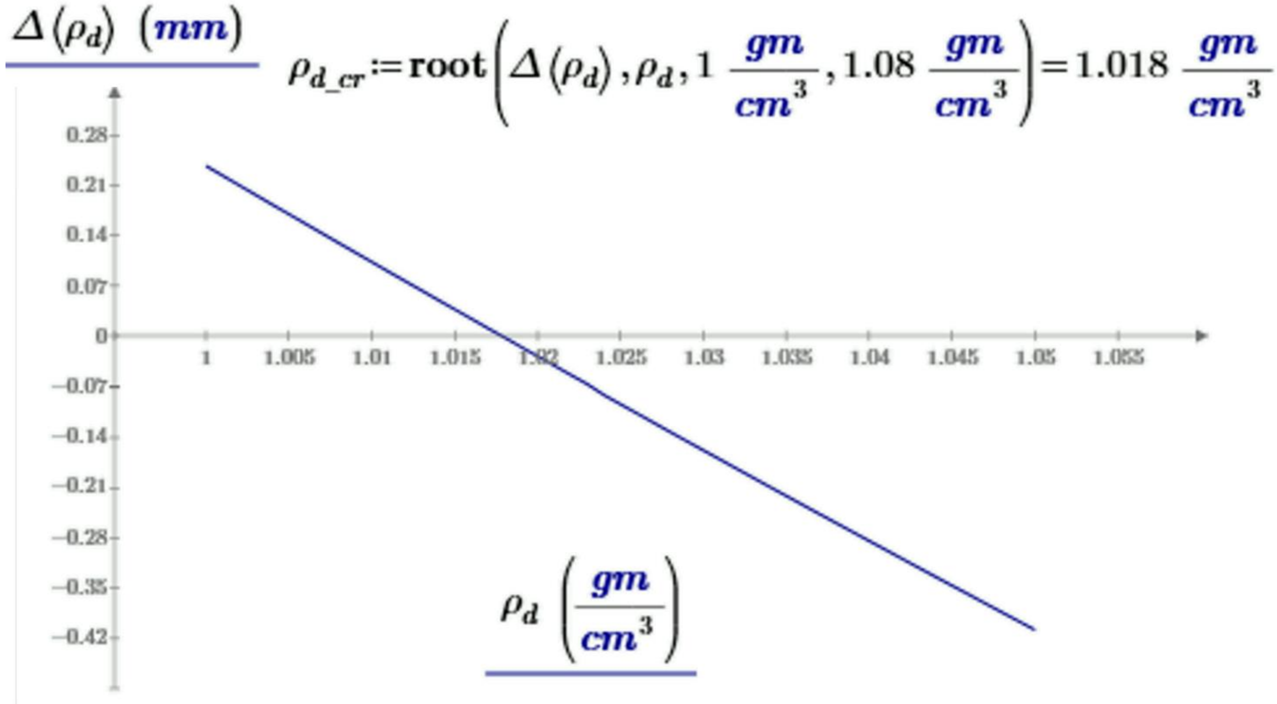

Fig. 23.7. Stability chart of the ship depending on water density

The calculation in Figure 23.7 shows that the critical density of water for our "ship" is 1.018 grams per milliliter. With the left graph in Figure 23.2 it is possible to estimate the salinity of water at such a density.

There is a legend about how ancient Rus (Old Russians) were saved from a raid of Vikings, the ancestors of the builders of the Vasa ship. When these pirates climbed on the river in their boats to the settlement of Rus and in full armor ready to land on-shore, the Rus began to pour in the water from the bags the most precious thing they had — salt. The density of water in the river increased, the Vikings' boats lost their stability and turned over. Vikings partly drowned, and some were taken prisoner.

This legend was invented by the authors when working on the book. But from the point of view of physics and mathematics, it is no worse than the ancient Greek legend of how the enemy fleet was burned with the help of mirrors that focused sun rays on wooden ships.

However truly, we can propose such a **laboratory work**.

A "ship" is produced in the form of a ruler made of wooden and metal rails (see Figure 23.4, center and right). Such a "battleship" descends into the water — it drops into a container of water. At the same time, the "ship" is designed so (see Figure 23.6) that it is steadily held vertically in fresh water. Then the table salt is poured into the water, the water is gently mixed and watched when the "ship" starts ... to fall to one side.

Then it will be possible to heat this aqueous solution and watch how the ship will return to its original vertical position due to the fact that the aqueous solution with a higher temperature has a lower density (see the central graph in Figure 23.2). All this can be supplemented by calculations on the ship's stability computer (solving algebraic equations — see Figures 23.6 and 23.7) and heat and mass transfer processes (solving differential equations). The shape of the ship can be made more complex, close to the real, etc. This will be a good educational theme linking math, physics, chemistry and computer science[1] at school. This can be written in an study. In the lesson, you can try creating an animation of turning the ship or swinging it with the drawing of the vectors of forces acting on the ship.

Of course, the calculation of the stability of real ships is much more complicated. This is a very

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[1] Here we use the term computer science, instead of informatics because it is more commonly applied in English speaking countries. However, Informatics is the science of methods of working with information. It arose long before the advent of computers. But now work with information is inconceivable without a computer. Therefore, these two terms strongly overlap, but do not merge.

beautiful engineering problem. To solve it, almost all great mathematicians have put their hand. Today we can only admire how they could have done it without computers. By the way, on modern ships, computers control special "fins" and other devices that reduce rolling even with strong sea waves and prevent a dangerous roll of the ship. Surprisingly, in a real ship, the center of displacement is still below the center of mass, but its stability is created, among other things, by the complex shape of the underwater part of the ship, which compensates for the deviation of the center of mass by offsetiing the location of the displacement center. The simplest example. If our ship-bar without ballast (Figure 23.4, left) lies on one side, then it is easy to calculate that the center of mass will still be higher than the center of application of the Archimedean forces. But if the ship (or rather, the raft) is tilted from the side wind, for example, the center of application of the Archimedean forces (displacement center) will shift so as to return the ship to a horizontal position — into a global potential pit.

By the way, our ship-ruler ("linear" ship) is a kind of hydrometer [1], a device for measuring the density of liquid, the principle of operation of which is based on the law of Archimedes. The hydrometer is a glass tube, the lower part of which, when calibrated, is filled with shot or mercury (ballast) to achieve the required mass. At the top of the tube is a scale that is graduated in terms of the density of the solution or the concentration of the dissolved substance[1]. A similar "scale" can be seen on the sides of ships to control its loading. Ships of the river-sea class have several such scales. The upper mark on this scale is called a load line, which should not be submerged in water when loading a ship in a sea or river port. Otherwise, the ship can turn over. And there were many such cases, alas.

[1] As reported in a letter by Synesius of Cyrene, bishop of Ptolemais, the invention of the Hydrometer is credited to Hypazia of Alexandria: http://www.livius.org/sources/content/synesius/synesius-letter-015

Paraphrasing AP Chekhov [2], we can say that everything should be fine in the ship — both the silhouette, the name and ... the calculations for which it was designed. Such a small, but instructive calculation we have just conducted.

Center of mass, center of gravity.

How to calculate the center of mass if the figure is not a rectangle (see Figure 23.4), but a more complex plane figure? If you make an appropriate request on the Internet, you will receive either formulas for simple figures (triangle, semicircle, circle sector, etc.) or a "terrible" formula with a double integral, which makes it practically impossible to make a calculation for a particular case. But this is a concrete case.

The figure (say, this is the cross section of the ship) has the form of a semicircle with an outer radius of 1 m and an inner radius of 0.9 m — see Figure 23.8. It is necessary to determine the coordinates of the center of mass of such a figure.

Let's do this — uniformly fill our figure with a set of points and determine the ordinate of the point, so that the total distance from this point to the remaining points will be minimal [3]. Whether this point will be the center of mass is a separate issue. We already instruct the reader how to answer this question. We did something similar in study 7, when we determined the moments of the forces on the sagging chain (see Figures 7.14 and 7.15). If this parameter is divided by the weight of the chain, then we obtain the coordinate of the center of gravity.

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[1] See an example of the tabular and graphical dependence of density and concentration in Study 2 in Figure 2.3.
[2] From the play "Uncle Vanya" (1897) by Anton Pavlovich Chekhov (1860-1904), the words of Dr. Astrov (Act 2): “A human being should be entirely beautiful: the face, the clothes, the mind, the thoughts”

[3] The center of mass x=0, because of a clear symmetry.

Figure 23.8 calculates the center of mass with a method, which can be called the Monte Carlo method. The Monte Carlo method, by the way, is often used in integrating — when determining, for example, areas of complex figures.

$ORIGIN := 1 \quad n := 10^5 \quad X := runif(n, -1, 1)\,m \quad Y := runif(n, -1, 0)\,m$

$$(X \ \ Y) := \left| \begin{array}{l} j \leftarrow 1 \\ \text{for } \ i \in 1..n \\ \quad \text{if } \ (X_i)^2 + (Y_i)^2 \le (1m)^2 \wedge (X_i)^2 + (Y_i)^2 \ge (90cm)^2 \\ \qquad \left| \begin{array}{l} (XX_j \leftarrow X_i \ \ YY_j \leftarrow Y_i) \\ j \leftarrow j + 1 \end{array} \right. \\ (XX \ \ YY) \end{array} \right.$$

$n_1 := last(X) = 15101$

$$M(y) := \left| \begin{array}{l} M \leftarrow 0 \\ \text{for } \ i \in 1..n_1 \\ \quad M \leftarrow M + \sqrt{(X_i - 0m)^2 + (Y_i - y)^2} \\ M \end{array} \right.$$

$y := -0.5 \qquad y := Minimize(M, y) = -750 \cdot mm$

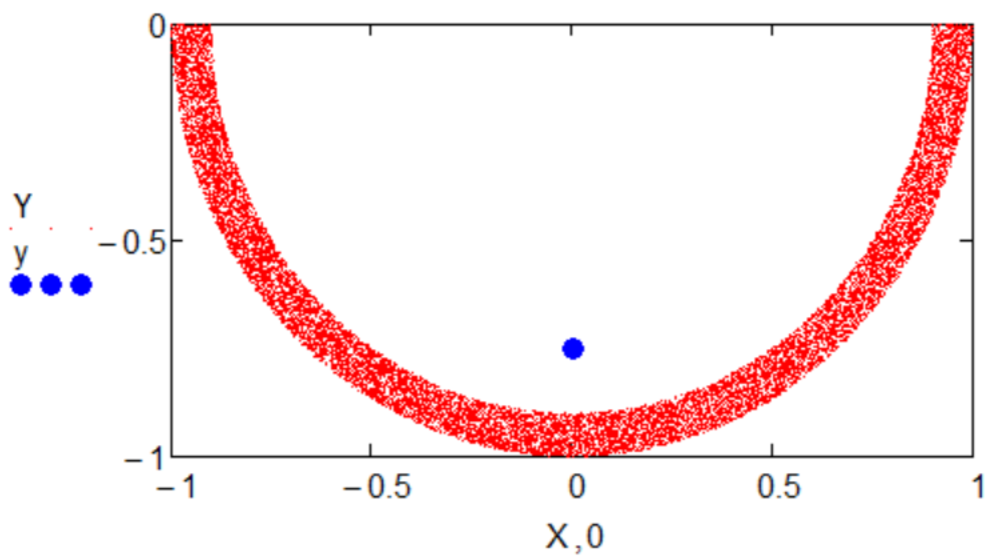

Fig. 23.8. Approximate calculation of the center of mass of a plane figure

We take a rectangle measuring 1 m by 2 m and fill it with n points with random coordinates. We did this work in study 16, when we calculated the number π by the Monte Carlo method — see Figure 16.3. You can also fill the rectangle with points located in the nodes of a uniform grid. Then we filter the array of points — we leave in X and Y vectors only those points that hit the "bottom of the ship" — a semicircle 10 cm thick. These points are shown in the graph of Figure 23.8, forming the given figure. In the if statement, you can add new conditions, forming a complex shape and watching the correctness of the calculations, looking at the points of the graph. The initial points in our particular task were 100,000 (n), and 15,101 points (n_1) hit the "bottom". Next, a target function M(y) is created that returns the total distance from the point with the coordinates (0, y) to the remaining points. Our figure in Figure 23.8 is symmetrical about the central vertical axis, and it is clear that the desired center of mass must be on this axis. But in general, a function named M must have two arguments x and y, and in its formula, instead of zero, there must be a variable (independent optimization parameter) x.. If the problem is not flat, but a volumetric one, then the function M will have three arguments: x, y and z, and then everything is done as described here — only the memory and speed of the computer would suffice. Further, the function M(y) is found numerically near the given value (0, -0.5). Answer: The center of mass of our thickened semicircle is 0.75 m away from its center — see the point on the graph of Figure 25.8. As we already wrote in study 19, remembering one famous film character[1]: "It's not aesthetic, but cheap, reliable and practical". And did we make this determination exactly? Let us verify this in a figure whose coordinates of the center of mass are known.

Figure 23.9 shows the calculation of the center of mass of the hemisphere. It is obtained when,

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[1] https://www.youtube.com/watch?v=9lumF8I69Io

for example, it is necessary to determine the center of application of the Archimedean forces to our "ship" (see Figure 23.8), as much as possible immersing it in water. For the hemisphere, the exact formula for calculating the center of mass is known (see the expression for the variable y_t). The accuracy of our approximate calculation turned out to be satisfactory — within a fraction of a percent (5 mm — see also the point and circle in Figure 23.9).

$y := -0.5 \qquad y := \mathrm{Minimize}(M, y) = -429\,\mathrm{mm}$

$$y_t := -\frac{1\mathrm{m}\cdot 4}{3\pi} = -424\,\mathrm{mm}$$

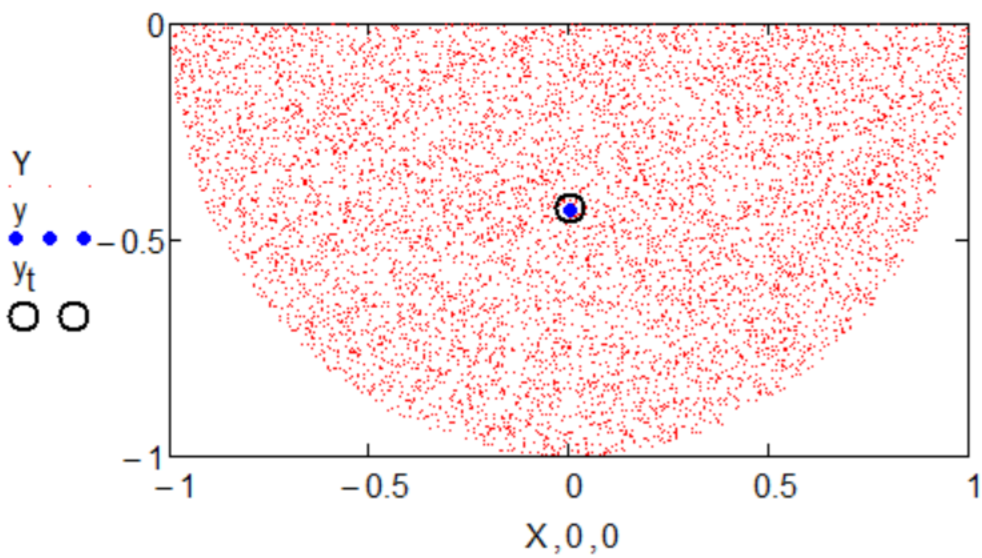

Fig. 23.9. Verification of the approximate calculation of the center of mass of a plane figure

And here is another laboratory work at the intersection of physics and computer science. Take a flat solid figure of complex shape, cut out of cardboard. In this, it is drilled a hole, through which a nail hangs. A weight is hung on this nail along the thread (plumb line). The thread draws a straight line on the figure (option: all this is photographed, sent to the computer and digitized). Then all the actions are repeated for another drilled hole. You can do all this for the third, fourth hole, it will be superfluous. The point of intersection of these two straight lines will be the center of gravity (center of mass) of our flat figure. Animation of such manipulations is located, for example, in the article "Center of Masses" of Wikipedia. This property is due to the fact that the potential energy of a freely suspended body will be minimized — the body will try to settle as low as possible. We examined this phenomenon in study 21 when we calculated the shape of the sagging thread with weights (see Figure 21.1.). Next, a real cardboard figure with two straight drawn lines is photographed, digitized to obtain an array of coordinates of its points along which center of mass according to the above described computer technique. The laboratory work is finished by comparing the coordinates of the two points obtained from physical and computer experiments. (A simplified version of the laboratory work: A flat figure is drawn on a screen that uses the method described above to determine the center of mass.) This figure is then printed on thick paper and cut from the paper with scissors. The cut is determined by the center of gravity, by double hanging it with a plumb on a stud. Verification of the coincidence of the calculated and empirically determined points.)

Assignments for readers

1. Write a program for determining the center of mass of bodies of complex shapes, consisting of simple elements with a known formula.
2. Calculate as far the cannon of a ship will continue to shoot, if you raise it from the lower deck to the top. A tip in Study 7.
3. Try to prove validity and accuracy of the method of calculating the center of mass shown in Figures 23.8 and 23.9.
4. Calculate the stability of the ship, whose cross section is shown in Figure 23.8, using different materials of the body (steel, wood, foam, etc.) and different liquids (mercury, glycerine, water with different salinity, etc.). Show how such a ship will behave if it is

tilted.

5. Take a picture of your friends and acquaintances and determine their center of "masses". It is believed that it should be located in the navel area (see the navel in the Pokémon in Figure 19.7). But is it really so?