Appendix 2

Calculating the threshold value of a 3-parameter Weibull distribution using MathCAD[826]

FIND PARAMETERS OF 3-PARAMETER WEIBULL DISTRIBUTION USING LEAST SQUARES

Example data:

Number of samples tested: $n := 10$

Number of samples failed: $r := 9$

Labelling each breakdown "i" $\quad i := 1, \ldots, r$

Find cumulative probability of breakdown, p, for each sample, i, using median rank approximation:

$$p_i := \frac{i - 0{\cdot}3}{n + 0{\cdot}4}$$

Define w for each "p" such that w values correspond to the linearised vertical axis scale on the Weibull plot:

$\text{Weibull}(p) := \log(-\ln(1-p)) \qquad w_i := \text{Weibull}[p_i]$

The example data points, x, corresponding to nine times-to-breakdown are:

$x_1 := 240 \quad x_2 := 300 \quad x_3 := 340 \quad x_4 := 390 \quad x_5 := 490$

$x_6 := 530 \quad x_7 := 590 \quad x_8 := 750 \quad x_9 := 900$

Now guess initial values for parameters:

Characteristic value: $\text{alpha} := 400$
Shape parameter: $\text{beta} := 1$
Threshold value: $\text{gamma} := 200$

Putting these into a form of $y = mx + c$, we have $w = m \cdot \log(x - g) + c$. The values of m, c, and g are therefore:

$m := \text{beta} \qquad c := -\text{beta} \cdot \log(\text{alpha}) \qquad g := \text{gamma}$

Find the best values of the parameters m, c, and g by solving Normal equations

Given

$$\sum_i [m \cdot [\log[x_i - g]]^2 + \log[x_i - g] \cdot [c - w_i]] \approx 0$$

$$\sum_i [c - w_i + m \cdot \log[x_i - g]] \approx 0$$

$$\sum_i \left[\frac{\log(e)}{x_i - g}[w_i - [m \cdot \log[x_i - g]] - c]\right] \approx 0$$

$g < x_1$

parameters := find (m, c, g) (This assigns the vector called "parameters" to contain the found values of m, c, and g)

$m :=$ parameters$_0$ $m = 1{\cdot}16$ $c :=$ parameters$_1$ $c = -3{\cdot}03$

$g :=$ parameters$_2$ $g = 199$

The best values of the parameters are:

$\text{alpha} := 10^{-[\frac{c}{m}]}$ alpha = 413

beta := m beta = 1·16

gamma := g gamma = 199

The best fitting, least squares line is given by f:

$$f_i := \text{weibull}\left[1 - \exp\left[-\left[\frac{x_i - \text{gamma}}{\text{alpha}}\right]^{\text{beta}}\right]\right]$$

Plotting these on 2-parameter Weibull plot scales gives a good straight line if the threshold value is first subtracted from the data values. Otherwise a convex curve is obtained: