Summary:

This file was created to check simple dynamic response models in FEA.

$Hz := \frac{2 \cdot \pi}{s}$ Mathcad's Hz unit is incorrect. Therefore, redefining it.

$mass := 1kg$ $k := 1 \frac{N}{m}$ $x := 4.3m$ $w := 1m$

$E = \frac{F}{area} = \frac{k \cdot x}{area}$ Young's Modulus

$E := \frac{k \cdot x}{w^2} = 4.3\,Pa$ Young's Modulus

$\omega_n := \sqrt{\frac{k}{mass}} = 1 \cdot \frac{rad}{s}$ Natural Angular Frequency

$\omega_n = 0.159155 \cdot Hz$ Natural Frequency

$area := w^2 = 1\,m^2$

$x1 := 1m$ $x2 := 1m$

$area1 := 1m^2$ $area2 := 1m^2$

$E1 := .5Pa$ $E2 := 5Pa$

$rho1 := .5 \frac{kg}{m^3}$ $rho2 := 1 \frac{kg}{m^3}$

$k1 := \frac{E1 \cdot area1}{x1} = 0.5 \frac{kg}{s^2}$ $k2 := \frac{E2 \cdot area2}{x2} = 5 \frac{kg}{s^2}$

$k1 = 0.5 \cdot \frac{N}{m}$ $k2 = 5 \cdot \frac{N}{m}$

$keq := \frac{k1 \cdot k2}{k1 + k2} = 0.454545 \frac{kg}{s^2}$

$$keq = 0.454545 \cdot \frac{N}{m}$$

$m1 := rho1 \cdot area1 \cdot x1 = 0.5\,kg$ $m2 := rho2 \cdot area2 \cdot x2 = 1\,kg$

$meq := m1 + m2 = 1.5\,kg$

Some of the FEA models use a continous mass matrix, while others use point masses. You can switch the positions of the M3 matrix, to match the given situation.

$$\omega_{eq} := \sqrt{\frac{keq}{meq}} = 0.550482 \cdot \frac{rad}{s}$$

$$K3 := \begin{pmatrix} k1 + k2 & -k2 \\ -k2 & k2 \end{pmatrix}$$

2 Beam Elements

$$\omega_{eq} = 0.087612 \cdot Hz$$

Where:

$$M3 := \begin{pmatrix} \frac{meq}{3} & \frac{meq}{6} \\ \frac{meq}{6} & \frac{meq}{3} \end{pmatrix}$$

First element has near zero mass or is a spring element

$$K3 := \begin{pmatrix} k1 + k2 & -k2 \\ -k2 & k2 \end{pmatrix}$$

2 Point Masses, 2 Springs

$$F = k \cdot x = m \cdot a$$

$$M3 := \begin{pmatrix} m1 & 0 \\ 0 & m2 \end{pmatrix}$$

$$a = \frac{F}{m} = \left(\frac{k}{m}\right) \cdot x$$

$$A1 := \frac{K3}{M3} = \begin{pmatrix} 11 & -5 \\ -10 & 5 \end{pmatrix} \frac{1}{s^2}$$

$$A1 = \frac{k}{m}$$

$$A2 := A1 \cdot s^2$$

$$m1a := \frac{meq}{3} + \frac{meq}{6} = 0.75\,kg$$

$$A2 = \begin{pmatrix} 11 & -5 \\ -10 & 5 \end{pmatrix}$$

$$m2a := \frac{meq}{6} + \frac{meq}{3} = 0.75\,kg$$

$$freq := \sqrt{eigenvals(A1)} = \begin{pmatrix} 3.959943 \\ 0.564672 \end{pmatrix} \frac{1}{s}$$ Eigenvalues (rad/s)

$$\phi := eigenvecs(A1) = \begin{pmatrix} 0.73 & 0.423962 \\ -0.683447 & 0.90568 \end{pmatrix}$$ Eigenvectors (non-dim)

Sorting eigenvalues and eigenvectors (from lowest to highest):

$$i1 := 0 .. (length(freq) - 1)$$

$$order1_{i1} := order(freq \cdot s)_{i1}$$

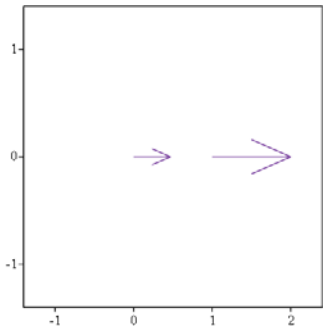
$$Freq_{i1} := freq_{order1_{i1}}$$ Eigenvalues (rad/s) - Sorted

$$eig^{\langle i1 \rangle} := \phi^{\langle order1_{i1} \rangle}$$ Eigenvectors (non-dim) - Sorted

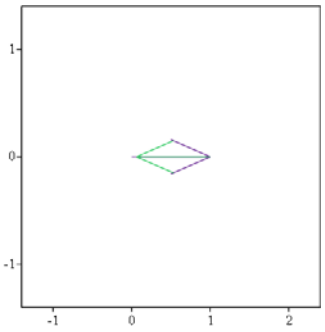
$$Freq = \begin{pmatrix} 0.564672 \\ 3.959943 \end{pmatrix} \cdot \frac{rad}{s} \qquad Freq = \begin{pmatrix} 0.08987 \\ 0.630244 \end{pmatrix} \cdot Hz \qquad eig = \begin{pmatrix} 0.423962 & 0.73 \\ 0.90568 & -0.683447 \end{pmatrix}$$

$$mode1 := eig^{\langle 0 \rangle} = \begin{pmatrix} 0.423962 \\ 0.90568 \end{pmatrix} \qquad mode2 := eig^{\langle 1 \rangle} = \begin{pmatrix} 0.73 \\ -0.683447 \end{pmatrix}$$

$$mode1a := mode1 + \begin{pmatrix} 0 \\ 0 \end{pmatrix} j \qquad mode2a := mode2 + \begin{pmatrix} 0 \\ 0 \end{pmatrix} j$$



mode1a



mode2a

Note; genvals, genvecs, and eigenvec can fail sometimes. Whereas, eigenvals and eigenvecs always seem to give the correct result, in my testing.