# COMPUTATION OF THE MAXIMUM I2t FOR AN ASYNCHRONOUS MOTOR IN A SPECIFIC STATIONARY POINT

**Project:** XXXX
**User:** XXXXX **Date:** dd.mm.yy

**Parameters defining the numerical computation of the analytical relationships**

tmax = maximum time for the calculation of the system of differential equations after the short-circuit

npoints= number of points used in the solution of the current after the short-circuit

num_short_circuit_points = number of explored short circuit instants inside one fourth of a fundamental period of the stator frequenccy (1/(4*fe))

Kdes = index of the short circuit instant whose differential equations and solutions for the corresponding I2t waveforms are desired to be plotted.

w = rotational speed of the qd0 reference frame (usually and by default here is w=0)

$tmax := 300 \cdot 10^{-3} \cdot s$ $npoints := 2000$ $num\_short\_circuit\_points := 20$ $Kdes := 3$ $\omega := 0$

Vectors' index for the time in every solution of the differential equations $n := 1 .. npoints + 1$

Vector's index for the short circuit instants $k := 1 .. num\_short\_circuit\_points$

$$Short\_circuit\_instant_k := \frac{k}{num\_short\_circuit\_points} \cdot \frac{1}{4 \cdot fe}$$

Sampling time (for currents over time): $t\_step := \frac{tmax}{npoints}$

Sampling time (for short circuit instants): $t\_step\_SC := \frac{1}{num\_short\_circuit\_points} \cdot \frac{1}{4 \cdot fe}$

## DYNAMIC MODEL IN SHORT CIRCUIT AT THE CHOSEN OPERATING POINT.

**General transformation matrix**

**dq-Transformed state variables of the system**
(wr is the mechanical speed referred to the field (electrical))

$var = \begin{bmatrix} iqs \\ ids \\ iqr \\ idr \\ \omega r \end{bmatrix}$

$poise := 3$

$n0 := 16.883 \cdot s^{-1}$

$ia\_s0 := \begin{bmatrix} -109.439 \\ -94.06 \\ -78.1 \\ -61.66 \\ -44.838 \\ -27.741 \\ -10.472 \\ 6.861 \\ 24.152 \\ 41.293 \\ 58.181 \\ 74.709 \\ 90.777 \\ 106.286 \\ 121.139 \\ 135.245 \\ 148.517 \\ 160.874 \\ 172.239 \\ 182.542 \end{bmatrix} \cdot A$

$ib\_s0$

Definition fehlernder Einheiten:

$ms \equiv 10^{-3} \cdot s$ $\quad m\Omega \equiv 10^{-3} \cdot \Omega$ $\quad °C \equiv K$ $\quad mH \equiv 10^{-3} \cdot H$ $\quad uH \equiv 10^{-6} \cdot H$ $\quad nH \equiv 10^{-9} \cdot H$

$kN \equiv 10^{3} \cdot N$ $\quad kA2s \equiv 10^{3} \cdot A^{2} \cdot s$ $\quad rpm \equiv \frac{1}{60 \cdot s}$ $\quad kNm \equiv 10^{3} \cdot N \cdot m$ $\quad kW \equiv 10^{3} \cdot W$ $\quad kA \equiv 10^{3} \cdot A$

$k\_sat \equiv 0.6$

$p := 3$ $\quad$ 3

$k\_rs := 1$

$rr := 192.066 \cdot m\Omega$

$k\_rr := 1$

$Tmech := 2.029 \cdot kN \cdot m$

$rs := 194.054 \cdot m\Omega$

$Llr := 3.26 \cdot mH$

$k\_Lls := k\_sat$

$rs = 194.054\ m\Omega$

$rr = 192.066\ m\Omega$

$Lh := 47.138 \cdot mH$

$Lls := 1.953 \cdot mH$

$rs\_dio := 0.259 \cdot m\Omega$

$k\_Llr := k\_sat$

$Lls\_add\_short := 400 \cdot nH$

$fe := 51.834 \cdot Hz$

$V\_dio := 1.48 \cdot V$

$k\_Lh := 1$

$V\_short := 10 \cdot V$

$rs\_add\_short := 0 \cdot 10^{-3} \cdot \Omega$

$$Tqd0(\theta) := \frac{2}{3} \cdot \begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{2 \cdot \pi}{3}\right) & \cos\left(\theta + \frac{2 \cdot \pi}{3}\right) \\ \sin(\theta) & \sin\left(\theta - \frac{2 \cdot \pi}{3}\right) & \sin\left(\theta + \frac{2 \cdot \pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

**Initial values of the stator qd currents, rotor qd currents and state variables**

$$iqd\_s_k := Tqd0(0) \cdot \begin{bmatrix} ia\_s0_k \\ ib\_s0_k \\ -\left(ia\_s0_k + ib\_s0_k\right) \end{bmatrix} \qquad iqd\_r_k := Tqd0(0) \cdot \begin{bmatrix} ia\_r0_k \\ ib\_r0_k \\ -\left(ia\_r0 + ib\_r0\right)_k \end{bmatrix}$$

mech. angular frequency:

$\omega mech\_0 := n0 \cdot 2 \cdot \pi$

$$var\_initial_k := \begin{bmatrix} \left(\frac{iqd\_s_k}{A}\right)_1 \\ \left(\frac{iqd\_s_k}{A}\right)_2 \\ \left(\frac{iqd\_r_k}{A}\right)_1 \\ \left(\frac{iqd\_r_k}{A}\right)_2 \\ \frac{p \cdot \omega mech\_0}{\frac{1}{s}} \end{bmatrix} = \begin{bmatrix} [5 \times 1] \\ [5 \times 1] \\ [5 \times 1] \\ [5 \times 1] \\ [5 \times 1] \\ [5 \times 1] \\ [5 \times 1] \\ [5 \times 1] \\ [5 \times 1] \\ [5 \times 1] \\ [5 \times 1] \\ [5 \times 1] \\ \vdots \end{bmatrix}$$

$Vd(ik) := V\_dio \cdot \text{signum}(ik)$

**System of non-linear differential equations**

$$Ans(var) := \left\| \begin{array}{l} iqd \leftarrow \begin{bmatrix} var_1 \\ var_2 \\ var_3 \\ var_4 \end{bmatrix} \cdot A \\ iqs \leftarrow var_1 \cdot A \\ ids \leftarrow var_2 \cdot A \\ iqr \leftarrow var_3 \cdot A \\ idr \leftarrow var_4 \cdot A \\ \omega r \leftarrow var_5 \cdot s^{-1} \\ ias \leftarrow iqs \\ ibs \leftarrow -\frac{1}{2} \cdot iqs - \frac{\sqrt{3}}{2} \cdot ids \\ ics \leftarrow -(ias + ibs) \\ v \leftarrow V\_short - rs\_add\_short \cdot (ias \cdot \Phi(-ias) + ibs \cdot \Phi(-ibs) + ics \cdot \Phi(-ics)) \\ Vab \leftarrow v \cdot \frac{\text{signum}\left(\frac{ibs}{A}\right) - \text{signum}\left(\frac{ias}{A}\right)}{2} \\ Vbc \leftarrow v \cdot \frac{\text{signum}\left(\frac{ics}{A}\right) - \text{signum}\left(\frac{ibs}{A}\right)}{2} \\ Vcm \leftarrow v \cdot \frac{2 \cdot \text{signum}\left(\frac{ibs}{A}\right) - \text{signum}\left(\frac{ias}{A}\right) - \text{signum}\left(\frac{ics}{A}\right)}{6} - \frac{Vd\left(\frac{ias}{A}\right) + Vd\left(\frac{ibs}{A}\right) + Vd\left(\frac{ics}{A}\right)}{3} \\ trasf\_volt \leftarrow Tqd0(0) \cdot \begin{bmatrix} Vab - Vcm - Vd\left(\frac{ias}{A}\right) \\ -Vcm - Vd\left(\frac{ibs}{A}\right) \\ -Vbc - Vcm - Vd\left(\frac{ics}{A}\right) \end{bmatrix} \\ u\_t\_iqd \leftarrow \text{stack}(trasf\_volt_1, trasf\_volt_2, 0, 0) \\ Lext \leftarrow \begin{bmatrix} Lls\_add\_short & 0 & 0 & 0 \\ 0 & Lls\_add\_short & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ Rext \leftarrow \begin{bmatrix} rs\_dio & 0 & 0 & 0 \\ 0 & rs\_dio & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ Ind\_matrix \leftarrow \begin{bmatrix} k\_Lls \cdot Lls + k\_Lh \cdot Lh & 0 & Lh & 0 \\ 0 & k\_Lls \cdot Lls + k\_Lh \cdot Lh & 0 & Lh \\ Lh & 0 & k\_Llr \cdot Llr + k\_Lh \cdot Lh & 0 \\ 0 & Lh & 0 & k\_Llr \cdot Llr + k\_Lh \cdot Lh \end{bmatrix} \\ Res\_matrix \leftarrow \begin{bmatrix} k\_rs \cdot rs & 0 & 0 & 0 \\ 0 & k\_rs \cdot rs & 0 & 0 \\ 0 & 0 & k\_rr \cdot rr & 0 \\ 0 & 0 & 0 & k\_rr \cdot rr \end{bmatrix} \end{array} \right.$$

$$(var\_initial_1) = \begin{bmatrix} -109.439 \\ 191.719 \\ 39.304 \\ -192.211 \\ 318.237 \end{bmatrix}$$

$\text{signum}(1 \cdot A) = ?$

$var := (var\_initial_1)$

$iqs := var_1 \cdot A$ $\quad ids := var_2 \cdot A$ $\quad iqr := var_3 \cdot A$ $\quad idr := var_4 \cdot A$

$$iqd := \begin{bmatrix} var_1 \\ var_2 \\ var_3 \\ var_4 \end{bmatrix} \cdot A$$

$ias := iqs = -109.439\ A$ $\quad \omega r := var_5 \cdot s^{-1}$ $\quad \Phi(1 \cdot A) = 1$

$$ibs := -\frac{1}{2} \cdot iqs - \frac{\sqrt{3}}{2} \cdot ids = -111.314\ A$$

$$ics := -(ias + ibs) = 220.753\ A$$

$$v := V\_short - rs\_add\_short \cdot (ias \cdot \Phi(-ias) + ibs \cdot \Phi(-ibs) + ics \cdot \Phi(-ics)) = 10\ V$$

$$Vab := v \cdot \frac{\text{signum}\left(\frac{ibs}{A}\right) - \text{signum}\left(\frac{ias}{A}\right)}{2} = 0\ V$$

$$Vbc := v \cdot \frac{\text{signum}\left(\frac{ics}{A}\right) - \text{signum}\left(\frac{ibs}{A}\right)}{2} = 10\ V$$

$$Vcm := v \cdot \frac{2 \cdot \text{signum}\left(\frac{ibs}{A}\right) - \text{signum}\left(\frac{ias}{A}\right) - \text{signum}\left(\frac{ics}{A}\right)}{6} - \frac{Vd\left(\frac{ias}{A}\right) + Vd\left(\frac{ibs}{A}\right) + Vd\left(\frac{ics}{A}\right)}{3} = -2.84\ V$$

$Vab = 0\ V$

$$Vab - Vcm - Vd\left(\frac{ias}{A}\right) = 4.32\ \frac{kg \cdot m^2}{s^3 \cdot A}$$

$$trasf\_volt := Tqd0(0) \cdot \begin{bmatrix} Vab - Vcm - Vd\left(\frac{ias}{A}\right) \\ -Vcm - Vd\left(\frac{ibs}{A}\right) \\ -Vbc - Vcm - Vd\left(\frac{ics}{A}\right) \end{bmatrix} = \begin{bmatrix} 4.32 \\ -7.482 \\ 0 \end{bmatrix} V$$

$$u\_t\_iqd := \text{stack}(trasf\_volt_1, trasf\_volt_2, 0, 0) = \begin{bmatrix} 4.32 \\ -7.482 \\ 0 \\ 0 \end{bmatrix} V$$

$$Lext := \begin{bmatrix} Lls\_add\_short & 0 & 0 & 0 \\ 0 & Lls\_add\_short & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 4 \cdot 10^{-7} & 0 & 0 & 0 \\ 0 & 4 \cdot 10^{-7} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \frac{s}{A} \cdot V$$

$$Rext := \begin{bmatrix} rs\_dio & 0 & 0 & 0 \\ 0 & rs\_dio & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2.59 \cdot 10^{-4} & 0 & 0 & 0 \\ 0 & 2.59 \cdot 10^{-4} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \frac{1}{A} \cdot V$$

$$Speed\_matrix \leftarrow \begin{bmatrix} 0 & \omega & 0 & 0 \\ -\omega & 0 & 0 & 0 \\ 0 & 0 & 0 & (\omega-\omega r) \\ 0 & 0 & -(\omega-\omega r) & 0 \end{bmatrix}$$

$$sup\_part \leftarrow -(Ind\_matrix + Lext)^{-1} \cdot ((Res\_matrix + Rext + Speed\_matrix \cdot Ind\_matrix) \cdot iqd - u\_t\_iqd)$$

$$Moment\_el \leftarrow \frac{3}{2} \cdot p \cdot Lh \cdot (var_4 \cdot var_1 - var_3 \cdot var_2) \cdot A^2$$

$$inf\_part \leftarrow \left( \frac{Moment\_el - \text{signum}\left(\frac{Moment\_el}{s \cdot A \cdot V}\right) \cdot min(|Moment\_el|, |Tmech|)}{J} \right) \cdot p$$

$$D(time, var) \leftarrow \text{stack}\left( \frac{sup\_part}{\frac{A}{s}}, \frac{inf\_part}{s^{-0}} \right)$$

$$solution \leftarrow \text{rkfixed}\left(var, 0, \frac{tmax}{s}, npoints, D\right)$$

$$solution$$

$$Ind\_matrix := \begin{bmatrix} k\_Lls \cdot Lls + k\_Lh \cdot Lh & 0 & Lh & 0 \\ 0 & k\_Lls \cdot Lls + k\_Lh \cdot Lh & 0 & Lh \\ Lh & 0 & k\_Llr \cdot Llr + k\_Lh \cdot Lh & 0 \\ 0 & Lh & 0 & k\_Llr \cdot Llr + k\_Lh \cdot Lh \end{bmatrix} = \begin{bmatrix} 0.048 & 0 & 0 \\ 0 & 0.048 & 0 \\ 0.047 & 0 & 0 \\ 0 & 0.047 & 0 \end{bmatrix}$$

$$Res\_matrix := \begin{bmatrix} k\_rs \cdot rs & 0 & 0 & 0 \\ 0 & k\_rs \cdot rs & 0 & 0 \\ 0 & 0 & k\_rr \cdot rr & 0 \\ 0 & 0 & 0 & k\_rr \cdot rr \end{bmatrix} = \begin{bmatrix} 0.194 & 0 & 0 & 0 \\ 0 & 0.194 & 0 & 0 \\ 0 & 0 & 0.192 & 0 \\ 0 & 0 & 0 & 0.192 \end{bmatrix} \frac{kg \cdot m^2}{s^3 \cdot A^2}$$

$$\omega = 0$$

$$Speed\_matrix := \begin{bmatrix} 0 & \omega & 0 & 0 \\ -\omega & 0 & 0 & 0 \\ 0 & 0 & 0 & (\omega-\omega r) \\ 0 & 0 & -(\omega-\omega r) & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -318.237 \\ 0 & 0 & 318.237 & 0 \end{bmatrix} \frac{1}{s} \qquad \omega r = 318.237 \frac{1}{s}$$

$$sup\_part := -(Ind\_matrix + Lext)^{-1} \cdot ((Res\_matrix + Rext + Speed\_matrix \cdot Ind\_matrix) \cdot iqd - u\_t\_iqd) = \begin{bmatrix} 5.075 \cdot 10^4 \\ -3.498 \cdot 10^5 \\ -5.147 \cdot 10^4 \\ 3.575 \cdot 10^5 \end{bmatrix}$$

$$Moment\_el := \frac{3}{2} \cdot p \cdot Lh \cdot (var_4 \cdot var_1 - var_3 \cdot var_2) \cdot A^2 = (2.864 \cdot 10^3)\ N \cdot m$$

$$inf\_part := \left( \frac{Moment\_el - \text{signum}\left(\frac{Moment\_el}{s \cdot A \cdot V}\right) \cdot min(|Moment\_el|, |Tmech|)}{J} \right) \cdot p = 2.504 \cdot 10^3$$

$$Ergebnis := \text{stack}\left( \frac{sup\_part}{\frac{A}{s}}, \frac{inf\_part}{s^{-0}} \right) = \begin{bmatrix} 5.075 \cdot 10^4 \\ -3.498 \cdot 10^5 \\ -5.147 \cdot 10^4 \\ 3.575 \cdot 10^5 \\ 2.504 \cdot 10^3 \end{bmatrix} \qquad \frac{sup\_part}{\frac{A}{s}} = \begin{bmatrix} 5.075 \cdot 10^4 \\ -3.498 \cdot 10^5 \\ -5.147 \cdot 10^4 \\ 3.575 \cdot 10^5 \end{bmatrix}$$

## Numerical solution of the non-linear system

$$Solution_k := Ans(var\_initial_k)$$

$$time_{n,k} := (Solution_k \cdot s)_{n,1} \qquad iqs\_sol_{n,k} := (Solution_k \cdot A)_{n,2} \qquad ids\_sol_{n,k} := (Solution_k \cdot A)_{n,3}$$

$$iqr\_sol_{n,k} := (Solution_k \cdot A)_{n,4} \qquad idr\_sol_{n,k} := (Solution_k \cdot A)_{n,5} \qquad \omega r_{n,k} := \left(Solution_k \cdot \frac{1}{s}\right)_{n,6}$$

$Solution_1 =$

| | | | | | |
|---|---|---|---|---|---|
| 0 | −109.439 | 191.719 | 39.304 | −192.211 | 318.237 |
| $1.5 \cdot 10^{-4}$ | −101.826 | 139.254 | 31.583 | −138.585 | 318.613 |
| $3 \cdot 10^{-4}$ | −94.214 | 86.79 | 23.863 | −84.958 | 318.988 |
| $4.5 \cdot 10^{-4}$ | −86.601 | 34.325 | 16.142 | −31.331 | 319.364 |
| $6 \cdot 10^{-4}$ | −78.988 | −18.139 | 8.421 | 22.296 | 319.739 |
| $7.5 \cdot 10^{-4}$ | −71.375 | −70.604 | 0.701 | 75.923 | 320.115 |
| $9 \cdot 10^{-4}$ | −63.763 | −123.068 | −7.02 | 129.55 | 320.491 |
| 0.001 | −56.15 | −175.533 | −14.74 | 183.176 | 320.866 |
| 0.001 | −48.537 | −227.997 | −22.461 | 236.803 | 321.242 |
| 0.001 | −40.924 | −280.462 | −30.182 | 290.43 | 321.617 |
| 0.002 | −33.312 | −332.926 | −37.902 | 344.057 | 321.993 |
| 0.002 | −25.699 | −385.391 | −45.623 | 397.684 | 322.369 |
| | | | | | ⋮ |

$$\omega mech_{n,k} := \frac{\omega r_{n,k}}{p}$$