COMPUTATION OF THE MAXIMUM I2t FOR AN ASYNCHRONOUS MOTOR IN A SPECIFIC STATIONARY POINT

Project: XXXX
User: XXXXX **Date:** dd.mm.yy

Parameters defining the numerical computation of the analytical relationships

tmax = maximum time for the calculation of the system of differential equations after the short-circuit

npoints= number of points used in the solution of the current after the short-circuit

num_short_circuit_points = number of explored short circuit instants inside one fourth of a fundamental period of the stator frequenccy (1/(4*fe))

Kdes = index of the short circuit instant whose differential equations and solutions for the corresponding I2t waveforms are desired to be plotted.

w = rotational speed of the qd0 reference frame (usually and by default here is w=0)

$tmax := 300 \cdot 10^{-3} \cdot s$ $npoints := 2000$ $num\_short\_circuit\_points := 20$ $Kdes := 3$ $\omega := 0$

Vectors' index for the time in every solution of the differential equations $n := 1 .. npoints + 1$

Vector's index for the short circuit instants $k := 1 .. num\_short\_circuit\_points$

$$Short\_circuit\_instant_k := \frac{k}{num\_short\_circuit\_points} \cdot \frac{1}{4 \cdot fe}$$

Sampling time (for currents over time): $t\_step := \frac{tmax}{npoints}$

Sampling time (for short circuit instants): $t\_step\_SC := \frac{1}{num\_short\_circuit\_points} \cdot \frac{1}{4 \cdot fe}$

DYNAMIC MODEL IN SHORT CIRCUIT AT THE CHOSEN OPERATING POINT.

General transformation matrix

dq-Transformed state variables of the system
(wr is the mechanical speed referred to the field (electrical))

$\left[iqs \right]$

$var = \begin{bmatrix} ids \\ iqr \\ idr \\ \omega r \end{bmatrix}$

$poise := 3$

$n0 := 16.883 \cdot s^{-1}$

Definition fehlernder Einheiten:

$ms \equiv 10^{-3} \cdot s$ $m\Omega \equiv 10^{-3} \cdot \Omega$ $°C \equiv K$ $mH \equiv 10^{-3} \cdot H$ $uH \equiv 10^{-6} \cdot H$ $nH \equiv 10^{-9} \cdot H$

$kN \equiv 10^{3} \cdot N$ $kA2s \equiv 10^{3} \cdot A^{2} \cdot s$ $rpm \equiv \frac{1}{60 \cdot s}$ $kNm \equiv 10^{3} \cdot N \cdot m$ $kW \equiv 10^{3} \cdot W$ $kA \equiv 10^{3} \cdot A$

$k\_sat \equiv 0.6$

$p := 3$ 3

$k\_rs := 1$

$rr := 192.066 \cdot m\Omega$

$k\_rr := 1$ $Tmech := 2.029 \cdot kN \cdot m$ $rs := 194.054 \cdot m\Omega$

$Llr := 3.26 \cdot mH$

$k\_Lls := k\_sat$ $Lh := 47.138 \cdot mH$ $rs = 194.054\ m\Omega$ $rr = 192.066\ m\Omega$

$Lls := 1.953 \cdot mH$ $rs\_dio := 0.259 \cdot m\Omega$

$k\_Llr := k\_sat$ $Lls\_add\_short := 400 \cdot nH$

$fe := 51.834 \cdot Hz$ $V\_dio := 1.48 \cdot V$

$k\_Lh := 1$ $V\_short := 10 \cdot V$ rs_add_s

$\left[\cos(\theta) \quad \cos\left(\theta - \frac{2 \cdot \pi}{3}\right) \quad \cos\left(\theta + \frac{2 \cdot \pi}{3}\right) \right]$

$$Tqd0(\theta) := \frac{2}{3} \cdot \begin{bmatrix} \sin(\theta) & \sin\left(\theta - \frac{2 \cdot \pi}{3}\right) & \sin\left(\theta + \frac{2 \cdot \pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Initial values of the stator qd currents, rotor qd currents and state variables

$$iqd\_s_k := Tqd0(0) \cdot \begin{bmatrix} ia\_s0_k \\ ib\_s0_k \\ -\left(ia\_s0_k + ib\_s0_k\right) \end{bmatrix} \qquad iqd\_r_k := Tqd0(0) \cdot \begin{bmatrix} ia\_r0_k \\ ib\_r0_k \\ -\left(ia\_r0 + ib\_r0\right)_k \end{bmatrix}$$

mech. angular frequency:

$$\omega mech\_0 := n0 \cdot 2 \cdot \pi$$

$$var\_initial_k := \begin{bmatrix} \left(\frac{iqd\_s_k}{A}\right)_1 \\ \left(\frac{iqd\_s_k}{A}\right)_2 \\ \left(\frac{iqd\_r_k}{A}\right)_1 \\ \left(\frac{iqd\_r_k}{A}\right)_2 \\ p \cdot \omega mech\_0 \end{bmatrix}$$

$Vd(ik) := V\_dio \cdot \text{sign}(ik)$

$$\begin{bmatrix} \frac{1}{s} \end{bmatrix}$$

System of non-linear differential equations

$V\_dio = 1.48\ V$

$$D(x,y) := \left\| \begin{array}{l} iqd \leftarrow \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \cdot A \\ iqs \leftarrow y_1 \cdot A \\ ids \leftarrow y_2 \cdot A \\ iqr \leftarrow y_3 \cdot A \\ idr \leftarrow y_4 \cdot A \\ \omega r \leftarrow y_5 \cdot s^{-1} \\ ias \leftarrow iqs \\ ibs \leftarrow -\frac{1}{2} \cdot iqs - \frac{\sqrt{3}}{2} \cdot ids \\ ics \leftarrow -(ias + ibs) \\ v \leftarrow V\_short - rs\_add\_short \cdot (ias \cdot \Phi(-ias) + ibs \cdot \Phi(-ibs) + ics \cdot \Phi(-ics)) \\ Vab \leftarrow v \cdot \frac{\text{sign}\left(\frac{ibs}{A}\right) - \text{sign}\left(\frac{ias}{A}\right)}{2} \\ Vbc \leftarrow v \cdot \frac{\text{sign}\left(\frac{ics}{A}\right) - \text{sign}\left(\frac{ibs}{A}\right)}{2} \end{array} \right.$$

$\text{signum}(-1) = -1$

$$Vcm \leftarrow v \cdot \frac{2 \cdot \operatorname{sign}\left(\frac{ibs}{A}\right) - \operatorname{sign}\left(\frac{ias}{A}\right) - \operatorname{sign}\left(\frac{ics}{A}\right)}{6} - \frac{Vd\left(\frac{ias}{A}\right) + Vd\left(\frac{ibs}{A}\right) + Vd\left(\frac{ics}{A}\right)}{3}$$

$$trasf\_volt \leftarrow Tqd0(0) \cdot \begin{bmatrix} Vab - Vcm - Vd\left(\frac{ias}{A}\right) \\ -Vcm - Vd\left(\frac{ibs}{A}\right) \\ -Vbc - Vcm - Vd\left(\frac{ics}{A}\right) \end{bmatrix}$$

$$u\_t\_iqd \leftarrow \begin{bmatrix} trasf\_volt_1 \\ trasf\_volt_2 \\ 0 \cdot V \\ 0 \cdot V \end{bmatrix}$$

$$Lext \leftarrow \begin{bmatrix} Lls\_add\_short & 0 \cdot H & 0 \cdot H & 0 \cdot H \\ 0 \cdot H & Lls\_add\_short & 0 \cdot H & 0 \cdot H \\ 0 \cdot H & 0 \cdot H & 0 \cdot H & 0 \cdot H \\ 0 \cdot H & 0 \cdot H & 0 \cdot H & 0 \cdot H \end{bmatrix}$$

$$Rext \leftarrow \begin{bmatrix} rs\_dio & 0 \cdot \Omega & 0 \cdot \Omega & 0 \cdot \Omega \\ 0 \cdot \Omega & rs\_dio & 0 \cdot \Omega & 0 \cdot \Omega \\ 0 \cdot \Omega & 0 \cdot \Omega & 0 \cdot \Omega & 0 \cdot \Omega \\ 0 \cdot \Omega & 0 \cdot \Omega & 0 \cdot \Omega & 0 \cdot \Omega \end{bmatrix}$$

$$\omega = 0$$

$$Ind\_matrix \leftarrow \begin{bmatrix} k\_Lls \cdot Lls + k\_Lh \cdot Lh & 0 \cdot H & Lh & 0 \cdot H \\ 0 \cdot H & k\_Lls \cdot Lls + k\_Lh \cdot Lh & 0 \cdot H & Lh \\ Lh & 0 \cdot H & k\_Llr \cdot Llr + k\_Lh \cdot Lh & 0 \cdot H \\ 0 \cdot H & Lh & 0 \cdot H & k\_Llr \cdot Llr + k\_Lh \cdot Lh \end{bmatrix}$$

$$Res\_matrix \leftarrow \begin{bmatrix} k\_rs \cdot rs & 0 \cdot \Omega & 0 \cdot \Omega & 0 \cdot \Omega \\ 0 \cdot \Omega & k\_rs \cdot rs & 0 \cdot \Omega & 0 \cdot \Omega \\ 0 \cdot \Omega & 0 \cdot \Omega & k\_rr \cdot rr & 0 \cdot \Omega \\ 0 \cdot \Omega & 0 \cdot \Omega & 0 \cdot \Omega & k\_rr \cdot rr \end{bmatrix}$$

$$var\_initial_1 = \begin{bmatrix} -109.439 \\ 191.719 \\ 39.304 \\ -192.211 \\ 318.237 \end{bmatrix}$$

$$Speed\_matrix \leftarrow \begin{bmatrix} 0 \cdot \frac{1}{s} & \omega \cdot \frac{1}{s} & 0 \cdot \frac{1}{s} & 0 \cdot \frac{1}{s} \\ -\omega \cdot \frac{1}{s} & 0 \cdot \frac{1}{s} & 0 \cdot \frac{1}{s} & 0 \cdot \frac{1}{s} \\ 0 \cdot \frac{1}{} & 0 \cdot \frac{1}{} & 0 \cdot \frac{1}{} & \left(\frac{1}{} \cdot \omega - \omega r\right) \end{bmatrix}$$

$$\begin{bmatrix} \cdots & \cdots & \cdots & \cdots \\ 0 \cdot \frac{1}{s} & 0 \cdot \frac{1}{s} & -\left(\frac{1}{s} \cdot \omega - \omega r\right) & 0 \cdot \frac{1}{s} \end{bmatrix}$$

$$sup\_part \leftarrow -(Ind\_matrix + Lext)^{-1} \cdot ((Res\_matrix + Rext + Speed\_matrix \cdot Ind\_matrix) \cdot iqd - u\_t\_iqd)$$

$$Moment\_el \leftarrow \frac{3}{2} \cdot p \cdot Lh \cdot \left(y_4 \cdot y_1 - y_3 \cdot y_2\right) \cdot A^2$$

$$inf\_part \leftarrow \left(\frac{Moment\_el - \operatorname{sign}\left(\frac{Moment\_el}{s \cdot A \cdot V}\right) \cdot min\left(|Moment\_el|, |Tmech|\right)}{J}\right) \cdot p$$

$$Ergebnis \leftarrow \operatorname{stack}\left(\frac{sup\_part}{\frac{A}{s}}, \frac{inf\_part}{s^{-0}}\right)$$

$$Ergebnis$$

Numerical solution of the non-linear system

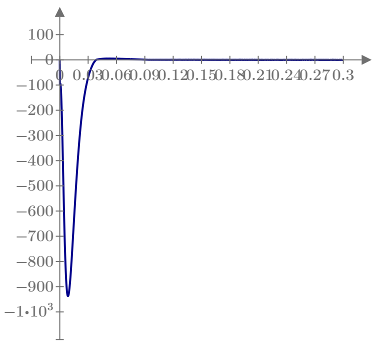
$$Solution_k := \begin{array}{l} y_1 \leftarrow var\_initial_k \\ \Delta t \leftarrow \frac{tmax}{s \cdot npoints} \\ \text{for } i \in 1, 2 .. npoints \\ \quad y_{i+1} \leftarrow y_i + D\left(i \cdot \Delta t, y_i\right) \cdot \Delta t \\ \quad output^{\langle i+1 \rangle} \leftarrow \operatorname{augment}\left(\left[i \cdot \Delta t\right], y_{i+1}{}^{T}\right) \\ output \end{array}$$

$Solution_1 =$

0	0	0	0	0	0
$1.5 \cdot 10^{-4}$	−101.826	139.254	31.583	−138.585	318.613
$3 \cdot 10^{-4}$	−96.751	87.417	26.459	−85.568	318.627
$4.5 \cdot 10^{-4}$	−94.168	36.367	23.885	−33.326	318.627
$6 \cdot 10^{-4}$	−93.802	−13.423	23.596	17.679	318.627
$7.5 \cdot 10^{-4}$	−95.805	−62.228	25.734	67.706	318.627
$9 \cdot 10^{-4}$	−100.322	−109.586	30.436	116.304	318.627
0.001	−107.061	−155.781	37.419	163.74	318.627
0.001	−115.949	−200.727	46.607	209.923	318.423
0.001	−126.897	−244.31	57.913	254.738	317.906
0.002	−139.804	−286.407	71.234	298.057	317.084
0.002	−154.56	−326.906	86.457	339.764	315.967
					⋮

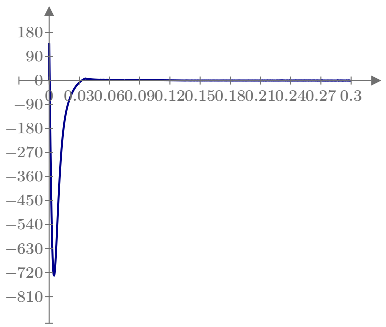
$count := 1, 2 .. npoints$

$$\left(\left(Solution_1^{\langle 10 \rangle}\right)^{\langle 1 \rangle}\right)_1 = 0.001$$



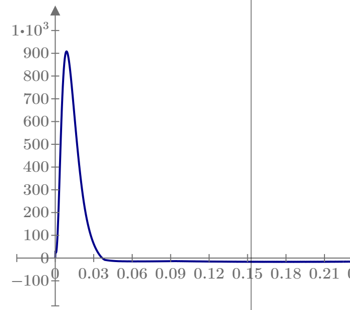
$$\left(\left(Solution_1^{\langle count \rangle}\right)^{\langle 2 \rangle}\right)_1$$

$$\left(\left(Solution_1^{\langle count \rangle}\right)^{\langle 1 \rangle}\right)_1$$

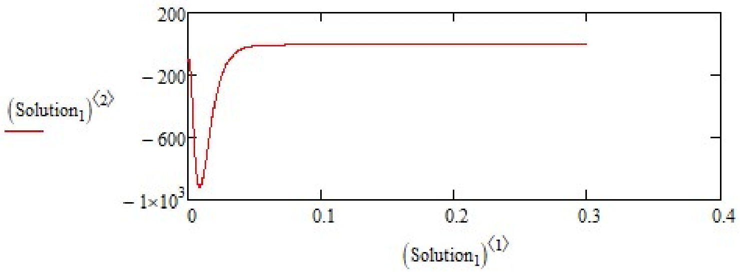


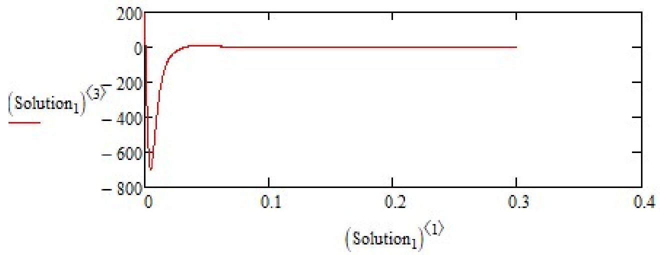
$$\left(\left(Solution_1^{\langle count \rangle}\right)^{\langle 3 \rangle}\right)_1$$

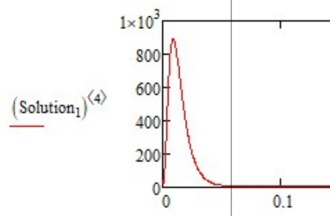
$$\left(\left(Solution_1^{\langle count \rangle}\right)^{\langle 1 \rangle}\right)_1$$

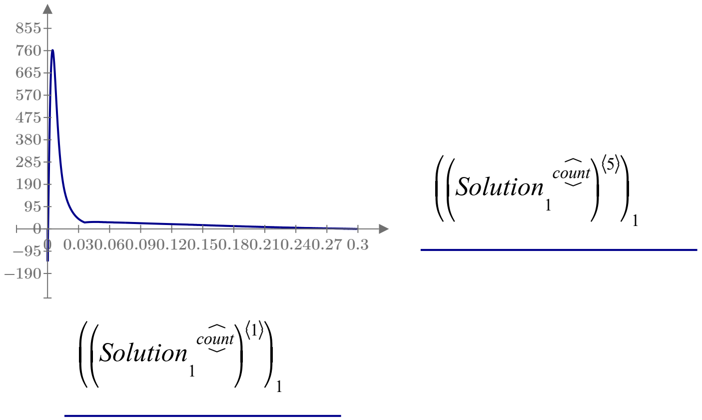


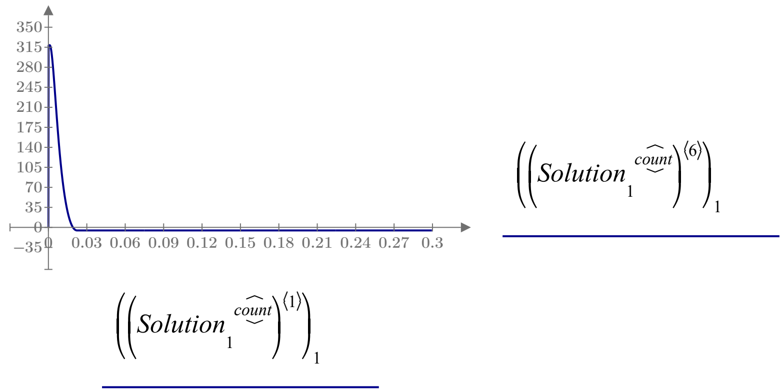
$$\left(\left(Solution_1^{\langle count \rangle}\right)^{\langle 1 \rangle}\right)_1$$

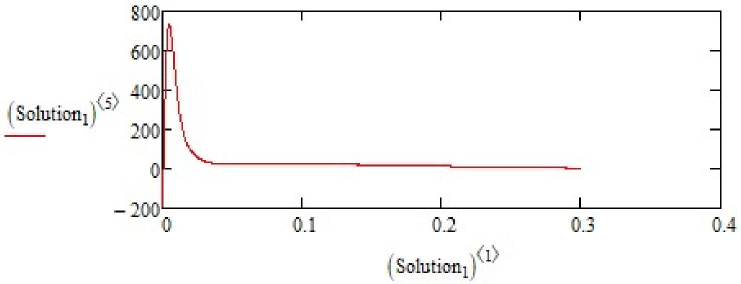


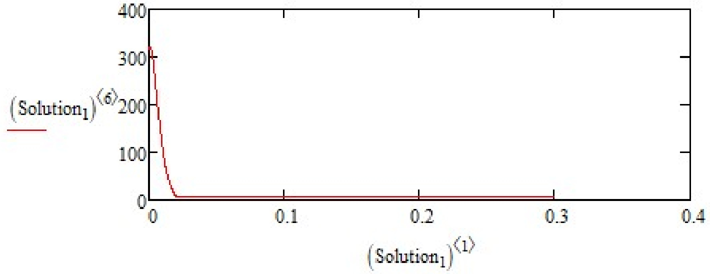





855
760
665
570
475
380
285
190
95
0
−95
−190
0 0.030.060.090.120.150.180.210.240.27 0.3
$\left(\left(\overset{\frown}{Solution_1}^{count}\right)^{\langle 5 \rangle}\right)_1$
$\left(\left(\overset{\frown}{Solution_1}^{count}\right)^{\langle 1 \rangle}\right)_1$


350
315
280
245
210
175
140
105
70
35
0
−35
0 0.03 0.06 0.09 0.12 0.15 0.18 0.21 0.24 0.27 0.3
$\left(\left(\overset{\frown}{Solution_1}^{count}\right)^{\langle 6 \rangle}\right)_1$
$\left(\left(\overset{\frown}{Solution_1}^{count}\right)^{\langle 1 \rangle}\right)_1$


800
600
400
200
0
− 200
0 0.1 0.2 0.3 0.4
$\left(Solution_1\right)^{\langle 5 \rangle}$
$\left(Solution_1\right)^{\langle 1 \rangle}$


400
300
200
100
0
0 0.1 0.2 0.3 0.4
$\left(Solution_1\right)^{\langle 6 \rangle}$
$\left(Solution_1\right)^{\langle 1 \rangle}$

$time_{n,k} := \left(Solution_k \cdot \mathbf{s}\right)_{n,1}$ $\quad iqs\_sol_{n,k} := \left(Solution_k \cdot \mathbf{A}\right)_{n,2}$ $\quad ids\_sol_{n,k} := \left(Solution_k \cdot \mathbf{A}\right)_{n,3}$

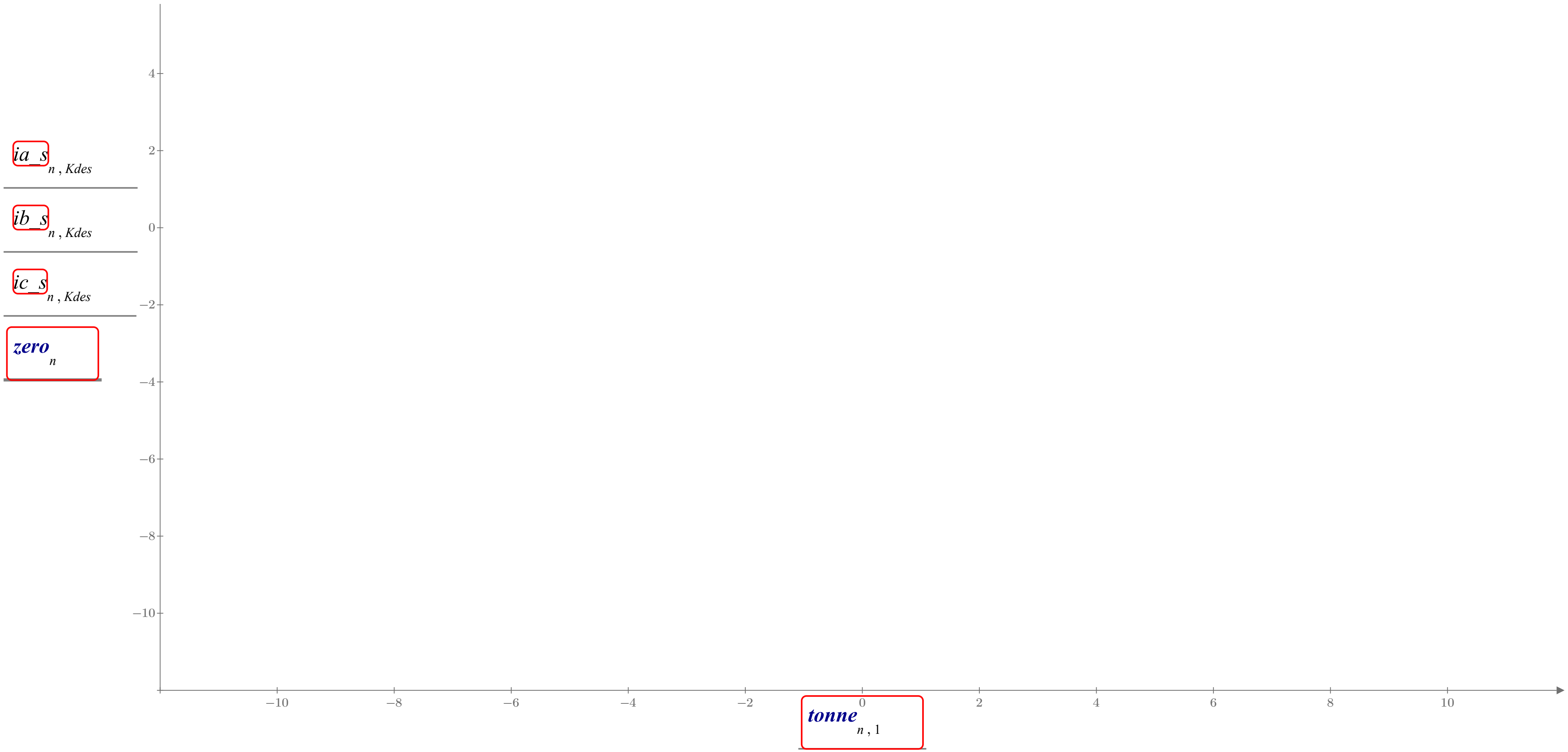
$iqr\_sol_{n,k} := \left(Solution_k \cdot \mathbf{A}\right)_{n,4}$ $\quad idr\_sol_{n,k} := \left(Solution_k \cdot \mathbf{A}\right)_{n,5}$ $\quad \omega r_{n,k} := \left(Solution_k \cdot \frac{1}{\mathbf{s}}\right)_{n,6}$ $\quad \omega mech_{n,k} := \frac{\omega r_{n,k}}{p}$

$$Solution_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1.5 \cdot 10^{-4} & -101.826 & 139.254 & 31.583 & -138.585 & 318.613 \\ 3 \cdot 10^{-4} & -96.751 & 87.417 & 26.459 & -85.568 & 318.627 \\ 4.5 \cdot 10^{-4} & -94.168 & 36.367 & 23.885 & -33.326 & 318.627 \\ 6 \cdot 10^{-4} & -93.802 & -13.423 & 23.596 & 17.679 & 318.627 \\ 7.5 \cdot 10^{-4} & -95.805 & -62.228 & 25.734 & 67.706 & 318.627 \\ 9 \cdot 10^{-4} & -100.322 & -109.586 & 30.436 & 116.304 & 318.627 \\ 0.001 & -107.061 & -155.781 & 37.419 & 163.74 & 318.627 \\ 0.001 & -115.949 & -200.727 & 46.607 & 209.923 & 318.423 \\ 0.001 & -126.897 & -244.31 & 57.913 & 254.738 & 317.906 \\ 0.002 & -139.804 & -286.407 & 71.234 & 298.057 & 317.084 \\ 0.002 & -154.56 & -326.906 & 86.457 & 339.764 & 315.967 \\ & & & & & \vdots \end{bmatrix}$$

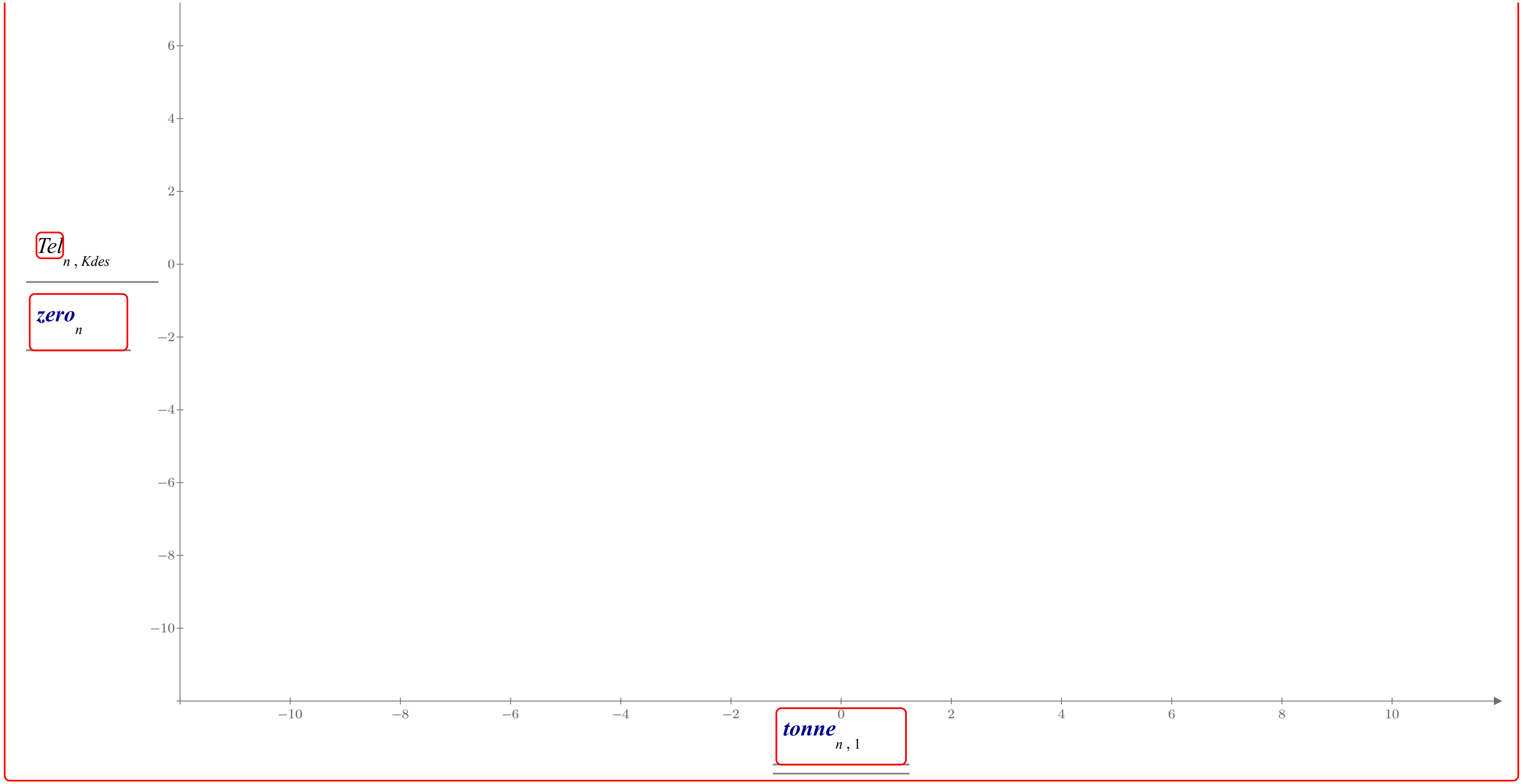
Current in motor phases after the short-circuit at one specific short-circuit time over time:

$k\_$

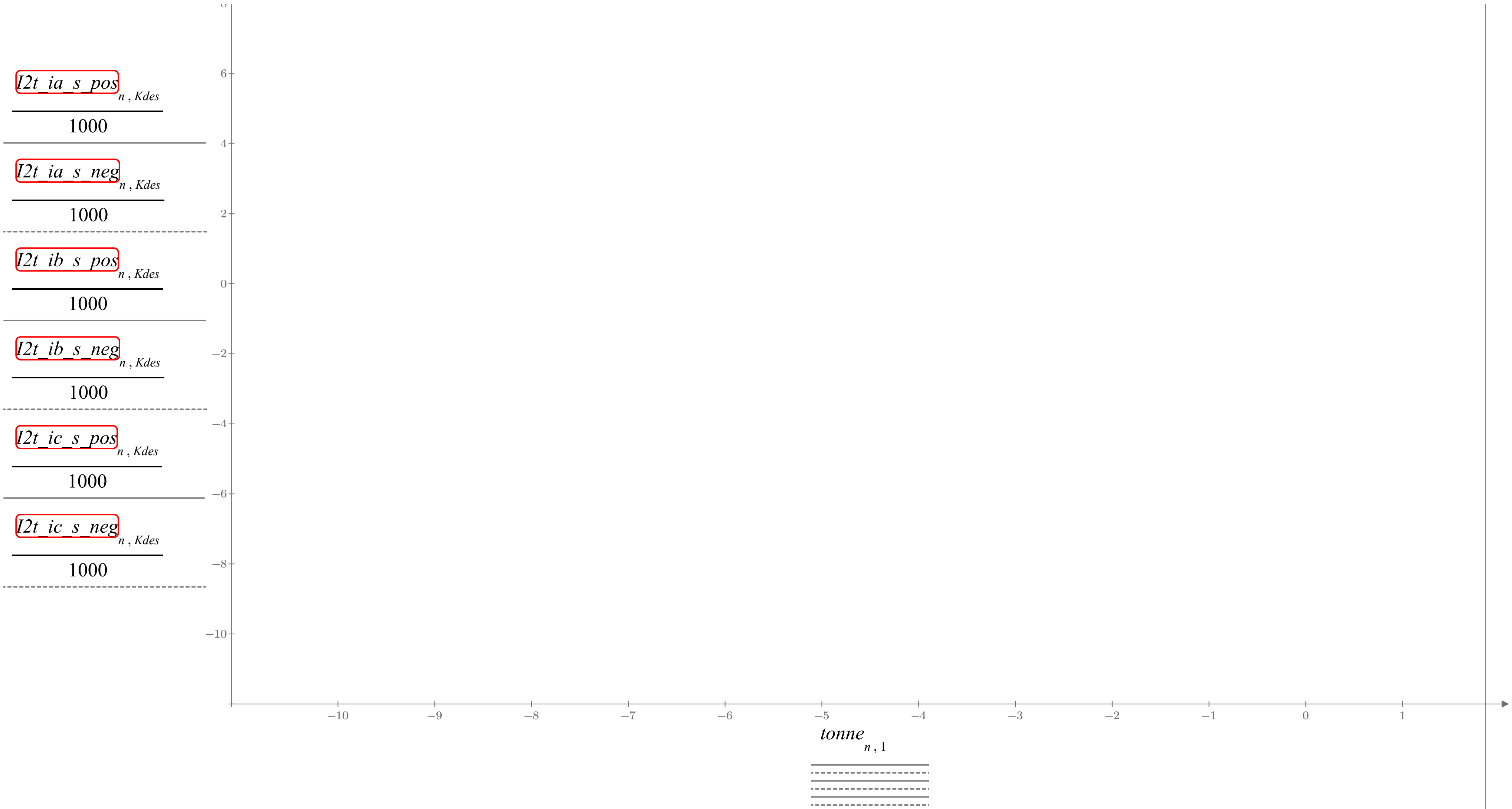
$k\_$



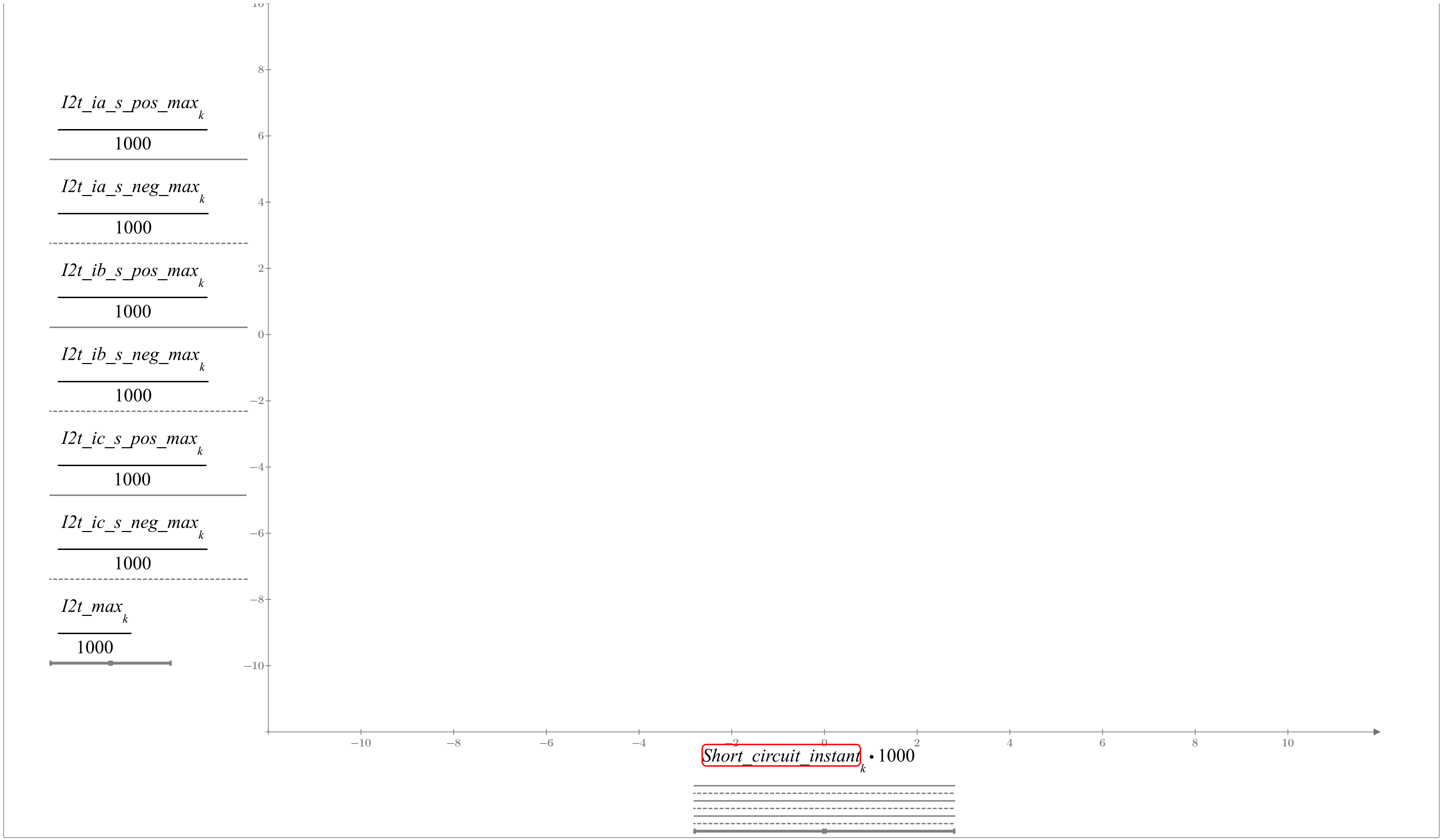
Motor torque after the short-circuit at one specific short-circuit time over time:



I2t-values in the different diodes at one specific short-circuit time over time:



I2t-values in the different diodes for different short circuit time instants:


I2t_ia_s_pos_max_k / 1000
I2t_ia_s_neg_max_k / 1000
I2t_ib_s_pos_max_k / 1000
I2t_ib_s_neg_max_k / 1000
I2t_ic_s_pos_max_k / 1000
I2t_ic_s_neg_max_k / 1000
I2t_max_k / 1000
Short_circuit_instant_k · 1000
8
6
4
2
0
−2
−4
−6
−8
−10
10

$$Excel(M) := \begin{array}{l} ze \leftarrow 1 \\ \text{while } M_{ze+1,1} > M_{ze,1} \\ \quad ze \leftarrow ze + 1 \\ \text{submatrix}(M, 1, ze, 1, \text{cols}(M)) \end{array}$$