Mathcad Community Challenge May 2023 - Optimize Trajectory for Maximum Horizontal Distance

Potential Energy of mass m_b at height h: $W_{pot}(h, m_b) := m_b \cdot g \cdot h$

Kinetic Energy of mass m_b at height h: $W_{kin}(h, h_0, m_b) := W_{pot}(h_0, m_b) - W_{pot}(h, m_b) \text{ simplify } \rightarrow -g \cdot m_b \cdot (h - h_0)$

Velocity at height h: $v(h, h_0) := \sqrt{\dfrac{2 \cdot W_{kin}(h, h_0, m_{ball})}{m_{ball}}} \text{ simplify } \rightarrow \sqrt{-2 \cdot g \cdot (h - h_0)}$

Speed of ball when it leaves the ramp: $v(h_1, h_0) \rightarrow \sqrt{2 \cdot g \cdot (h_0 - h_1)}$

Horizontal distance after leaving the ramp: $s_x(t, h_1, h_0, \varphi) := v(h_1, h_0) \cdot \cos(\varphi) \cdot t$ #1

Vertical distance after leaving the ramp $s_y(t, h_1, h_0, \varphi) := h_1 + v(h_1, h_0) \cdot \sin(\varphi) \cdot t - \dfrac{g}{2} \cdot t^2$

Time after which the ball lands on ground $t_2(h_1, h_0, \varphi) := \text{root}(s_y(t, h_1, h_0, \varphi), t, 0s, 1min)$

Horizontal distance wher the ball lands $dist(h_1, h_0, \varphi) := s_x(t_2(h_1, h_0, \varphi), h_1, h_0, \varphi)$

Point mass of "ball": $m_{ball} := 1kg$

Intiial height: $h0 := 10m$

Final ramp angle: $\varphi_{ramp} := 30°$

Final ramp height: $h1 := 2m$

$dist(h1, h0, \varphi_{ramp}) = 16.726\,m$ #2

Optimal launch ramp angle $\varphi_{opt} := \text{Maximize}(dist(h1, h0), \varphi_{ramp})$

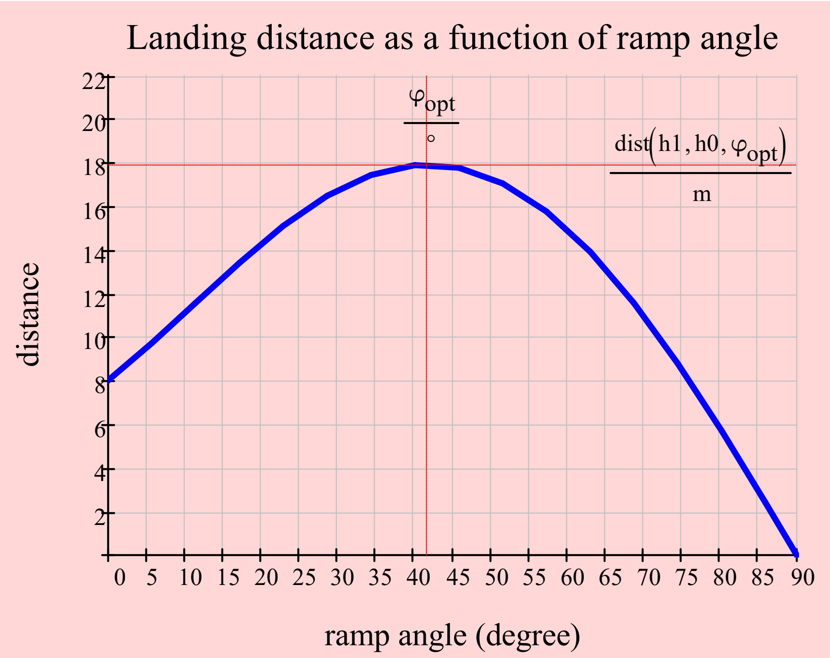
$\varphi_{opt} = 41.81 \cdot °$ #3

The horizontal distance where the ball touches the floor the first time is independent on gravity (as long as its not zero). It would be necessary to change the functions above, making them dependent on g as well and solving symbolically to make Mathcad show that gravity cancels out in the distance function.
Couldn't be bothered doing so. #4

But you could set $g := \dfrac{g}{6} = 1.634 \dfrac{m}{s^2}$ at the top of the sheet to see the same distance will be calculated.

BTW, the symbolic function would be

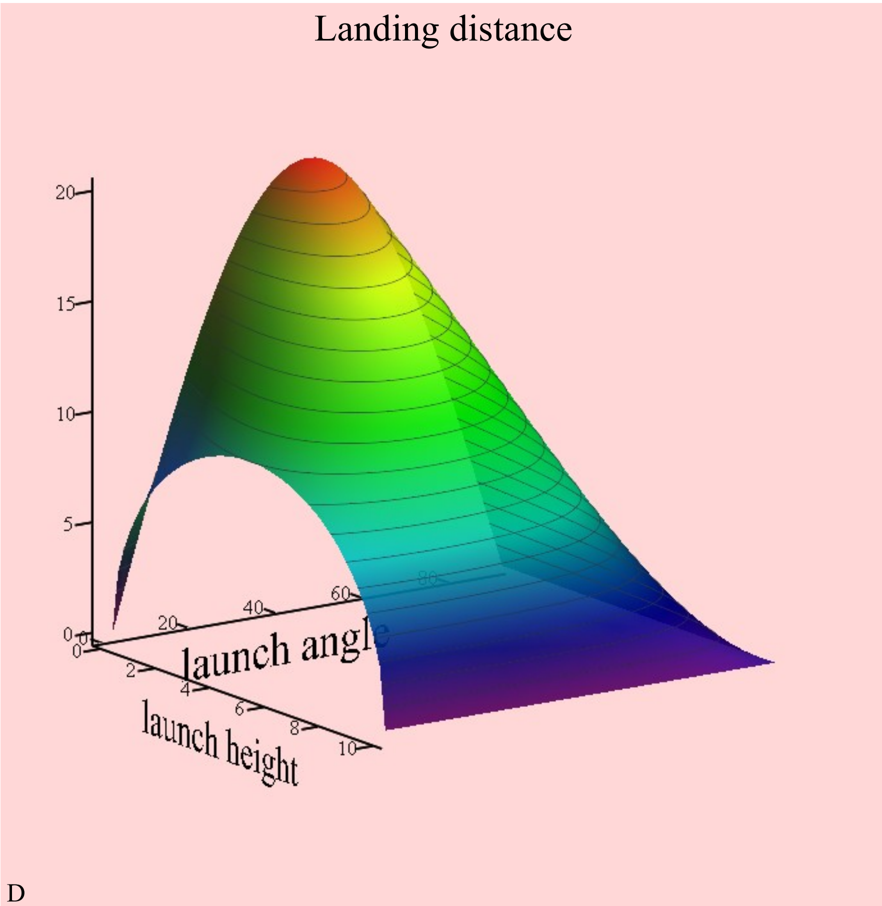
$$dist(h_1, h_0, \varphi) := (h_0 - h_1) \cdot \sin(2 \cdot \varphi) + 2 \cdot \cos(\varphi) \cdot \sqrt{h_0 - h_1} \cdot \sqrt{h_0 - (h_0 - h_1) \cdot \cos(\varphi)^2}$$



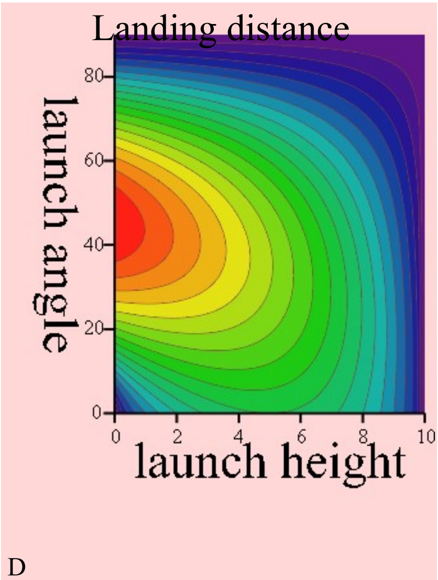
#5

$$\mathrm{dist2}(h_1,\varphi) := \mathrm{dist}(h_1\cdot m, 10m, \varphi\cdot deg)\cdot\frac{1}{m}$$

$$D := \mathrm{CreateMesh}(\mathrm{dist2}, 0, 10, 0, 90, 50, 45)$$



#6



The lower the launch height, the higher the speed (because no kinetic energy is lost for rolling (slinding) up the launch ramp