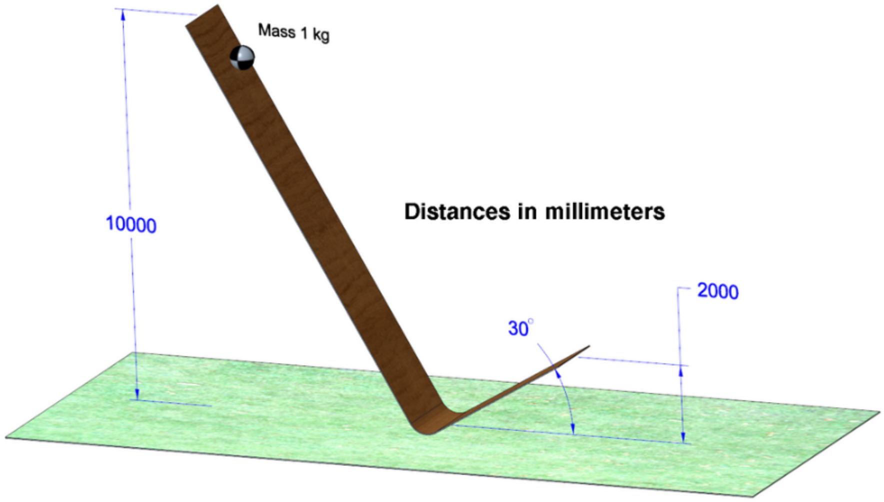
A ball with a mass of 1 kilogram is at the top of a frictionless ramp 10 meters above the ground. The ball rolls down the incline and launches from a height of 2 meters and an angle of 30 degrees above the ground.



(This picture was created in Creo.)

1. Create a function that calculates the horizontal distance as a function of initial height, launch height, and launch angle.
2. Calculate the horizontal distance the ball will land from the end of the ramp.
3. Solve for the angle that will optimize the horizontal distance.
4. How will the horizontal distance change if this were performed on the Moon instead of on the Earth's surface? Assume the acceleration due to gravity on the Moon's surface is 1/6 that of Earth.
5. Use the Chart Component to depict how the horizontal landing distance changes as a function of angle.
6. Use a 3D Plot to show how the horizontal landing distance changes as a function of ramp height and launch angle. Assume the ball starts at a height of 10 meters.

$m_b := 1\ kg$ — Mass of Ball (Solid Sphere)

$h_0 := 10\ m$ — Drop Height

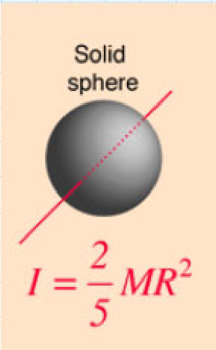
$h_1 := 2\ m$ — Launch Height

$\theta_l := 30\ deg$

$A := \frac{2}{5}$ — Moment of Inertia factor

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

From gsu.edu



$$\omega = \frac{v}{r}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left[\frac{2}{5}mr^2\right]\left[\frac{v}{r}\right]^2$$

1. CREATE A FUNCTION THAT CALCULATES THE HORIZONTAL DISTANCE AS A FUNCTION OF LAUNCH HEIGHT AND LAUNCH ANGLE.

$$H_{dist}\left(h_0, h_1, A, \theta_l\right) := \frac{2 \cdot \sqrt{2} \cdot h_1 \cdot \cos\left(\theta_l\right) \cdot \sqrt{\frac{-\left(g \cdot \left(h_1 - h_0\right)\right)}{A+1}}}{\sqrt{\frac{g \cdot \left(h_1 - h_0\right) \cdot \left(\cos\left(2 \cdot \theta_l\right) - 1\right)}{A+1} + 2 \cdot h_1 \cdot g - \sqrt{2} \cdot \sin\left(\theta_l\right) \cdot \sqrt{\frac{-\left(g \cdot \left(h_1 - h_0\right)\right)}{A+1}}}}$$

2. CALCULATE THE HORIZONTAL DISTANCE THE BALL WILL LAND FROM THE END OF THE RAMP

$$H_{dist}\left(h_0, h_1, A, \theta_l\right) = 12.615\ m$$

3. SOLVE FOR THE ANGLE THAT WILL OPTIMISE THE HORIZONTAL DISTANCE

$$\text{root}\left(\frac{d}{d\theta_l}H_{dist}\left(h_0, h_1, A, \theta_l\right), \theta_l, 0, \frac{\pi}{2}\right) = 40.717\ deg$$

or : The Maximum Horizontal Reach is

$$H_{max}\left(h_0, h_1, A\right) := 2 \cdot \sqrt{\frac{-\left(\left(h_1 - h_0\right) \cdot \left(h_1 \cdot A + h_0\right)\right)}{(A+1)^2}}$$

$$Horiz_{max} := H_{max}\left(h_0, h_1, A\right) = 13.279\ m$$

$$Opr_{ang} := \frac{1}{2}\ \text{atan}\left(\frac{Horiz_{max}}{h_1}\right) = 40.717\ deg$$

4. HOW WILL THE HORIZONTAL DISTANCE CHANGE CHANGE IF PERFORMED ON THE MOON

Lets Check for the Special case above - The Maximum Reach

$$H_{max}\left(h_0, h_1, A\right) := 2 \cdot \sqrt{\frac{-\left(\left(h_1 - h_0\right) \cdot \left(h_1 \cdot A + h_0\right)\right)}{(A+1)^2}}$$

Unaffected by g

$$Horiz_{max} := H_{max}\left(h_0, h_1, A\right) = 13.279\ m$$

$$Opr_{ang} := \frac{1}{2}\ \text{atan}\left(\frac{Horiz_{max}}{h_1}\right) = 40.717\ deg$$

4. CHART COMPONENT - REFER TO PREVIOUS POST(S)