

$$A = e^{-\zeta\omega_n \Delta t} \left( \frac{\zeta}{\sqrt{1-\zeta^2}} \sin \omega_D \Delta t + \cos \omega_D \Delta t \right)$$

$$B = e^{-\zeta\omega_n \Delta t} \left( \frac{1}{\omega_D} \sin \omega_D \Delta t \right)$$

$$C = \frac{1}{k} \left\{ \frac{2\zeta}{\omega_n \Delta t} + e^{-\zeta\omega_n \Delta t} \left[ \left( \frac{1-2\zeta^2}{\omega_D \Delta t} - \frac{\zeta}{\sqrt{1-\zeta^2}} \right) \sin \omega_D \Delta t - \left( 1 + \frac{2\zeta}{\omega_n \Delta t} \right) \cos \omega_D \Delta t \right] \right\}$$

$$D = \frac{1}{k} \left[ 1 - \frac{2\zeta}{\omega_n \Delta t} + e^{-\zeta\omega_n \Delta t} \left( \frac{2\zeta^2-1}{\omega_D \Delta t} \sin \omega_D \Delta t + \frac{2\zeta}{\omega_n \Delta t} \cos \omega_D \Delta t \right) \right]$$

$$A' = -e^{-\zeta\omega_n \Delta t} \left( \frac{\omega_n}{\sqrt{1-\zeta^2}} \sin \omega_D \Delta t \right)$$

$$B' = e^{-\zeta\omega_n \Delta t} \left( \cos \omega_D \Delta t - \frac{\zeta}{\sqrt{1-\zeta^2}} \sin \omega_D \Delta t \right)$$

$$C' = \frac{1}{k} \left\{ -\frac{1}{\Delta t} + e^{-\zeta\omega_n \Delta t} \left[ \left( \frac{\omega_n}{\sqrt{1-\zeta^2}} + \frac{\zeta}{\Delta t \sqrt{1-\zeta^2}} \right) \sin \omega_D \Delta t + \frac{1}{\Delta t} \cos \omega_D \Delta t \right] \right\}$$

$$D' = \frac{1}{k \Delta t} \left[ 1 - e^{-\zeta\omega_n \Delta t} \left( \frac{\zeta}{\sqrt{1-\zeta^2}} \sin \omega_D \Delta t + \cos \omega_D \Delta t \right) \right]$$


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$$\mathbf{u}_{i+1} = \mathbf{A}u_i + \mathbf{B}\dot{u}_i + \mathbf{C}p_i + \mathbf{D}p_{i+1}$$

$$\dot{\mathbf{u}}_{i+1} = \mathbf{A}'u_i + \mathbf{B}'\dot{u}_i + \mathbf{C}'p_i + \mathbf{D}'p_{i+1}$$

### Example Problem

**TABLE E5.1a** NUMERICAL SOLUTION USING LINEAR INTERPOLATION OF EXCITATION

$t_i$ sec	$p_i$ kN	$Cp_i$ cm	$Dp_{i+1}$ cm	$B\dot{u}_i$ cm	$\dot{u}_i$ cm/sec	$Au_i$ cm	$u_i$ cm	Theoretical $u_i$ cm
0.0	0.0000	0.0000	0.0882	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	25.0000	0.1716	0.1528	0.2356	2.5982	0.0717	0.0882	0.0911
0.2	43.3013	0.2972	0.1764	0.7727	8.5221	0.5135	0.6317	0.6477
0.3	50.0000	0.3432	0.1528	1.2230	13.4884	1.4307	1.7599	1.8021
0.4	43.3013	0.2972	0.0882	1.1918	13.1440	2.5605	3.1497	3.2236
0.5	25.0000	0.1716	0.0000	0.4870	5.3708	3.3636	4.1377	4.2338
0.6	0.0000	0.0000	0.0000	-0.7596	-8.3777	3.2698	4.0222	4.1149
0.7	0.0000	0.0000	0.0000	-1.8797	-20.7311	2.0406	2.5102	2.5681
0.8	0.0000	0.0000	0.0000	-2.2357	-24.6571	0.1308	0.1609	0.1648
0.9	0.0000	0.0000	0.0000	-1.7423	-19.2155	-1.7111	-2.1049	-2.1532
1.0	0.0000				-6.9914		-3.4534	-3.5329