The problem discussed here (1) involves a network of factories, warehouses and sales outlets. We need to find the least expensive flow of products from factories to warehouses to stores. One particularity is that each store gets its products from one warehouse.

Mathematical Model

We use the following indices:

- p: products
- f: factories
- w: warehouses
- s: stores

We introduce the following variables:

- $x_{p,f,w} \geq 0$: shipments of product p from factory f to warehouse w,
- $y_{s,w} \in \{0,1\}$: links each store s to a single warehouse w.

The data associated with the model is:

- $pcost_{f,p}$: unit production cost
- $tcost_p$: unit transportation cost
- $dist_{f,w}, dist_{s,w}$: distances
- $pcap_{f,p}$: factory production capacities
- $wcap_w$: warehouse capacity
- $d_{s,p}$: demand for product p at store s
- $turn_p$: product turnover rate

The optimization model looks like:

$$
\begin{aligned}
\min \sum_{p,f,w} (pcost_{f,p} + tcost_p \cdot dist_{f,w}) \cdot x_{p,f,w} + \sum_{s,w,p} d_{s,p} \cdot tcost_p \cdot dist_{s,w} \cdot y_{s,w} \\
& \sum_{w} x_{p,f,w} \leq pcap_{f,p} \;\; \forall f,p \;\; \text{(production capacity)} \\
& \sum_{f} x_{p,f,w} = \sum_{s} d_{s,p} \cdot y_{s,w} \;\; \forall p,w \;\; \text{(demand)} \\
& \sum_{p,s} \frac{d_{s,p}}{turn_p} y_{s,w} \leq wcap_w \;\; \forall w \;\; \text{(warehouse capacity)} \\
& \sum_{w} y_{s,w} = 1 \;\; \forall s \;\; \text{(one warehouse for a store)} \\
& x_{p,f,w} \geq 0 \\
& y_{s,w} \in \{0,1\}
\end{aligned}
$$