Attemp at getting correct $\phi$, example problem from Illinois University

$$M := \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \qquad K := \begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix} \qquad F := \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

$$\Omega := \sqrt{\text{genvals}(K, M)} = \begin{bmatrix} 2.236 \\ 1.414 \end{bmatrix} \qquad \Omega^2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$P := \text{genvecs}(K, M) = \begin{bmatrix} -0.5 & 1 \\ 1 & 1 \end{bmatrix}$$

We wish to redefine the eigenvalues so that our system of equations are easilly recognized (general FEA practice)

$$\phi := P \cdot \text{cholesky}\left(P^{\mathrm{T}} \cdot M \cdot P\right)^{-1} = \begin{bmatrix} -0.408 & 0.577 \\ 0.816 & 0.577 \end{bmatrix}$$

$$m := \phi^{\mathrm{T}} \cdot M \cdot \phi = \begin{bmatrix} 1 & -5.551 \cdot 10^{-17} \\ -5.551 \cdot 10^{-17} & 1 \end{bmatrix}$$

$$k := \phi^{\mathrm{T}} \cdot K \cdot \phi = \begin{bmatrix} 5 & 4.441 \cdot 10^{-16} \\ 3.331 \cdot 10^{-16} & 2 \end{bmatrix}$$

$$f := \phi^{\mathrm{T}} \cdot F = \begin{bmatrix} 8.165 \\ 5.774 \end{bmatrix}$$

As demonstrated below, this method will graph correctly for a forcing function. U(t) is our transformed system. A general solved form of U(t) can be derived:

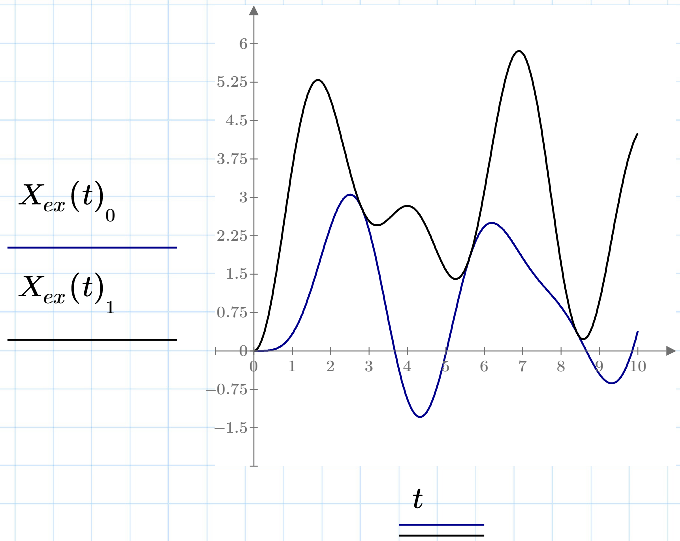
$$C := k^{-1} \cdot f = \begin{bmatrix} 1.633 \\ 2.887 \end{bmatrix} \qquad U(t) := \overrightarrow{-C \cdot \cos(\Omega \cdot t) + C} \qquad X(t) := \phi \cdot U(t)$$

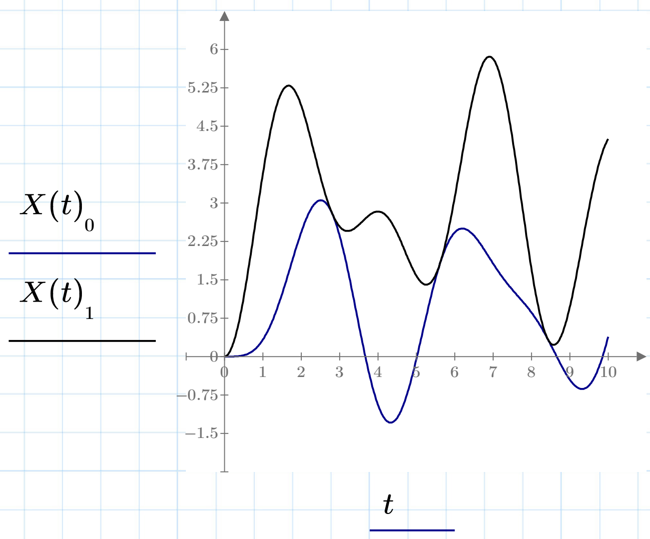
Defining Exact Answer as given:

$$u_1(t) := \frac{5}{\sqrt{3}} \left(1 - \cos\left(\Omega_1 \cdot t\right)\right) \qquad u_2(t) := 2 \cdot \sqrt{\frac{2}{3}} \cdot \left(-1 + \cos\left(\Omega_0 \cdot t\right)\right)$$

$$U(t) := \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \qquad \phi := \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{2} \cdot \sqrt{\frac{2}{3}} \\ \frac{1}{\sqrt{3}} & -\sqrt{\frac{2}{3}} \end{bmatrix} = \begin{bmatrix} 0.577 & 0.408 \\ 0.577 & -0.816 \end{bmatrix} \qquad X_{ex}(t) := \phi \cdot U(t)$$

The generalized FEA Modal Superposition is given by the following:
http://ocw.mit.edu/resources/res-2-002-finite-element-procedures-for-solids-and-structures-spring-2010/linear/lecture-11/MITRES2_002S10_lec11.pdf


$X_{ex}(t)_0$
$X_{ex}(t)_1$
6
5.25
4.5
3.75
3
2.25
1.5
0.75
0
−0.75
−1.5
0 1 2 3 4 5 6 7 8 9 10
$t$


$X(t)_0$
$X(t)_1$
6
5.25
4.5
3.75
3
2.25
1.5
0.75
0
−0.75
−1.5
0 1 2 3 4 5 6 7 8 9 10
$t$

Repeat for lecture Matrix, hand calculations in lecture notes

$$M := \begin{bmatrix} 50 & 0 \\ 0 & 20 \end{bmatrix} \qquad K := \begin{bmatrix} 450 & -350 \\ -350 & 350 \end{bmatrix} \qquad F := \begin{bmatrix} 0 \\ 1000 \end{bmatrix}$$

$$\Omega := \sqrt{\text{genvals}(K, M)} = \begin{bmatrix} 5.011 \\ 1.181 \end{bmatrix} \qquad \Omega^2 = \begin{bmatrix} 25.106 \\ 1.394 \end{bmatrix}$$

$$P := \text{genvecs}(K, M) = \begin{bmatrix} -0.435 & 0.92 \\ 1 & 1 \end{bmatrix}$$

$$\phi := P \cdot \text{cholesky}\left(P^{\mathrm{T}} \cdot M \cdot P\right)^{-1} = \begin{bmatrix} -0.08 & 0.117 \\ 0.184 & 0.127 \end{bmatrix}$$

$$m := \phi^{\mathrm{T}} \cdot M \cdot \phi = \begin{bmatrix} 1 & -1.11 \cdot 10^{-16} \\ -1.11 \cdot 10^{-16} & 1 \end{bmatrix}$$

$$k := \phi^{\mathrm{T}} \cdot K \cdot \phi = \begin{bmatrix} 25.106 & 1.776 \cdot 10^{-15} \\ 2.22 \cdot 10^{-15} & 1.394 \end{bmatrix}$$

$$f := \phi^{\mathrm{T}} \cdot F = \begin{bmatrix} 184.287 \\ 126.642 \end{bmatrix}$$

Expressing our general solution:

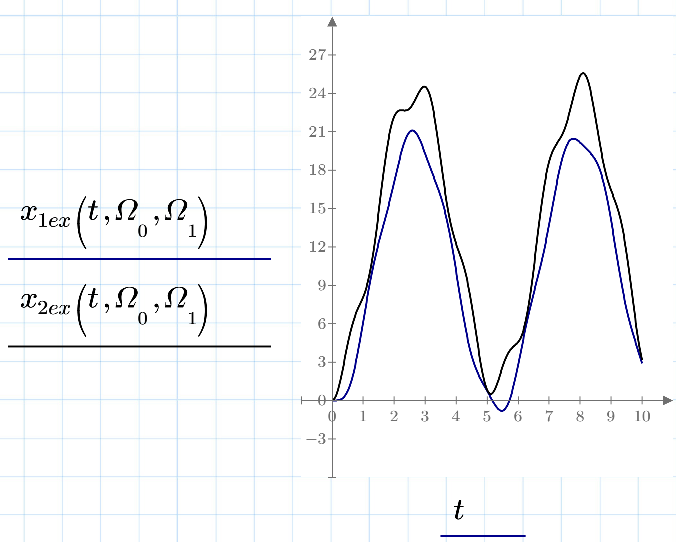
$$C := k^{-1} \cdot f = \begin{bmatrix} 7.34 \\ 90.842 \end{bmatrix} \qquad U(t) := \overrightarrow{-C \cdot \cos(\Omega \cdot t) + C} \qquad X(t) := \phi \cdot U(t)$$

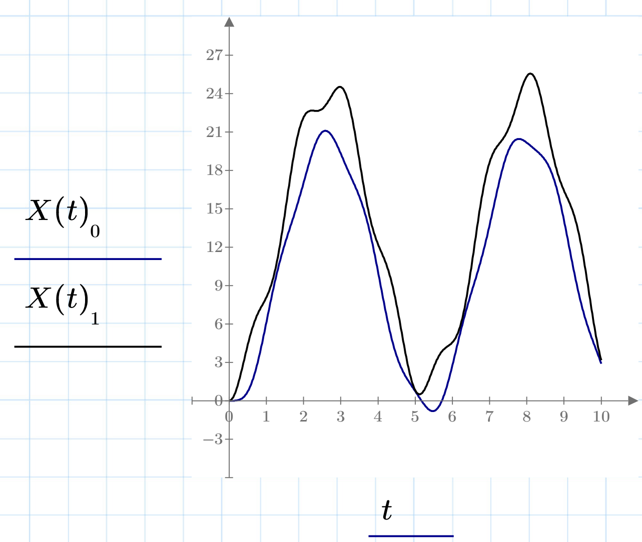
Hand solution to the equations:

$$x_{1ex}(t, w_1, w_2) := 10 + 0.588 \cos\left(\Omega_0 \cdot t\right) - 10.589 \cos\left(\Omega_1 \cdot t\right)$$

$$x_{2ex}(t, w_1, w_2) := 12.8626 - 1.3524 \cdot \cos\left(\Omega_0 \cdot t\right) - 11.5102 \cdot \cos\left(\Omega_1 \cdot t\right)$$

$x_1$ corresponds to mass 1 and $x_2$ corresponds to mass 2



University of Iowa Example: Multiple Vectors and Initial Conditions:

$$M := 4 \cdot \text{identity}(4) = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \qquad K := \begin{bmatrix} 10 & -5 & 0 & 0 \\ -5 & 10 & -5 & 0 \\ 0 & -5 & 10 & -5 \\ 0 & 0 & -5 & 5 \end{bmatrix} \qquad F := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

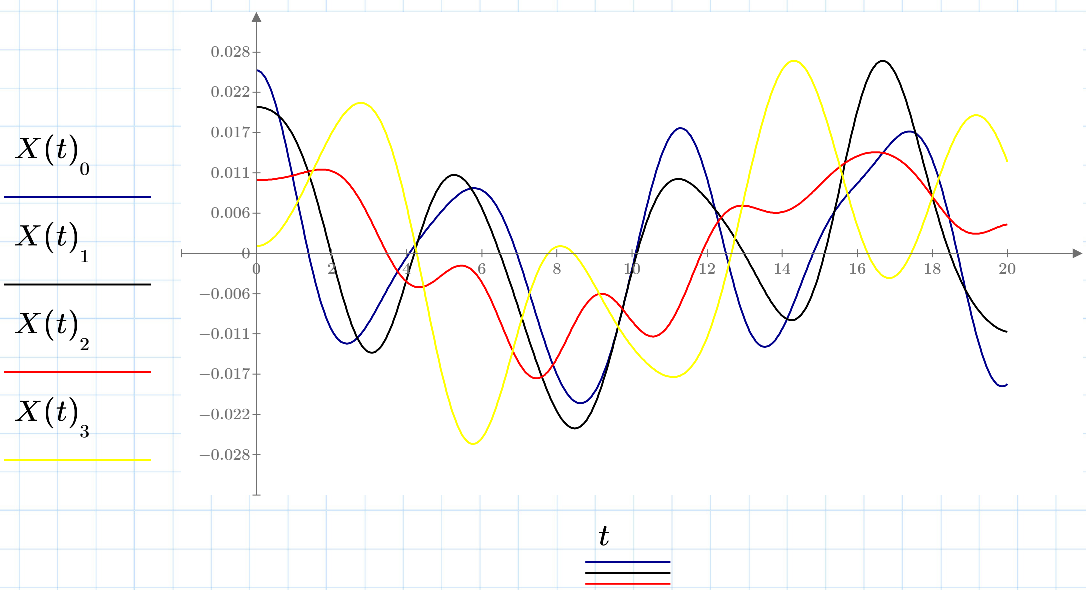
$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} := \text{genvals}(K, M) = \begin{bmatrix} 4.415 \\ 2.934 \\ 1.25 \\ 0.151 \end{bmatrix} \qquad \Omega := \sqrt{\text{genvals}(K, M)} = \begin{bmatrix} 2.101 \\ 1.713 \\ 1.118 \\ 0.388 \end{bmatrix}$$

$$P := \text{genvecs}(K, M) = \begin{bmatrix} 0.653 & -1 & 1 & -0.347 \\ -1 & 0.347 & 1 & -0.653 \\ 0.879 & 0.879 & 5.162 \cdot 10^{-17} & -0.879 \\ -0.347 & -0.653 & -1 & -1 \end{bmatrix}$$

$$m := P^{\mathrm{T}} \cdot M \cdot P \qquad k := P^{\mathrm{T}} \cdot K \cdot P \qquad f := P^{\mathrm{T}} \cdot F$$

$$x_0 := \begin{bmatrix} 0.025 \\ 0.02 \\ 0.01 \\ 0.001 \end{bmatrix} \qquad z_0 := P^{-1} \cdot x_0 = \begin{bmatrix} 0.00205 \\ -0.00427 \\ 0.01467 \\ -0.01359 \end{bmatrix}$$

$$C := k^{-1} \cdot f = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad U(t) := \overrightarrow{z_0 \cdot \cos(\Omega \cdot t)} \qquad X(t) := P \cdot U(t)$$




The above matches the Document. http://user.engineering.uiowa.edu/~sxiao/class/058-153/lecture-6.pdf

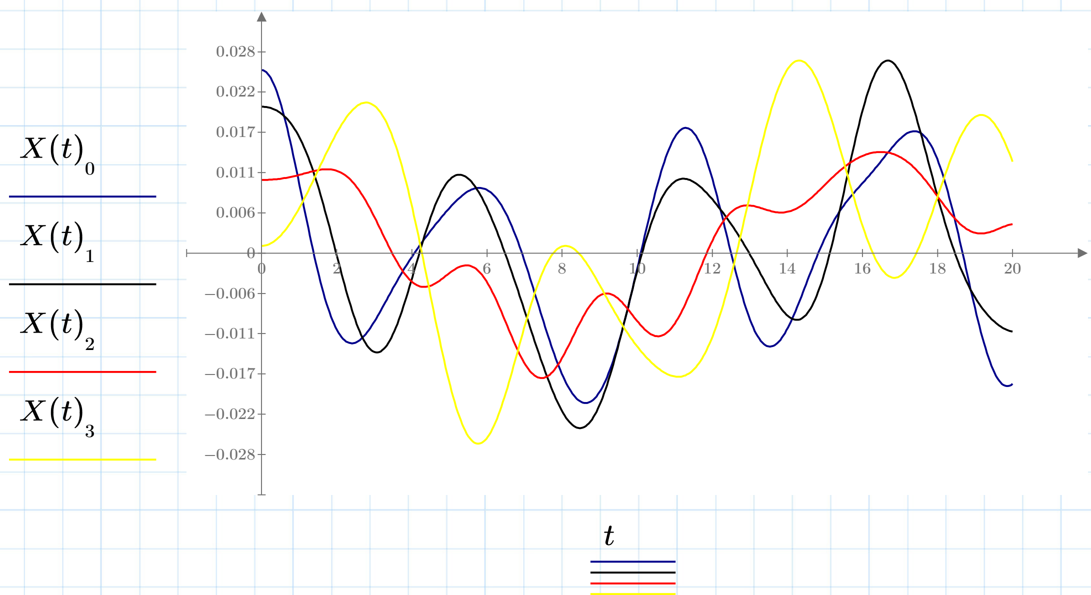
Using our definition of $\phi$

$$\phi := P \cdot \text{cholesky}\left(P^{T} \cdot M \cdot P\right)^{-1} = \begin{bmatrix} 0.214 & -0.328 & 0.289 & -0.114 \\ -0.328 & 0.114 & 0.289 & -0.214 \\ 0.289 & 0.289 & 1.49 \cdot 10^{-17} & -0.289 \\ -0.114 & -0.214 & -0.289 & -0.328 \end{bmatrix}$$

$$m := \phi^{T} \cdot M \cdot \phi \qquad k := \phi^{T} \cdot K \cdot \phi \qquad f := \phi^{T} \cdot F$$

$$x_0 := \begin{bmatrix} 0.025 \\ 0.02 \\ 0.01 \\ 0.001 \end{bmatrix} \qquad z_0 := \phi^{-1} \cdot x_0 = \begin{bmatrix} 0.00626 \\ -0.01302 \\ 0.05081 \\ -0.0414 \end{bmatrix}$$

$$C := k^{-1} \cdot f = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad U(t) := \overrightarrow{z_0 \cdot \cos(\Omega \cdot t)} \qquad X(t) := \phi \cdot U(t)$$




The above matches the Document information earlier. The generalized solution for a single DOF system is:

$$U(t) := e^{-\zeta \cdot \Omega \cdot t} \cdot \left( u_0 \cdot cos\left(\Omega_d \cdot t\right) + \frac{\Omega_{0\_rate} + \zeta \cdot \Omega \cdot u_o}{\Omega_d} \cdot sin\left(\Omega_d \cdot t\right) \right)$$

$$\Omega_d := \Omega \cdot \sqrt{1 - \zeta^2}$$

May use Rayliegh Damping to get a generalized Vector for $\zeta$ via C using $\alpha$ & $\beta$