

$$\lim_{n \rightarrow \infty} \sum_{r=1}^n \left( 2 + 2 \frac{r}{n} \right) \cdot \frac{2}{n} \quad \text{given}$$

$$\lim_{n \rightarrow \infty} \sum_{r=1}^n \left( \frac{4}{n} + \frac{4r}{n^2} \right) \quad \text{multiplied by } \frac{2}{n}$$

$$\lim_{n \rightarrow \infty} \sum_{r=1}^n \left( \frac{4}{n} \right) + \lim_{n \rightarrow \infty} \sum_{r=1}^n \left( \frac{4r}{n^2} \right) \quad \text{separating the terms}$$

$$\lim_{n \rightarrow \infty} \frac{4}{n} \cdot \sum_{r=1}^n (1) + \lim_{n \rightarrow \infty} \frac{4}{n^2} \cdot \sum_{r=1}^n (r) \quad \frac{4}{n} \text{ and } \frac{4}{n^2} \text{ separated}$$

$$\sum_{r=1}^n (1) \rightarrow n$$

$$\sum_{r=1}^n (r) \rightarrow \frac{n \cdot (n+1)}{2}$$

$r$  is the "range" variable from 1 to  $n$

$$\lim_{n \rightarrow \infty} \frac{4}{n} \cdot \sum_{r=1}^n (1) \rightarrow 4$$

$$\lim_{n \rightarrow \infty} \frac{4}{n^2} \cdot \sum_{r=1}^n (r) \rightarrow 2$$

$$\lim_{n \rightarrow \infty} \frac{4}{n} \cdot \sum_{r=1}^n (1) + \lim_{n \rightarrow \infty} \frac{4}{n^2} \cdot \sum_{r=1}^n (r) \rightarrow 6$$