Multiwavelength Code Ver 6c.mcd
May 11, 2009

Guy Beadie
Optical Sciences Division
US Naval Research Lab

In response to questions about propagation in lossy media, I've revisited the derivations of the matrix transfer equations with complex permitivities. In particular, I have changed the way I define the unit vector $\gamma(j)$ versus Offersgaard. *[Changed back, however, in ver6b]*

Based on these issues, I am also coming up with different forms of the transmission coefficient. This plus some cosmetic changes I'd like to make has prompted me to start a new version - version 6 alpha.

NOTE: As of Apr. 21 this should still be considered an alpha version - it needs to be tested against Born & Wolf's lossy medium curves, at the least.

<u>v6b notes Apr 30:</u>
I changed the definition of γ back to the original - it wasn't the problem in the transmission function; my definition of the transmission function didn't properly derive from the Poynting vector for the general case.

I've added several functions to evaluate fields on the low-x side of a layer.

I've added Poynting vector functions & field mapping.

I've added a large exposition section covering the theory of propagation, loss, and power conservation in lossy media. This is still a work in progress, but much of the section is there, and is used to check the numerical computations.

<u>v6c notes May 11:</u>
Because of the change in T(), I needed to change the peak finding functions for finding mode constants. I streamlined the searching, including the addition of an automatic Re & Im search via a brute-force search first along the real axis followed by a purely imaginary search - iterated 3 times.

<u>v6c notes Jul 14:</u>
Working on some global changes. In particular:
- Changing all distances (other than the wavelength) to unitless microns. So, the d vector, in particular, will be unitless
- Chaning all functions' input variable name for polarization to Pol, with global definitions defined for TE and Pol

Changes:
Units & constants section updated, though the Ebars function has been removed as unnecessary.

Other changes will be listed within collapsed areas.

$OL = 0.0010$

Unit Definitions:

$Hz := sec^{-1}$ $kHz := 10^3 \cdot Hz$ $MHz := 10^6 \cdot Hz$ $GHz := 10^9 \cdot Hz$ $THz := 10^3 \cdot GHz$

$ms := 10^{-3} \cdot sec$ $\mu s := 10^{-3} \cdot ms$ $ns := 10^{-3} \cdot \mu s$ $ps := 10^{-3} \cdot ns$ $fs := 10^{-3} \cdot ps$

$cm := 10^{-2} \cdot \mathbf{m}$ $\mu m := 10^{-6} \cdot \mathbf{m}$ $nm := 10^{-3} \cdot \mu m$ $pm := 10^{-3} \cdot nm$ $fm := 10^{-3} \cdot pm$

$watt := 1 \cdot \mathbf{W}$ $mW := 10^{-3} \cdot \mathbf{W}$ $\mu W := 10^{-6} \cdot \mathbf{W}$ $nW := 10^{-9} \cdot \mathbf{W}$ $pW := 10^{-3} \cdot nW$

$kW := 10^3 \cdot \mathbf{W}$ $MW := 10^6 \cdot \mathbf{W}$ $GW := 10^9 \cdot \mathbf{W}$ $J := 1 \cdot \mathbf{joule}$ $mJ := 10^{-3} \cdot J$

$\mu J := 10^{-3} \cdot mJ$ $nJ := 10^{-3} \cdot \mu J$ $pJ := 10^{-3} \cdot nJ$ $kJ := 10^3 \cdot J$ $MJ := 10^3 \cdot kJ$

$g := 10^{-3} \cdot \mathbf{kg}$ $ml := 10^{-3} \cdot \mathbf{liter}$ $eV := 1.60217733 \cdot 10^{-19} \cdot J$ $meV := 10^{-3} \cdot eV$

$mrad := 10^{-3} \cdot \mathbf{rad}$ $\mu rad := 10^{-6} \cdot \mathbf{rad}$ $fJ := 10^{-3} \cdot pJ$ $keV := 10^3 \cdot eV$ $MeV := 10^6 \cdot eV$ $GeV := 10^9 \cdot eV$

Fundamental constants:

$h := 6.6260755 \cdot 10^{-34} \cdot \mathbf{joule} \cdot sec$ $c := 299792458 \cdot \mathbf{m} \cdot Hz$ $k := 1.380658 \cdot 10^{-23} \cdot \mathbf{joule} \cdot \mathbf{K}^{-1}$

$\mu_o := 4 \cdot \pi \cdot 10^{-7} \cdot \mathbf{newton} \cdot \mathbf{amp}^{-2}$ $e_o := 1.60217733 \cdot 10^{-19} \cdot \mathbf{coul}$ $m_e := 9.1093897 \cdot 10^{-31} \cdot \mathbf{kg}$

$\varepsilon_o := (c \cdot c \cdot \mu_o)^{-1}$ $h := h \cdot (2 \cdot \pi)^{-1}$

Macros:

$\lambda to\omega(\lambda) := 2 \cdot \pi \cdot c \cdot \lambda^{-1}$ $\omega to\lambda(\omega) := 2 \cdot \pi \cdot c \cdot \omega^{-1}$ $Eto\lambda(E) := h \cdot c \cdot E^{-1}$ $\lambda toE(\lambda) := h \cdot c \cdot \lambda^{-1}$

$Get\_Range(st, ed, num) := st, st + (ed - st) \cdot (num - 1)^{-1} .. ed$

$$Get\_Vec(st, ed, num) := \left\| \begin{array}{l} \text{for } j \in 0 .. num - 1 \\ \quad \| \ vout_j \leftarrow st + (ed - st) \cdot j \cdot (num - 1)^{-1} \\ vout \end{array} \right.$$

CPUTicks:

$Speed := ReadCpuSpeed(0) \cdot MHz$

$$Get\_\Delta t := \left\| \begin{array}{l} t0 \leftarrow ResetCpuTicks(0) \\ \text{for } j \in 0 .. 100000 \\ \quad \| \ tmp \leftarrow j \end{array} \right.$$

$Get\_\Delta t = ?\ ms$

$tmp \leftarrow j$

$t1 \leftarrow ReadCpuTicks(0)$

$\dfrac{t1 - t0}{Speed}$

Definition of $Get\_Fit(\,)$:

$$FitAll := \begin{bmatrix} 12345678901234 \\ 23456789012345 \\ 34567890123456 \end{bmatrix}$$

Just a placeholder vector, to serve as a named constant. When Get_Fit() is called with this variable chosen for vmask, the routine will automatically vary all the fit parameters.

Note: $MyF(X, v)$ must take a ***vector*** *of input values X and generate a* ***vector*** *of outputs F.*

```
Residual_Vector(v, vin, vmask, X, Y, MyF) :=
    if vmask ≠ FitAll
        "Keep variables fixed when mask value=0:"
        for j ∈ 0 .. rows(vmask) − 1
            if vmask_j = 0
                v_j ← vin_j
    F ← MyF(X, v)
    "Linear weighting"
    Dist_V ← F − Y
    Dist_V
```

Начальные приближения
Ограничения
Решатель

$$Residual\_Vector(v, vin, vmask, X, Y, MyF) = 0$$

$$Get\_GenErr\_Fit(v, vin, vmask, X, Y, MyF) := \mathbf{Minerr}(\mathbf{v})$$

Simple fitting routine. Allows for selective fitting of model parameters via $vmask$: if $vmask_j = 0$ then fit variable v_j will always be fixed to the initial value $vguess_j$. If $vmask$ is set equal to $FitAll$ then all variables are varied in the fit.

$$Get\_Fit(vguess, vmask, X, Y, MyF) := Get\_GenErr\_Fit(vguess, vguess, vmask, X, Y, MyF)$$

eps1=n^2-k^2; eps2=2nk

0 1 2 3 4 5 6

$Col\_Hdr := [\text{"eV"} \quad \text{"wvl (nm)"} \quad \text{"eps1"} \quad \text{"eps2"} \quad \text{"eV"} \quad \text{"n"} \quad \text{"k"}]$

PMMA from
VASE

Disp_Dat :=

	0	1	2	3	4
0	$2.604\cdot10^{-4}$	$4.762\cdot10^{3}$	2.108	$3.327\cdot10^{-3}$	$2.604\cdot10^{-4}$
1	$2.6\cdot10^{-4}$	$4.769\cdot10^{3}$	2.108	$3.342\cdot10^{-3}$	$2.6\cdot10^{-4}$
2	$2.597\cdot10^{-4}$	$4.776\cdot10^{3}$	2.107	$3.357\cdot10^{-3}$	$2.597\cdot10^{-4}$
3	$2.593\cdot10^{-4}$	$4.782\cdot10^{3}$	2.107	$3.372\cdot10^{-3}$	$2.593\cdot10^{-4}$

$cs\_n := cspline\left(Disp\_Dat^{\langle 1\rangle}, Disp\_Dat^{\langle 5\rangle}\right)$

$cs\_k := cspline\left(Disp\_Dat^{\langle 1\rangle}, Disp\_Dat^{\langle 6\rangle}\right)$

$$
\begin{aligned}
n\_PMMA(\lambda) := \;& x \leftarrow \lambda \div nm \\
& \text{if } x < min\left(Disp\_Dat^{\langle 1\rangle}\right) \\
& \quad x \leftarrow min\left(Disp\_Dat^{\langle 1\rangle}\right) \\
& \text{if } x > max\left(Disp\_Dat^{\langle 1\rangle}\right) \\
& \quad x \leftarrow max\left(Disp\_Dat^{\langle 1\rangle}\right) \\
& interp\left(cs\_n, Disp\_Dat^{\langle 1\rangle}, Disp\_Dat^{\langle 5\rangle}, x\right) + 1i \cdot interp\left(cs\_k, Disp\_Dat^{\langle 1\rangle}, Disp\_Dat^{\langle 6\rangle}, x\right)
\end{aligned}
$$

0 1 2 3 4 5 6

$Col\_Hdr := [\text{"eV"} \quad \text{"wvl (nm)"} \quad \text{"eps1"} \quad \text{"eps2"} \quad \text{"eV"} \quad \text{"n"} \quad \text{"k"}]$

Au from Rakic

Disp_Dat := READTEXT ("C:\Users\materials constants\Au - Rakic 1998 - eV WL e1 e2 eV n k.txt", "delimited", "auto", "1-", "1-", NaN, "sk

$cs\_n := cspline\left(Disp\_Dat^{\langle 1\rangle}, Disp\_Dat^{\langle 5\rangle}\right)$

$cs\_k := cspline\left(Disp\_Dat^{\langle 1\rangle}, Disp\_Dat^{\langle 6\rangle}\right)$

$$n\_Au(\lambda) := \left\| \begin{array}{l} x \leftarrow \lambda \div nm \\ \text{if } x < min\left(Disp\_Dat^{\langle 1\rangle}\right) \\ \quad \| x \leftarrow min\left(Disp\_Dat^{\langle 1\rangle}\right) \\ \text{if } x > max\left(Disp\_Dat^{\langle 1\rangle}\right) \\ \quad \| x \leftarrow max\left(Disp\_Dat^{\langle 1\rangle}\right) \\ interp\left(cs\_n, Disp\_Dat^{\langle 1\rangle}, Disp\_Dat^{\langle 5\rangle}, x\right) + 1i \cdot interp\left(cs\_k, Disp\_Dat^{\langle 1\rangle}, Disp\_Dat^{\langle 6\rangle}, x\right) \end{array} \right.$$

Pt from Rakic

Disp_Dat :=

	0	1	2	3	4
0	6.002	206.6	0.929	5.284	6.002
1	5.88	210.895	0.82	5.177	5.88
2	5.76	215.279	0.69	5.088	5.76
3	5.643	219.755	0.54	5.019	5.643

$cs\_n := cspline\left(Disp\_Dat^{\langle 1\rangle}, Disp\_Dat^{\langle 5\rangle}\right)$

$cs\_k := cspline\left(Disp\_Dat^{\langle 1\rangle}, Disp\_Dat^{\langle 6\rangle}\right)$

$$n\_Pt(\lambda) := \left\| \begin{array}{l} x \leftarrow \lambda \div nm \\ \text{if } x < min\left(Disp\_Dat^{\langle 1\rangle}\right) \\ \quad \| x \leftarrow min\left(Disp\_Dat^{\langle 1\rangle}\right) \\ \text{if } x > max\left(Disp\_Dat^{\langle 1\rangle}\right) \\ \quad \| x \leftarrow max\left(Disp\_Dat^{\langle 1\rangle}\right) \\ interp\left(cs\_n, Disp\_Dat^{\langle 1\rangle}, Disp\_Dat^{\langle 5\rangle}, x\right) + 1i \cdot interp\left(cs\_k, Disp\_Dat^{\langle 1\rangle}, Disp\_Dat^{\langle 6\rangle}, x\right) \end{array} \right.$$

Cu from Rakic

$Disp\_Dat := READTEXT$(“C:\Users\materials constants\Cu - Rakic 1998 - eV WL e1 e2 eV n k.txt”, “delimited”, “auto”, “1-”, “1-”, N

$cs\_n := cspline\left(Disp\_Dat^{\langle 1\rangle}, Disp\_Dat^{\langle 5\rangle}\right)$

$cs\_k := cspline\left(Disp\_Dat^{\langle 1\rangle}, Disp\_Dat^{\langle 6\rangle}\right)$

$$n\_Cu(\lambda) := \begin{array}{|l} x \leftarrow \lambda \div nm \\ \text{if } x < min\left(Disp\_Dat^{\langle 1 \rangle}\right) \\ \quad x \leftarrow min\left(Disp\_Dat^{\langle 1 \rangle}\right) \\ \text{if } x > max\left(Disp\_Dat^{\langle 1 \rangle}\right) \\ \quad x \leftarrow max\left(Disp\_Dat^{\langle 1 \rangle}\right) \\ interp\left(cs\_n, Disp\_Dat^{\langle 1 \rangle}, Disp\_Dat^{\langle 5 \rangle}, x\right) + 1i \cdot interp\left(cs\_k, Disp\_Dat^{\langle 1 \rangle}, Disp\_Dat^{\langle 6 \rangle}, x\right) \end{array}$$

Si from drude in VASE

$Disp\_Dat :=$ READTEXT(“C:\Users\materials constants\Si eV nm e1 e2 eV n k.txt”, “delimited”, “auto”, “1-”, “1-”, NaN, “skip”) = ?

$$cs\_n := cspline\left(Disp\_Dat^{\langle 1 \rangle}, Disp\_Dat^{\langle 5 \rangle}\right)$$

$$cs\_k := cspline\left(Disp\_Dat^{\langle 1 \rangle}, Disp\_Dat^{\langle 6 \rangle}\right)$$

$$n\_Si(\lambda) := \begin{array}{|l} x \leftarrow \lambda \div nm \\ \text{if } x < min\left(Disp\_Dat^{\langle 1 \rangle}\right) \\ \quad x \leftarrow min\left(Disp\_Dat^{\langle 1 \rangle}\right) \\ \text{if } x > max\left(Disp\_Dat^{\langle 1 \rangle}\right) \\ \quad x \leftarrow max\left(Disp\_Dat^{\langle 1 \rangle}\right) \\ interp\left(cs\_n, Disp\_Dat^{\langle 1 \rangle}, Disp\_Dat^{\langle 5 \rangle}, x\right) + 1i \cdot interp\left(cs\_k, Disp\_Dat^{\langle 1 \rangle}, Disp\_Dat^{\langle 6 \rangle}, x\right) \end{array}$$

Ti from Johnson and Christy

Disp_Dat :=

	0	1	2	3	4
0	6.596	188	-1.414	3.564	6.596
1	6.458	192	-1.344	3.805	6.458
2	6.359	195	-1.267	4.05	6.359
3	6.231	199	-1.26	4.2	6.231

$$cs\_n := cspline\left(Disp\_Dat^{\langle 1 \rangle}, Disp\_Dat^{\langle 5 \rangle}\right)$$

$$cs\_k := cspline\left(Disp\_Dat^{\langle 1 \rangle}, Disp\_Dat^{\langle 6 \rangle}\right)$$

$$n\_Ti(\lambda) := \begin{array}{|l} x \leftarrow \lambda \div nm \\ \text{if } x < min\left(Disp\_Dat^{\langle 1 \rangle}\right) \\ \quad x \leftarrow min\left(Disp\_Dat^{\langle 1 \rangle}\right) \\ \text{if } x > max\left(Disp\_Dat^{\langle 1 \rangle}\right) \\ \quad x \leftarrow max\left(Disp\_Dat^{\langle 1 \rangle}\right) \end{array}$$

$$\| x \leftarrow max\left(Disp\_Dat^{\langle 1 \rangle}\right)$$

$$interp\left(cs\_n, Disp\_Dat^{\langle 1 \rangle}, Disp\_Dat^{\langle 5 \rangle}, x\right) + 1i \cdot interp\left(cs\_k, Disp\_Dat^{\langle 1 \rangle}, Disp\_Dat^{\langle 6 \rangle}, x\right)$$

Cr from Rakic

isp_Dat :=

	0	1	2	3	4	5
0	6.002	206.6	-3.29	2.105	6.002	0.55
1	5.88	210.895	-3.444	2.221	5.88	0.57
2	5.76	215.279	-3.601	2.343	5.76	0.5
3	5.643	219.755	-3.764	2.472	5.643	0.60

$$cs\_n := cspline\left(Disp\_Dat^{\langle 1 \rangle}, Disp\_Dat^{\langle 5 \rangle}\right)$$

$$cs\_k := cspline\left(Disp\_Dat^{\langle 1 \rangle}, Disp\_Dat^{\langle 6 \rangle}\right)$$

$$n\_Cr(\lambda) := \begin{array}{l} x \leftarrow \lambda \div nm \\ \text{if } x < min\left(Disp\_Dat^{\langle 1 \rangle}\right) \\ \quad x \leftarrow min\left(Disp\_Dat^{\langle 1 \rangle}\right) \\ \text{if } x > max\left(Disp\_Dat^{\langle 1 \rangle}\right) \\ \quad x \leftarrow max\left(Disp\_Dat^{\langle 1 \rangle}\right) \\ interp\left(cs\_n, Disp\_Dat^{\langle 1 \rangle}, Disp\_Dat^{\langle 5 \rangle}, x\right) + 1i \cdot interp\left(cs\_k, Disp\_Dat^{\langle 1 \rangle}, Disp\_Dat^{\langle 6 \rangle}, x\right) \end{array}$$

Layers(▮) outputs a four-column, unitless matrix whose rows represent individual layers. The first three columns contain the (nx,ny,nz) refractive index values for each layer; these values are all equal if the material is isotropic. Birefringent material can be added only if the principle axes of the index ellipsoid coincide with the xyz axes of the problem. The fourth column contains the thickness of each layer, in microns, without carrying unit information. All of the information concerning the multilayer is contained in such a matrix.

$$Layers(\lambda, Params) := \begin{array}{l} dAu \leftarrow Params_1 \\ dPt \leftarrow Params_2 \\ dTi \leftarrow Params_0 \\ nglass \leftarrow 2 \end{array}$$

$$
\begin{aligned}
& namb \leftarrow 1. \\
& nTi \leftarrow n\_Ti(\lambda) \\
& nAu \leftarrow n\_Au(\lambda) \\
& nPt \leftarrow n\_Pt(\lambda) \\
& lay1 \leftarrow [nglass \;\; nglass \;\; nglass \;\; 1] \\
& lay2 \leftarrow [nTi \;\; nTi \;\; nTi \;\; dTi] \\
& lay3 \leftarrow [nAu \;\; nAu \;\; nAu \;\; dAu] \\
& lay4 \leftarrow [nPt \;\; nPt \;\; nPt \;\; dPt] \\
& lay5 \leftarrow [namb \;\; namb \;\; namb \;\; 1] \\
& lays \leftarrow stack(lay1, lay2, lay3, lay4, lay5) \\
& lays
\end{aligned}
$$

$dTi := 2 \cdot nm$

$dAu := 45 \cdot nm$

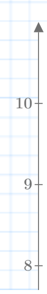
$dPt := 0 \cdot nm$

$Ps0 := [dTi \;\; dAu \;\; dPt \;\; 0 \;\; 0]^{T} \div \mu m$

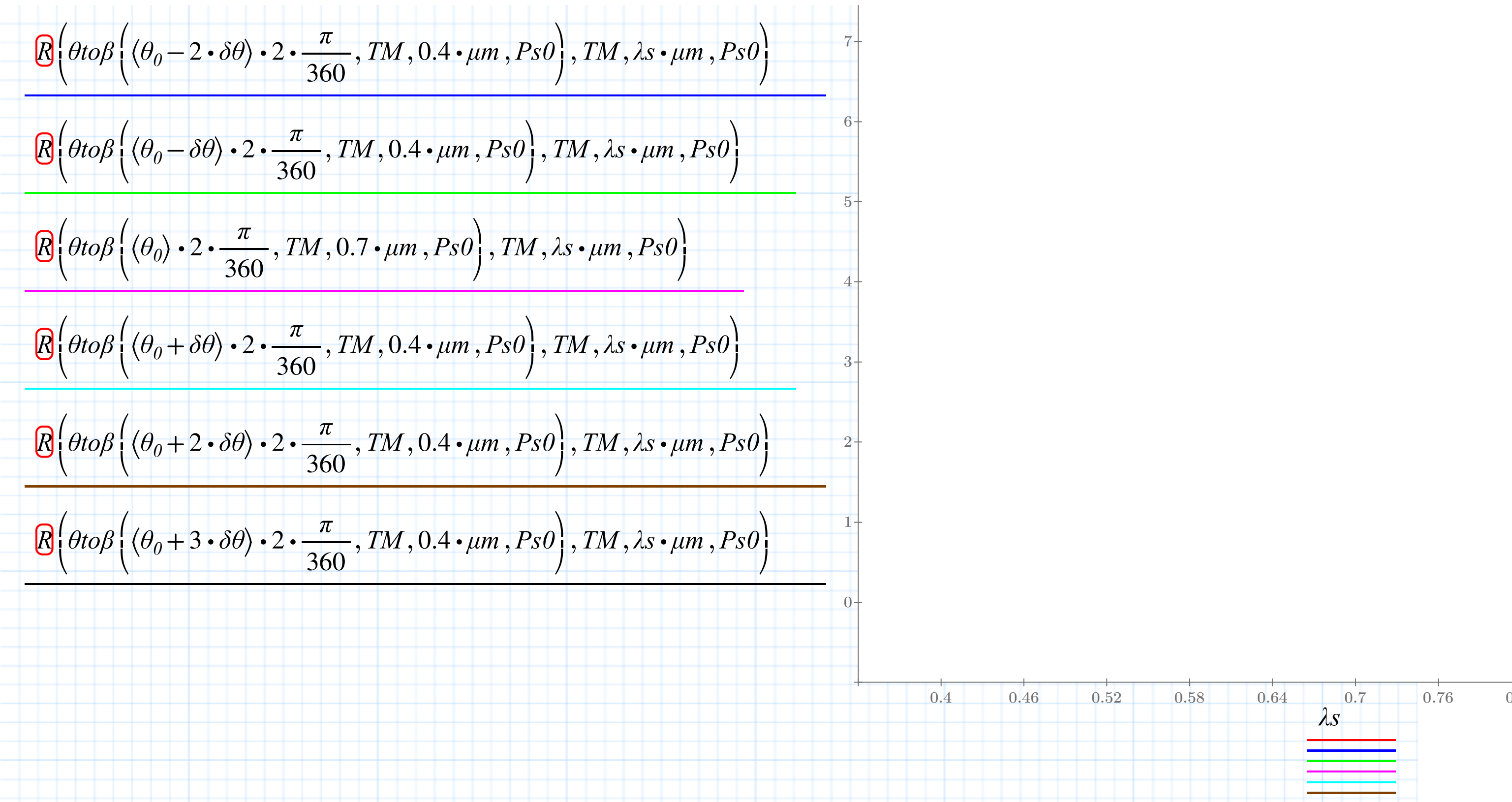
$\lambda s := Get\_Range(0.4, 1, 1001)$

$\theta_0 := 32$

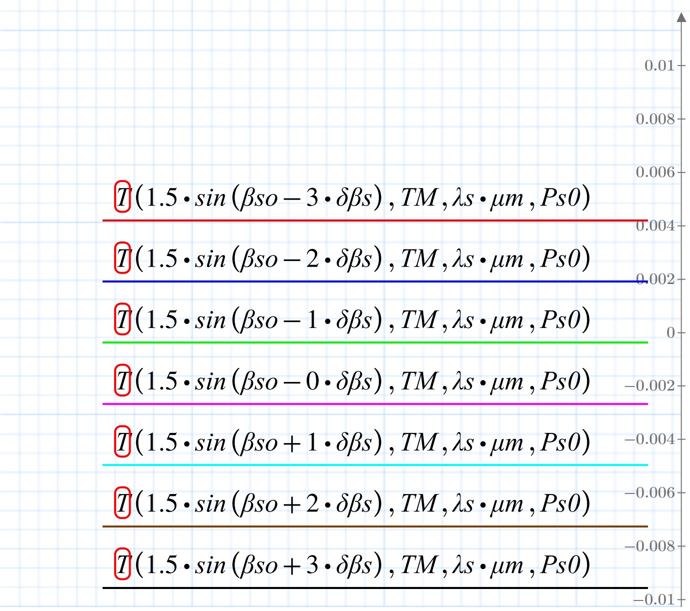
$\delta\theta := 0.5$

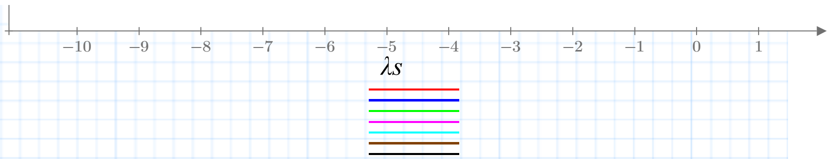


$R\left(\theta to\beta\left(\left(\theta_0 - 3 \cdot \delta\theta\right) \cdot 2 \cdot \frac{\pi}{360}, TM, 0.4 \cdot \mu m, Ps0\right), TM, \lambda s \cdot \mu m, Ps0\right)$



$\beta so := 44 \cdot \mathbf{deg}$ $\delta\beta s := 0.5 \cdot \mathbf{deg}$





$\lambda o := .6 \cdot \mu m$ $\delta\lambda := 20 \cdot nm$

$\beta s := Get\_Range\left(1.5 \cdot \sin\left(30 \cdot \boldsymbol{deg}\right), 1.5 \cdot \sin\left(60 \cdot \boldsymbol{deg}\right), 300\right)$