

We assume the following assumptions/relationships to be mandatory throughout the worksheet!!

$$k = \cos(Q) \quad m = \frac{1}{k} \quad s = (m - 1) \cdot \lambda + 1 \quad \text{so we have} \quad \lambda = \frac{s - 1}{m - 1}$$

$$uk1(s, Q) := \left[k \leftarrow \cos(Q) \right. \\ \left. \operatorname{Re} \left[\left(\operatorname{acoth} \left(\sqrt{\frac{1 - s^2 \cdot k^2}{1 - k^2}} \right) - s \cdot \operatorname{acoth} \left(\frac{1}{s} \cdot \sqrt{\frac{1 - s^2 \cdot k^2}{1 - k^2}} \right) \right) \cdot \frac{1}{\ln \left(\frac{1}{k} \right)} \right] \right]$$

$$\frac{\partial}{\partial s} uk1(s, Q) \left[\begin{array}{l} \text{simplify} \\ \text{rewrite, cos} \end{array} \right] \rightarrow -\operatorname{Re} \left[\frac{1}{\ln \left(\frac{1}{\cos(Q)} \right)} \cdot \operatorname{acoth} \left[\frac{1}{s} \cdot \sqrt{\frac{1}{\cos(Q)^2 - 1} \cdot [s^2 \cdot (\cos(Q)^2 - 1) + s^2 - 1]} \right] \right]$$

Used "rewrite,cos", otherwise simplify would prefer sin(Q)

Without "simplify" the result is a little bit bulky :-)

$$\frac{\partial}{\partial s} uk1(s, Q) \rightarrow -\operatorname{Re} \left[\frac{1}{\ln \left(\frac{1}{\cos(Q)} \right)} \cdot \operatorname{acoth} \left(\frac{1}{s} \cdot \sqrt{\frac{s^2 \cdot \cos(Q)^2 - 1}{\cos(Q)^2 - 1}} \right) + \frac{s}{\frac{1}{s^2} \cdot (s^2 \cdot \cos(Q)^2 - 1) - 1} \cdot \left(\frac{1}{s^2} \cdot \sqrt{\frac{s^2 \cdot \cos(Q)^2 - 1}{\cos(Q)^2 - 1}} - \frac{\frac{\cos(Q)^2}{\sqrt{s^2 \cdot \cos(Q)^2}}}{\cos(Q)^2} \right) \right]$$

But it seems you are not interested in the derivative of uk1 but rather in that of the function uk, which is derived from uk1 by some substitutions (but is not the same as uk1, so the different name)

Because Q is no external variable anymore you have to replace Q by acos(k) or better cos(Q) by k according to the global assumption at the beginning of this sheet:

$$uk(\lambda, m) := uk1(s, Q) \left\{ \begin{array}{l} \text{substitute, } s = [(m-1) \cdot \lambda + 1] \\ \text{substitute, } \cos(Q) = \frac{1}{m} \end{array} \right. \rightarrow \text{Re} \left[\frac{\text{acoth} \left[\frac{\left(\frac{1}{m}\right)^2 \cdot [\lambda \cdot (m-1) + 1]^2 - 1}{\left(\frac{1}{m}\right)^2 - 1} \right] - \text{acoth} \left[\frac{\left(\frac{1}{m}\right)^2 \cdot [\lambda \cdot (m-1)]}{\lambda \cdot (m-1) + \left(\frac{1}{m}\right)^2} \right]}{\ln \left(\frac{1}{m} \right)} \right]$$

$$uk(\lambda, m) := uk1(s, Q) \left\{ \begin{array}{l} \text{substitute, } s = [(m-1) \cdot \lambda + 1] \\ \text{substitute, } \cos(Q) = \frac{1}{m} \\ \text{simplify} \end{array} \right. \rightarrow$$

Without the assumption ALL > 0 it did not evaluate in an acceptable reasonable time, therefore the expression is disabled

$$uk(\lambda, m) := uk1(s, Q) \left\{ \begin{array}{l} \text{substitute, } s = [(m-1) \cdot \lambda + 1] \\ \text{substitute, } \cos(Q) = \frac{1}{m} \\ \text{assume, ALL} > 0 \\ \text{simplify} \end{array} \right. \rightarrow \frac{\text{Re} \left[\text{acoth} \left[\frac{\sqrt{-(\lambda-1) \cdot (m-\lambda + \lambda \cdot m + 1)}}{\sqrt{m+1} \cdot (\lambda \cdot m - \lambda + 1)} \right] \right] - \text{Re} \left[\text{acoth} \left[\frac{1}{\sqrt{m+1}} \cdot \sqrt{-(\lambda-1)} \right] \right]}{2}$$

$$\frac{\text{Re} \left[\text{acoth} \left[\frac{\sqrt{-(\lambda-1) \cdot (m-\lambda + \lambda \cdot m + 1)}}{\sqrt{m+1} \cdot (\lambda \cdot m - \lambda + 1)} \right] \right] - \frac{\text{Re} \left[\frac{m - \lambda + (\lambda-1) \cdot (m-1) + \lambda \cdot n}{\sqrt{m+1} \cdot \left[\frac{(\lambda-1) \cdot (m-\lambda + \lambda \cdot m + 1)}{m+1} + 1 \right] \cdot \sqrt{-(\lambda-1)}} \right]}{2}}$$

$$duk(\lambda, m) := \frac{\partial}{\partial \lambda} uk(\lambda, m) \rightarrow$$

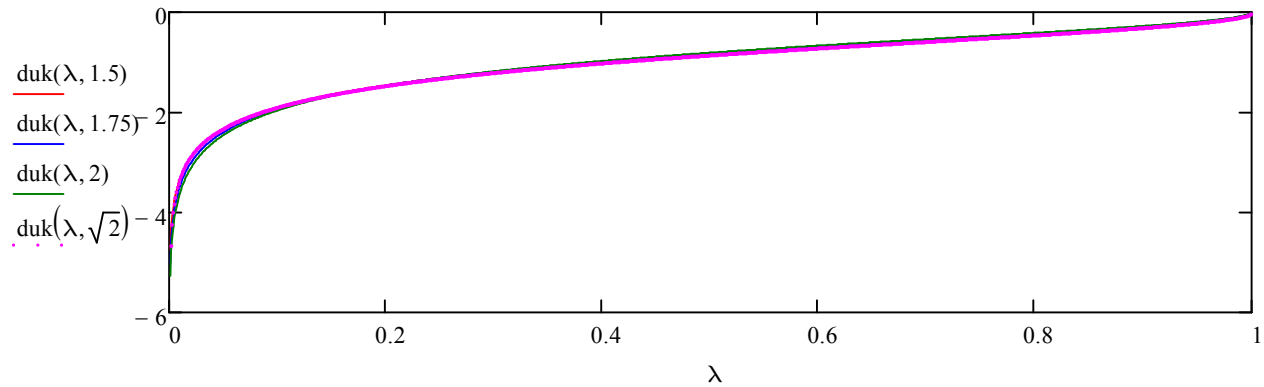
Again had not the time waiting for simplify to finish without the assumption ALL > 0

$$duk(\lambda, m) := \frac{\partial}{\partial \lambda} uk(\lambda, m) \left\{ \begin{array}{l} \text{assume, ALL} > 0 \\ \text{simplify} \end{array} \right. \rightarrow \frac{\text{Re} \left[\text{acoth} \left[\frac{\sqrt{-(\lambda-1) \cdot (m-\lambda + \lambda \cdot m + 1)}}{\sqrt{m+1} \cdot (\lambda \cdot m - \lambda + 1)} \right] \right] - m \cdot \text{Re} \left[\text{acoth} \left[\frac{\sqrt{-(\lambda-1) \cdot (m-\lambda + \lambda \cdot m + 1)}}{\sqrt{m+1} \cdot (\lambda \cdot m - \lambda + 1)} \right] \right]}{2}$$

If the following output is not what you are looking for, you have to reconsider your assumptions at the beginning of the page.

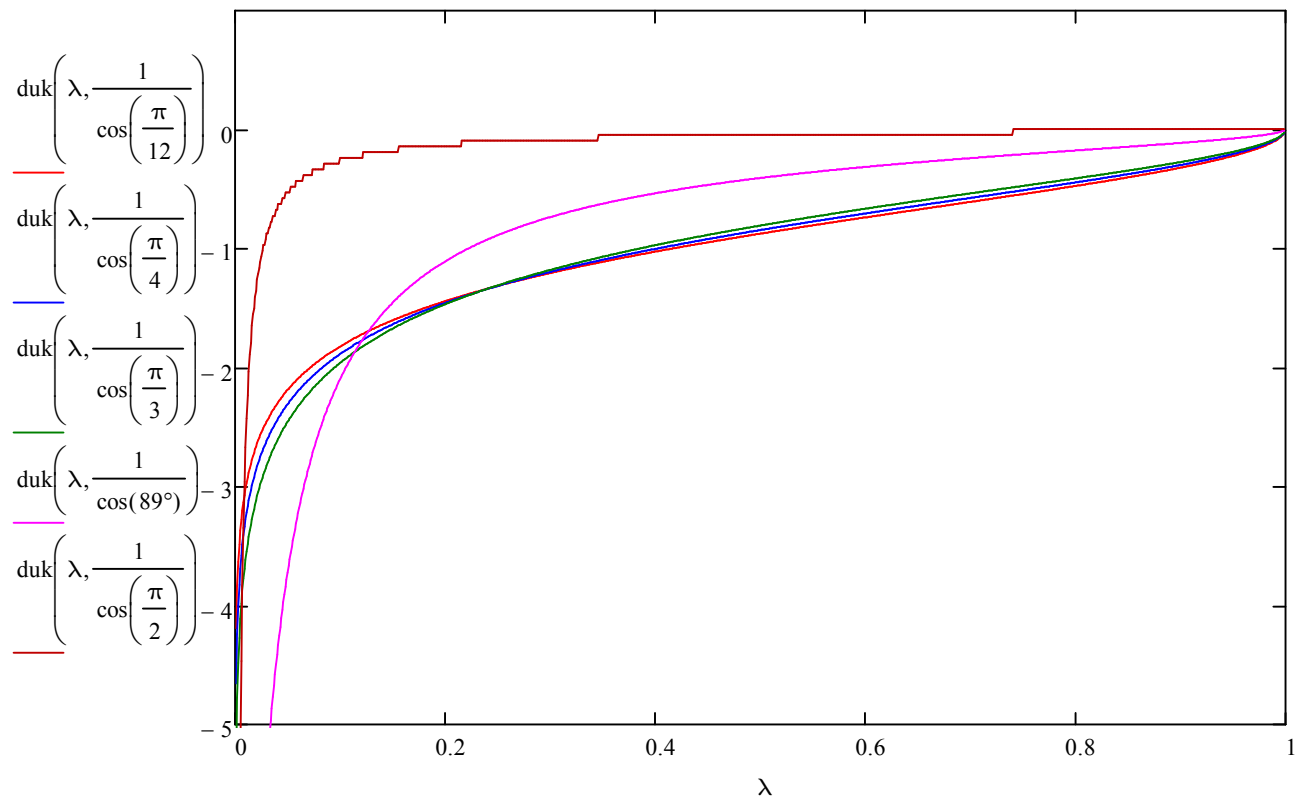
If uk and/or duk should be dependent on a value Q or k which is NOT in the given relationship to m , you should make that a third parameter of those functions.

$\lambda := 0, 0.001.. 1$



The graphs are very close, but still different. The one I added with $m = \sqrt{2}$ accords to $Q = \pi/4$.

As Q is assumed to run from 0 to $\pi/2$, the values of $m (= 1/\cos(Q))$ should be in the range from 1 to infinity



In our case it seems not be necessary but to get rid of the zero values which you dont want to see you could define a function uk_plot which is only intended for plotting and plot this instead of uk

```

uk_plot(λ, m) := | val ← uk(λ, m)
                  | return val if val > 0
                  | return NaN otherwise
    
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