

SPUR GEAR ANALYSIS USING MATHCAD

by

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ABSTRACT

SPUT GEAR ANALYSIS USING MATHCAD

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In gear manufacturing applications, it is required to calculate surface stress and bending stress with minimum error. This thesis represents engineering calculation of bending and surface compression of external teathed involute spur gear. One may change the input values to calculate the engineering data for gears. The freedom of changing input values gives the capability to program to calculate for factor, Y ; and the bending geometry factor, J to analyze spur gear geometry.

Here the engineering gear calculation has been done by developing program in Mathcad software. The program is designed to calculate surface and bending stress data by input such as pitch, face width, pressure angle, horse-power, and load. Also the program can calculate stresses for various pitches, face widths, and pressure angles.

Thus Mathcad program is useful for design and manufacturing application for various types of external involute spur gears. It can reduce the errors in engineering calculations. This program gives the advantages to engineers to design gears and get the values for form factors and geometry factors which are not available.

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CHAPTER 1

INTRODUCTION

1.1 Object

The primary objective of this thesis is the design of program for an analysis of external involute spur gear which can provide flexibility to choose input values and ease to use.

The analysis should be associated into the primary object for the engineering calculation of Lewis form factor, Y ; and the geometry factor, J for any involute tooth system. Thus the engineering calculation can be done for which tables are not available.

1.2 Scope

The engineering calculation encloses external spur gear which uses an involute tooth system. The program can calculate for standard and nonstandard tooth system. Thus unbalanced addendum gears can be analyzed. The program results can be helpful for practical use for manufacturing.

This thesis explains the calculation to determine the Lewis form factor, Y ; and geometry factor, J . These values can be used to calculate bending and surface stress for every external involute spur gears.

1.2.1 Lewis geometry factor, Y

In present manufacturing process, designers adopt to use diametral pitch instead of circular pitch for stress calculations. This can be done by calculating Lewis form factor, Y .

1.2.2 Geometry form factor, J

In this thesis mathematical calculation for geometry form factor, J has been done for external involute spur gear. The calculation accommodates root fillets produced by rack and pinion types of tools.

CHAPTER 2

THEORY

2.1 Introduction

The theory will contain brief study of fundamentals in gearing. The second step will be the classical derivation for bending and surface stress equation which are useful in present day. The American Gear Manufacturing Association (AGMA) equations are also included. The terms for bending and surface stress of AGMA equations are defined. In the final, theory engages equations for practical gear design.

2.2 Fundamentals

2.2.1. General

Gears have incalculable application from nut and bolts for transmission of rotary motion to power generating all big machines. Gear is geometric shape that has teeth uniformly spaced around the circumference. (Dudley, 1984) In this thesis, stress analysis is done for external spur gear for uniform rotary motion.

Spur gears are frequently used to transfer power in-between two parallel shafts. They have involute shape of teeth and impose only radial loads on their bearings. (Dudley, 1984) Figure 2.1 and definition will assist to study spur gear classification and elements.

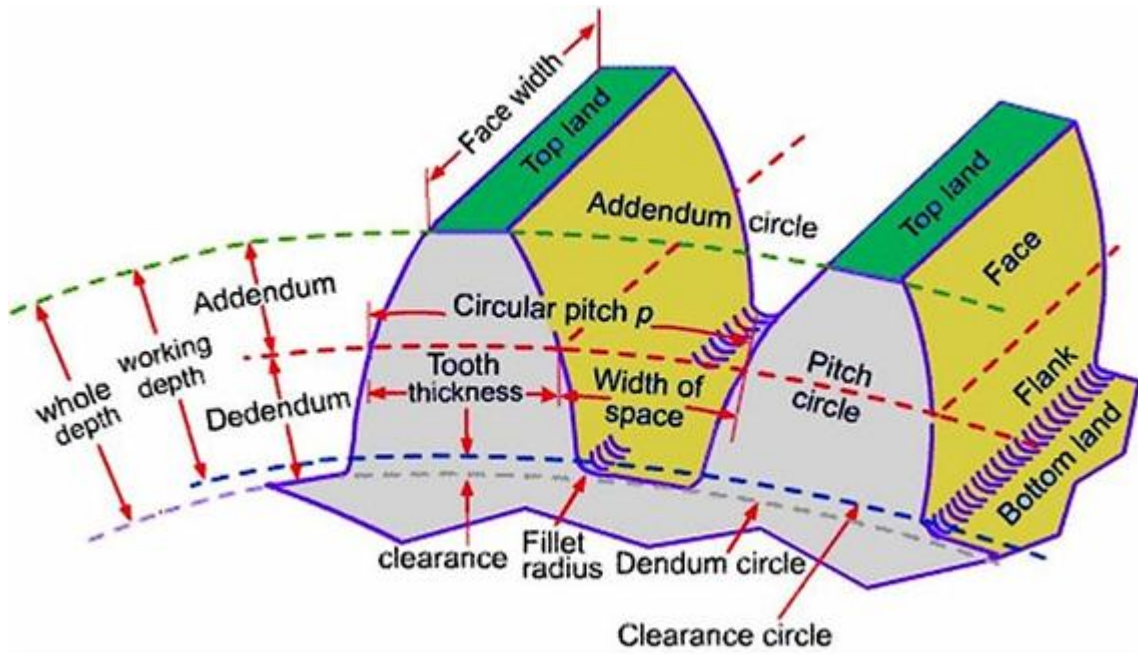


Figure 2.1 Spur Gear Terminologies (www.google.com/images)

2.2.2. Definitions

Definition of spur gear nomenclature is described as below.

Gear ratio: The ratio of gear teeth to pinion teeth is known as gear ratio. It is denoted by m symbol.

Pitch circle: Predefine diametral position on the gear from where pressure angle, and tooth thickness are measured is called pitch circle.

Circular pitch: The distance between one face of tooth to corresponding face of adjacent tooth including pitch circle. It is denoted by P_t .

Line of action, pressure line: A line which can direct force between two meshing gear is known as line of action. It is constant for involute tooth profile. It is denoted by Z .

Addendum: The radial distance between pitch surface and outer most point of the tooth is known as an addendum.

Dedendum: The radial distance between depths of tooth to pitch surface is known as Dedendum.

Module: The scaling factor that enlarges the tooth size is known as module. It is denoted by M symbol.

Backlash: It is an error in motion occurs during changing the gear direction. It is denoted by B symbol.

Pitch diameter: It is a ratio of number of teeth to pitch diameter. It is denoted by Pd symbol.

Interference: Teeth connect with each other also with the other parts of surface is known as interference. It is denoted by I symbol.

Face width: The length of gear teeth measured along the line of parallel to gear axis is called face width (Dudley, 1984). It is denoted by F symbol. Face width is taken as $\frac{8}{P} \leq F \leq \frac{12.5}{P}$.

Pitch-line velocity: The linear speed of a point on the pitch circle of gear when it rotates is called pitch-line velocity (Dudley, 1984).

2.3 Fundamental Law of Gearing

The fundamental law of gearing is to maintain angular velocity ratio and ratio of number of teeth of gear to pinion must be constant throughout the mesh.

$$\frac{\omega_1}{\omega_2} = \frac{N_1}{N_2} = C \dots\dots\dots 1$$

To manage the fundamental law of gearing, for all contact point in mesh, the common normal of tooth profile should pass from pitch point which is fixed point on line of centers.

2.4 Involute Gear and Conjugate Action

There is various gear tooth profile which can satisfy the fundamental law of gearing and make involute circle because it is conjugate with itself

2.4.1 Conjugate action

The rotary motion is produced when a couple of gear teeth act against each other. The rotary motion is transferred from the driver to the driven shaft. According to law of gearing gear teeth should be designed as angular velocity remain constant.

$$\frac{\omega_1}{\omega_2} = C \dots\dots\dots 2$$

ω_1 and ω_2 are angular velocity for driver and driven gear respectively. Thus, conjugate action can be represented as, “To transmit uniform rotary motion from one shaft to another by means of gear teeth, the normal to the profile of these teeth at all points of contact must pass through a fixed point in the common center line of the two shafts” (Buckingham, 1928).

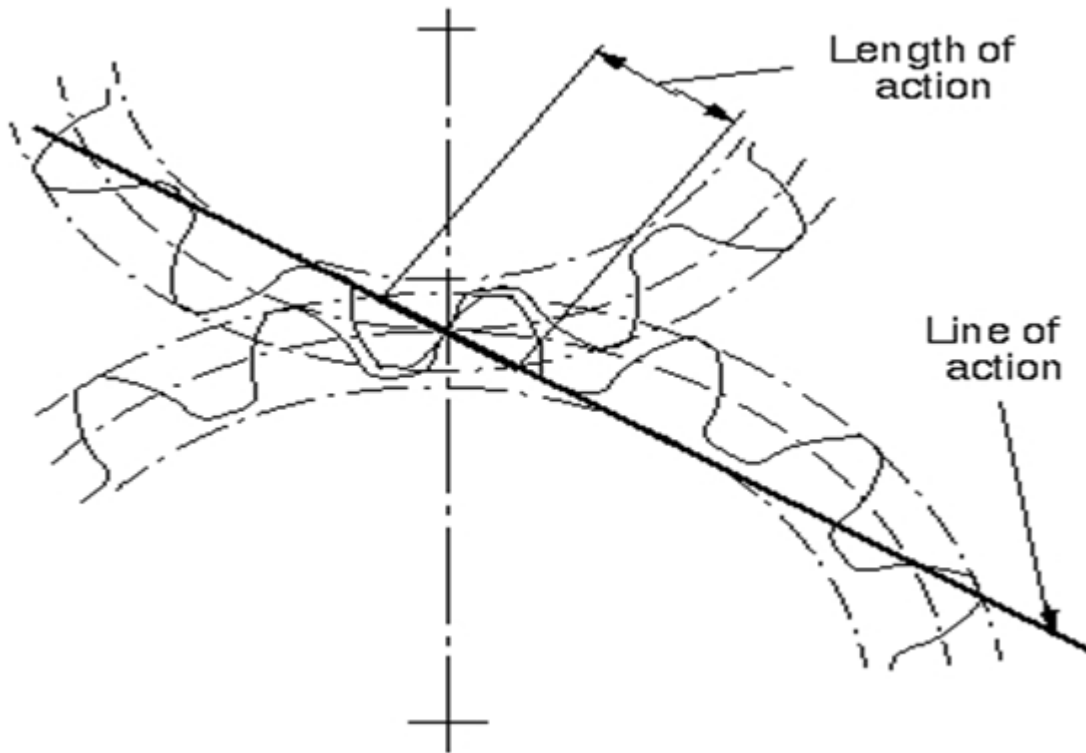


Figure.2.2 Conjugate Action (www.google.com/images)

2.4.2 Involute curve

The curve which is generated by the end point of rope which is kept stretched during the disentangled from a circle is called involute curve. The other point on the cord will also generate involute curve as rope is disentangle from the circle. This involute curve helps to maintain conjugate action and to follow the law of gearing. The geometry construction for involute curve is shown below (Maitra).

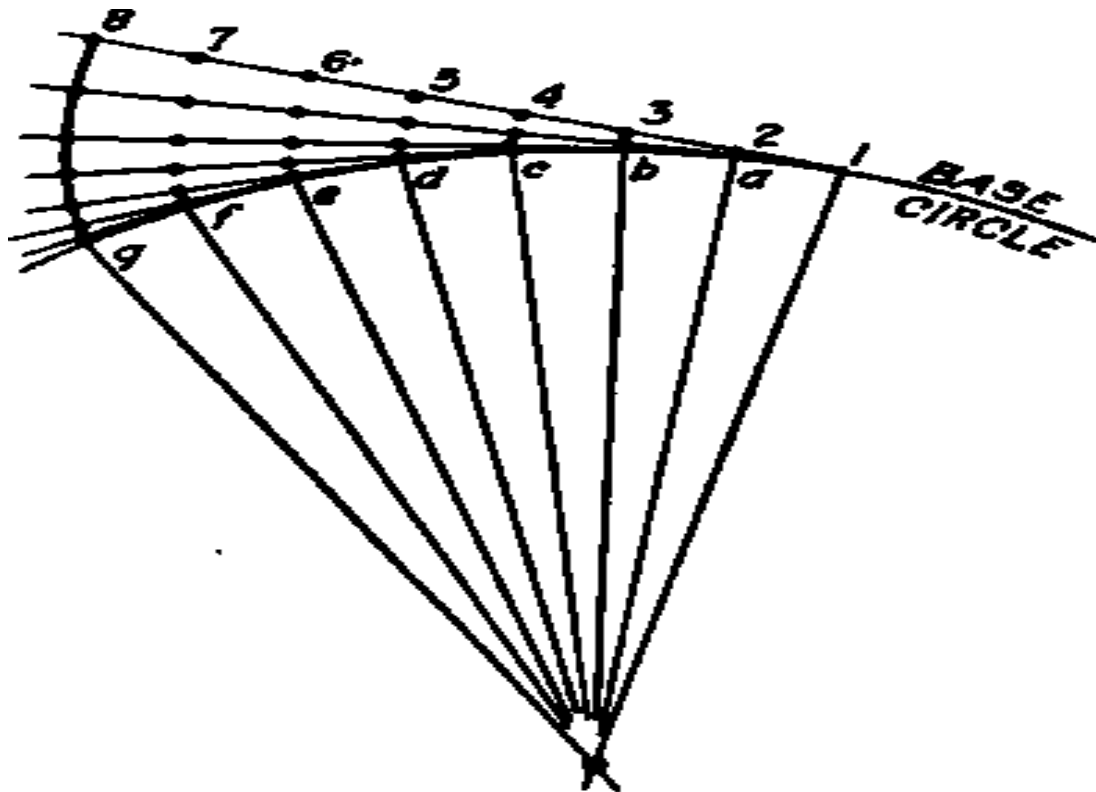


Figure.2.3 Involute Tooth (www.google.com/images)

Above method to draw involute profile is capable for standard representation. After drawing base circle, an arc is drawn with the center of point 1. The arc 8g is tolerable profile of involute profile for all practical purpose.

2.5 Gear Geometry Calculation

Spur gears are always designed by involute tooth profile. From many years addendum proportion is taken as $1.00 \times \text{module}$ and whole depth is taken as $2.250 \times \text{module}$. According to this design, a very small root fillet radius of curvature is possible and it is very difficult to design cutter for it. When gears are used for maximum load capacity application, it requires having larger root filler radius of curvature. Gears which are used in such application has larger pressure angle. The common pressure angle for spur gear is 20 degree. Vehicle and aircraft has 25 degree pressure angle which require larger root fillet radius of curvature. To avoid

interference and undercutting problem, root fillet radius of curvature and minimum number of teeth on gear and pinion are required.

In this thesis, diameter of pinion is required to enter by the applicant. Teeth on the pinion and gear are calculated by following equations

$$T = P \times D \quad \text{Pinion Teeth} \dots\dots\dots 3$$

$$T1 = Q \times T \quad \text{Gear Teeth} \dots\dots\dots 4$$

Where, Q is velocity ratio. The minimum number of teeth is calculated by below equation.

$$T_{\max} = \frac{4 \times Q^2 - T \times \sin(RHO)^2}{2 \times T \times \sin(RHO)^2 - 4 \times Q} \dots\dots\dots 5$$

Where, QK in taken as constant and it is related with the pressure angle. It is taken 1.0 for 14.5 degree and 0.8 for other pressure angles. The relation to check interference is given below.

$$Interface = T_{\max} < T \dots\dots\dots 6$$

2.5.1 Addendum and Dedendum Calculation

Addendum and dedendum are related with the module. In this thesis, the fundamental equations for both are given by below.

$$Addendum = 1.00 \times Module \dots\dots\dots 7$$

$$Dedendum = 1.25 \times Module \dots\dots\dots 8$$

The pinion and gear addendum radius are calculated by following equations respectively.

$$R01 = \frac{T + 2}{2 \times P} \dots\dots\dots 9$$

$$R02 = \frac{T1 + 2}{2 \times P} \dots\dots\dots 10$$

2.5.2 Length of Action

The point of leaving and beginning in contact defines the mesh of pinion and gear. The distance of these mesh points along the line of action is called length of action. It is denoted by Z symbol. It can also be represented by intersection of addendum circle within the line of action.

Figure 2.4 can describe the length of action. In this thesis length of action is found by following equation.

$$Z = \sqrt{Ra^2 - Rb^2} + \sqrt{Ra1^2 - Rb1^2} - CRD \times \sin(RHO) \dots\dots\dots 11$$

Where, Ra(Pitch), Ra1 are standard pinion and gear addendum radius and Rb, Rb1 are standard dedendum radius. CRD is center distance between gear and pinion. Ra(Pitch), Ra1, Rb, Rb1 are described by below equations.

$$Ra = R + \frac{Q}{P} \dots\dots\dots 12$$

$$Rb = R \times \cos(RHO) \dots\dots\dots 13$$

$$Ra1 = R1 + \frac{Q}{P} \dots\dots\dots 14$$

$$Rb1 = R1 \times \cos(RHO) \dots\dots\dots 15$$

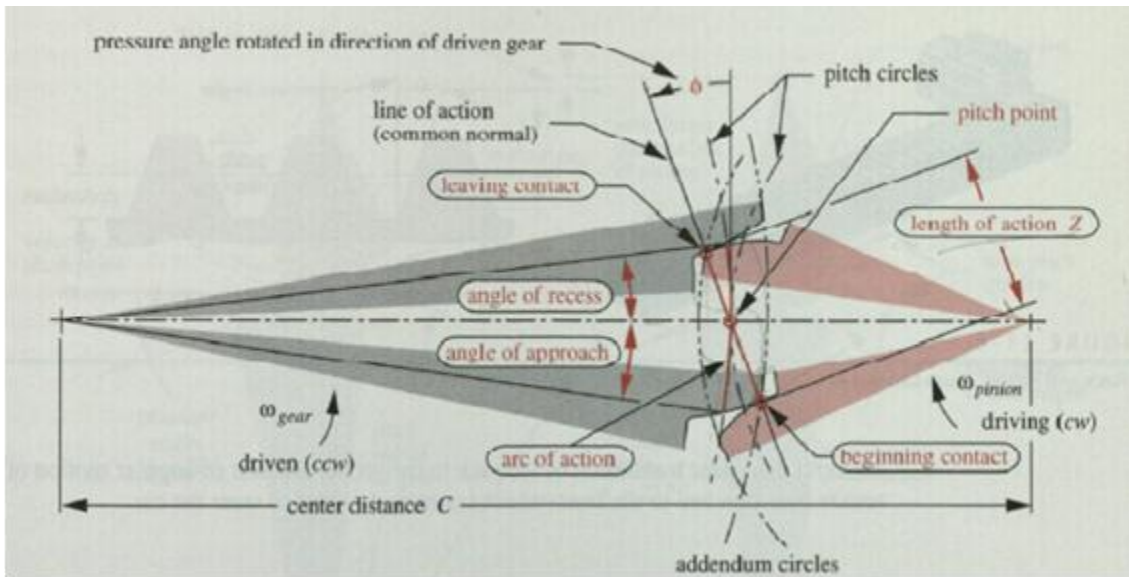


Figure 2.4 Length of Action (Nortan)

2.5.3 Contact Ratio

It has seen that during the teeth engagement that load is transmitted by one tooth of driving gear for the part of the time and by two teeth the rest of the time along the path of action.

Contact ratio has the relation between two angles, the angle of action and the angle subtended at the center by circular pitch. The equation used in this thesis is given bellow.

$$CR(\text{Pitch}) := \frac{Z(\text{Pitch})}{3.1415927 \cos(\text{RHO})} \dots\dots\dots 16$$

In other way, an average number of teeth in contact are known as contact ratio. Contact ratio should be higher to drive the gear smooth and continuous. Contact ratio 1.6 means during the engagement period, single tooth has 100% contacts and also during the same time two teeth are in contact by 60%

CHAPTER 3

GEAR STRENGTH CALCULATION

Strength and power transmission are still question for the gear design process. There are various rules and regulations, equations and different factors are derived for a single gear design from time to time.

There are two general rules are derived for the gear design. First is to check the design using equations and formulas for tooth strength and durability. Second is if it falls into practical trade standard like AGMA or DIN, it should meet trade standards (Dudley, 1984).

3.1 Introduction of Theoretical Bending Stress

Gear teeth are loaded with various types of load which create stresses high enough to break the teeth by overload or fatigue (Nicholson, 1978). A gear tooth is always considered as cantilever beam under loading condition. Tensile stress is loaded on the base of beam and compressive stress is loaded on the upper side. Gear teeth fail usually at the base of teeth on the tensile side. A gear tooth is always referred to as their beam strength or their flexural strength to resist tooth breakage.

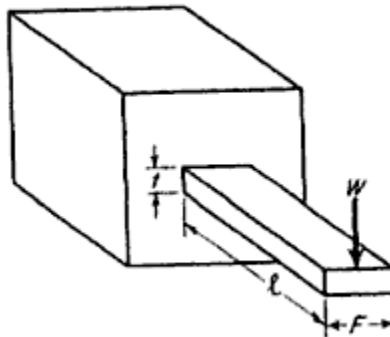


Figure 3.1 A loaded cantilever beam (Dudley, 1984)

Wilfred analyzed Lewis equation for bending stress which is now called as Lewis equation. He has included forty eight equations in it. One of those equations is given below (Buckingham, 1928). After that Buckingham derived new equation which has no relation with forty eight equations. Thus, Lewis form factor is the first factor which is included in the analysis of gear teeth.

$$X := 2000P \cdot F \dots\dots\dots 17$$

Where

x = gear teeth breaking load, pound

P = circular pitch of gear, inches

F = face width of tooth, inches

3.2 Lewis Equation for Bending Stress

The beam strength is calculated near to accurate first time by Lewis in 1893. He created the idea of imprinting a parabola inside a gear tooth profile because the stress is constant along the parabola. One can easily point out most critically stress point by imprinting maximum size parabola inside the gear tooth. This point is at which the position of parabola become tangent to the gear tooth surface. Lewis has derived the tensile stress equation at the root of cantilever beam.

$$St = \frac{6 \times W \times I}{F \times t^3} \dots\dots\dots 18$$

Considering involute tooth profile, there are two load conditions possible. When the load applied at the peak point of tooth and no load sharing occurs between two teeth is the first condition. Figure 3.2 describe this condition. The second condition is when irregular load condition is managed by the single pair of teeth at period of second pair of teeth are about to contact.

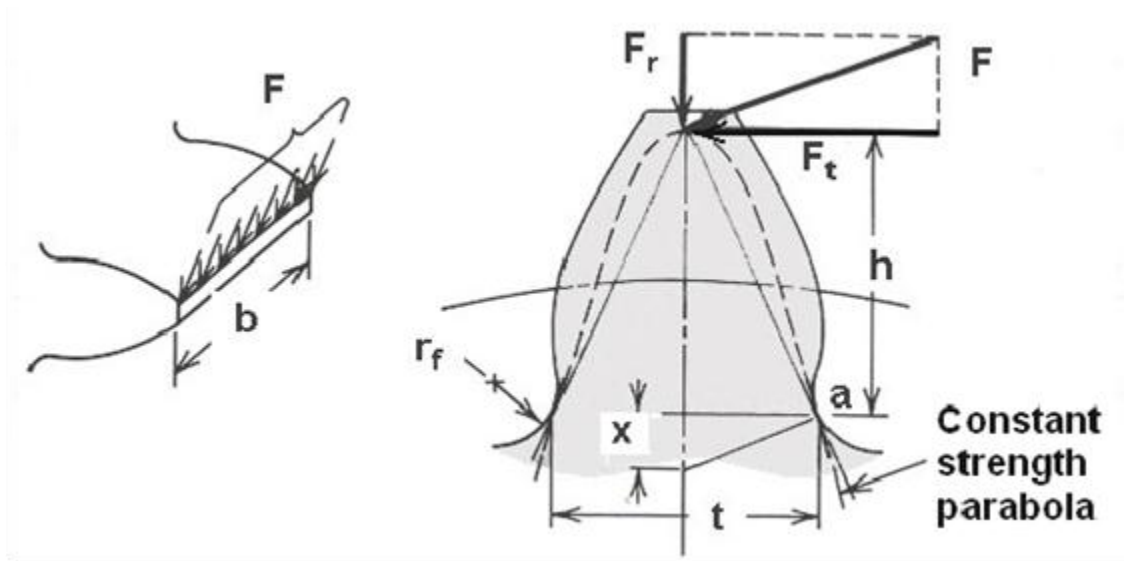


Figure 3.2 Tip load condition (M.M.Mayuram)

3.2.1 Worst Load Condition

The Tip load condition is not the most critical for the gear tooth break. The worst load condition occur when a single pair of teeth carry full load and the contact has rolled to a point at which a second pair of teeth is just ready to come into contact (Dudley, 1984). Figure 3.3 shows the worst load condition.

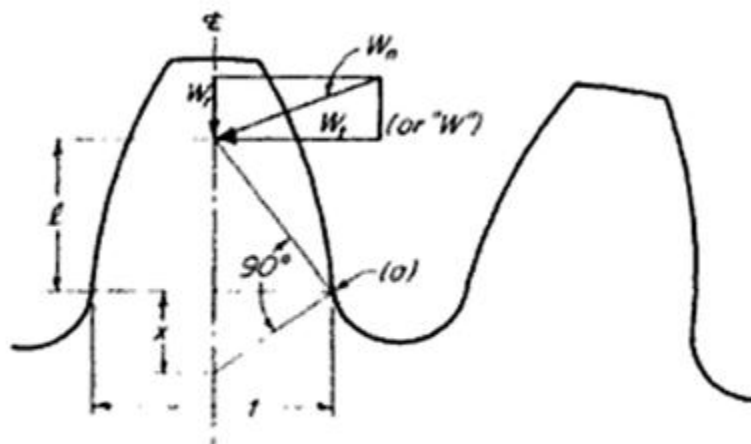


Figure 3.3 Worst load condition (Dudley, 1984)

3.2.2 Lewis Geometry Form Factor

As discussed above, Lewis derived the bending equation by considering gear tooth as cantilever beam and imprinting maximum parabola inside. The equation is shown below (Dudley, 1984).

$$\sigma = \frac{M \times c}{I} = \frac{W \times \cos(\phi) \times h \times \frac{t}{2}}{\frac{F \times t^3}{12}} \dots\dots\dots 19$$

Where

W = Normal load on tooth

t = Tooth thickness at highest stress section

F = Face-width

Φ = Pressure angle

h = Moment arm

For the parabola condition, moment arm is taken as below equation.

$$h = K \times t^2 \dots\dots\dots 20$$

From the figure 4.2 h can be neglected by below equation.

$$h = \frac{t^2}{4 \times X} \dots\dots\dots 21$$

Substituting (30) into (28) yields

$$\sigma = W \times \cos(\phi) \times \frac{6}{4 \times F \times X} \dots\dots\dots 22$$

Multiplying Pnd = Normal diametral pitch,

$$\sigma = W \times \cos(\phi) \times \frac{Pnd}{F} \times \frac{3}{2 \times X \times Pnd} \dots\dots\dots 23$$

The term $3 \div (2 \times X \times Pnd)$ in 23 is only based on tooth shape and load conditions. It is also called as Lewis form factor. It is denoted by Y symbol.

$$Y = \frac{3}{2 \times X \times Pnd} \dots\dots\dots 24$$

Finally, the equation for bending stress become like below.

$$\sigma = W \times \cos(\phi) \times \frac{Pnd}{F \times Y} \dots\dots\dots 25$$

Thus, Lewis form factor is depends on pressure angle and diametral pitches. In this work, below equation for form factor is used if pressure angle is 14.50 degree.

$$Y = 0.39 - \frac{1.99}{(D \times P - 0.28)^{0.98}} \dots\dots\dots 26$$

If the pressure angle is different than 14.50 degree, the below equation for form factor is used.

$$Y = 0.55 - \frac{1.41}{(D \times P - 1.86)^{0.756}} \dots\dots\dots 27$$

The above form factors equations are used for pinion. The same equations with same rules are used for gear with D1 (Gear diameter) instead of D (Pinion Diameter). The form factor value changes by changing teeth number and teeth numbers are depending on the diameter of pinion and gear. The different values are evaluated by graph which is evaluated by Mathcad program which is shown below.

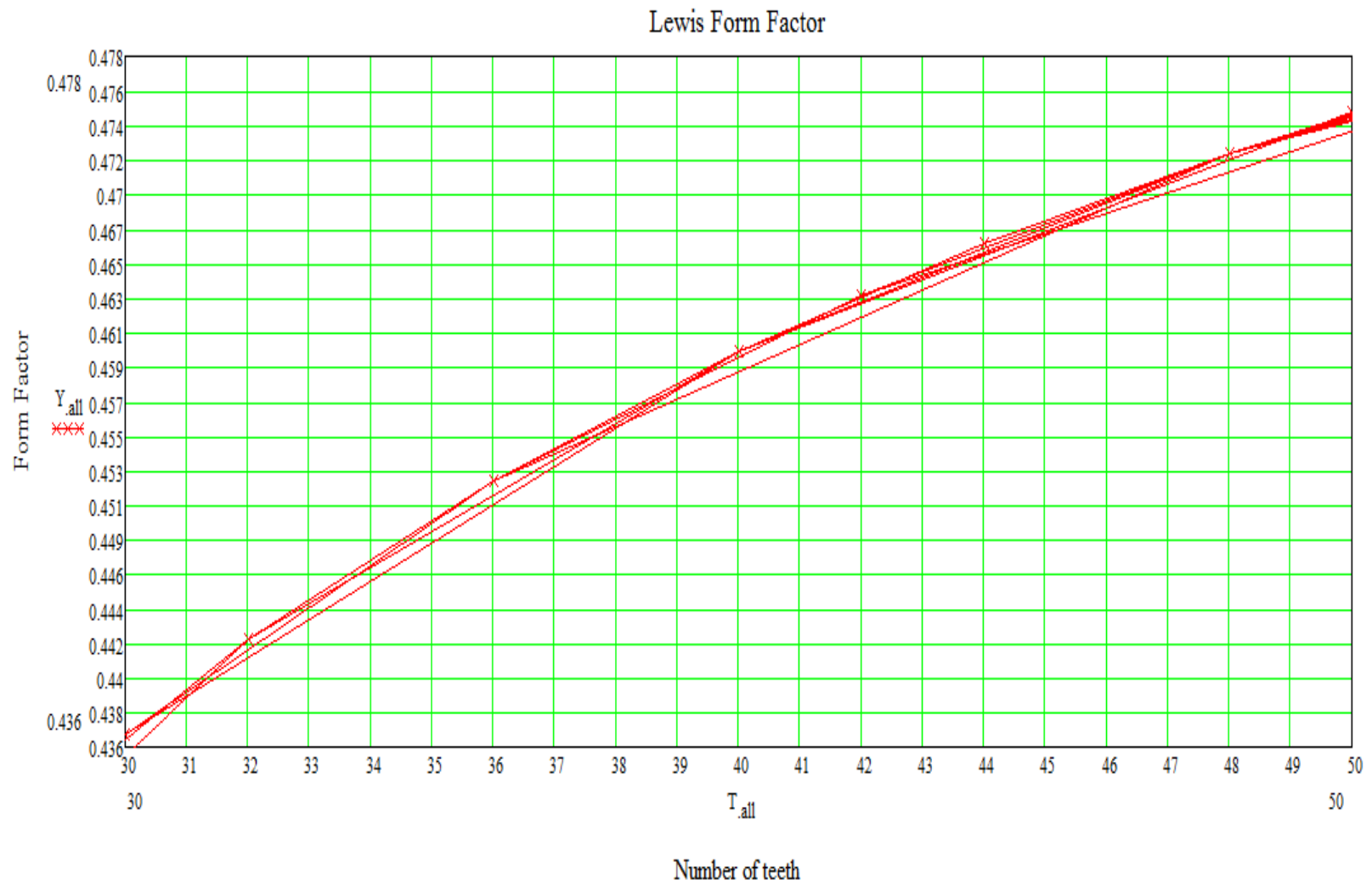


Figure 3.4 Lewis Form Factor Values for Different Teeth and Diameters of Pinion

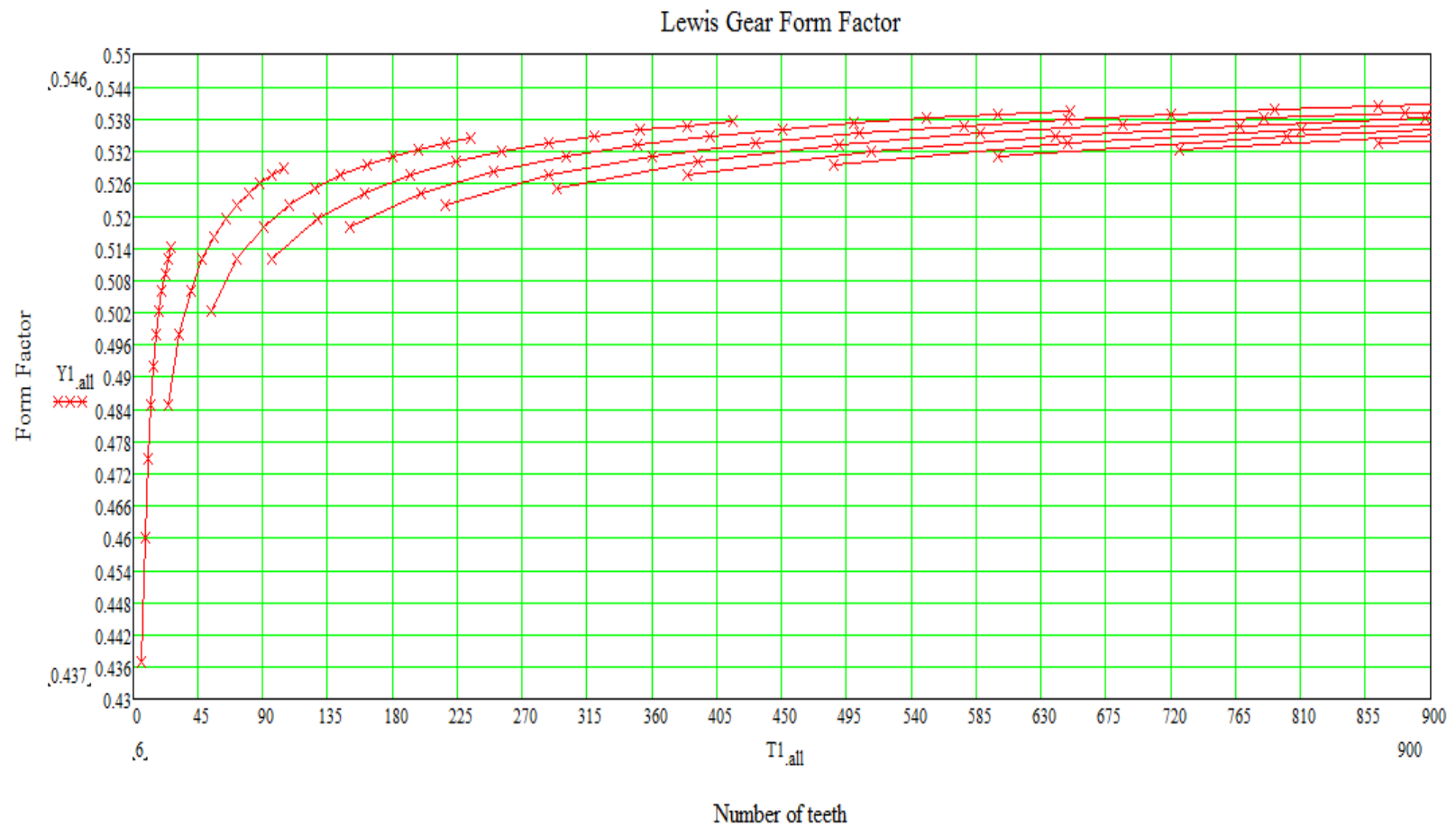


Figure 3.5 Lewis Form Factors for Different Teeth and Diameters of Gear

3.3 AGMA Bending Stress Calculation

AGMA bending equations are used with certain assumptions which are described below (Nortan). These equations are related to external gears only.

1 $1 \leq \text{Contact ratio} \leq 2$

2 No interference and undercutting between two mating teeth

3 No teeth are pointed

4 Backlash $\neq 0$

5 Root fillet should be smooth

6 Friction force = negligible

Contact ratio is in relation with tooth stiffness and tooth accuracy. To assume larger contact ratio because of larger load sharing ratio, the first assumption is taken. This assumption can reduce the problem of making assumption about contact ratio. Assumption 2 comes in spite to require minimum amount of tooth on gear and pinion. Assumption 3 takes care about unequal-addendum teeth requirement. For the packaging purpose, smaller amount of teeth required. to fulfill this condition unequal-addendum tooth requires. Gear with zero backlash will not run smoothly and that identified in assumption 4. Assumption 5 relates with stress concentration factor for root fillet and assumption 6 is self-explained. AGMA has described updated bending stress equation which still uses the base of Lewis bending stress equation. It is updated with the geometry form factor J instead of Lewis form factor.

$$\sigma = \frac{W_t \times P_{nd}}{F \times K_v \times J} \dots\dots\dots 28$$

Where

W_t = Tangential load

P_{nd} = Normal diametral pitch

F = Face width

K_v = dynamic load factor

J = Geometry form factor.

3.3.1 AGMA Geometry Form Factor

In AGMA standard 908-B89 information sheet, geometry form factor is calculated using complicated algorithms. The AGMA geometry factor J depends on the number of gear and pinion teeth and it can be used if gear teeth obey the assumption 2.

The actual stress or transmitted load on teeth can logically be reduced by load sharing ratio because the transmitted load is shared by teeth on the same gear. For the spur gears the load sharing ratio is one but if one tooth on each gear is in contact is called the critical point.

The root stress concentration factor also most important part to design AGMA geometry factor, J. The relation of these factors for spur gears is as below.

$$J = \frac{Y}{Kf} \dots\dots\dots 29$$

3.3.2 Root Fillet Stress Concentration Factor

This factor is derived from the photo elastic study done by T. J. Dolan and E. I. Broghamer (1942, University of Illinois). The factor Kf, include the stress concentration and load location on the tooth. The general form of the equation is derived by follow by Deutschman (1975) is

$$Kf = H + \left(\frac{t}{Rf}\right)^L + \left(\frac{t}{h}\right)^M \dots\dots\dots 30$$

Where

Kf = Root fillet stress concentration factor

t = Tooth thickness at point of maximum bending stress

rf = Minimum root fillet radius

H, L, M = Constant

The minimum root fillet radius which is taken from the AGMA information sheet 226.01 is as below

$$Rf = \frac{(bp - Rt)^2}{\frac{D}{2} + (bp - Rt)} + Rt \dots\dots\dots 31$$

The values of constant changes with the pressure angle changes. The table 3.1 shows the values for the constant for different pressure angles.

Table 3.1 Dolan and Broghamer Constants For Use in 30 (Deutschman,1975)

Pressure Angle	H	L	M
14	0.22	0.20	0.40
14.5	0.22	0.20	0.40
20	0.18	0.15	0.45

3.3.3 Final Update of Bending Stress Equation

In above both sections, new variables such as geometry factor and root fillet stress concentration factor are derived by AGMA. In the Lewis equation, radial component of load is neglected and AGMA included that in root fillet stress concentration factor, Kf.

After inserting these two factors in Lewis bending stress equation, new equation for geometry form factor and bending stress equation are derived.

$$\sigma = \frac{1.5 \times FD \times P}{CR \times FW \times J} \dots\dots\dots 32$$

3.3.4 Hardness Number

The easiest way to check the tensile strength of gear is to check harness. The hardening process of gear tooth is applied by various ways such as heating, carburizing, and nitriding. The hardness number shows tooth harness during the surface stress condition. In Mathcad program, the used equation to find out harness value is given as below.

$$BHNW = \frac{SS + 10^4}{400} \dots\dots\dots 33$$

If replacing surface stress value with AGMA surface stress value, we can find out the harness number by AGMA.

CHAPTER 4
PRACTICAL DESIGN OF SPUR GEAR

4.1 Backlash And Undercut

Backlash is the gap measured in-between mating teeth which is calculated along with the circumference of pitch circle. Gear teeth can not be the same dimensional and must mesh with each other. According to this, manufacturing tolerance prevent to have zero backlash. Applicable amount of backlash establish smooth run of gear set. Preventing the gear set from jamming and teeth on both side should not be in contact simultaneously are main purpose of backlash. Excess amount of backlash require to be avoided because it can increase the cost of gear allowances such as run-out, profile and mounting error. These allowances are kept smaller to set backlash. Table 4.1 shows values for backlashes for spur gear under normal application. The developed Mathcad program has module value from 1 to 0.1 so from the table 4.1 the backlash value is set for 0.10.

Table 4.1 Backlashes for Spur Gear for Normal Application (Maitra, Handbook of Gear Design , 1989)

Module	Min	Max
20	0.75	1.25
16	0.50	0.85
12	0.35	0.60
10	0.30	0.51
8	0.22	0.40
6	0.20	0.33
5	0.15	0.25
4	0.13	0.20
3	0.10	0.15

Table 4.1 Continued

2.5	0.08	0.13
2	0.08	0.13
1.5	0.00	0.10

In figure 4.1, the backlash can be visualized between mating pair of gears because the two gears have been apart and the center distance has now been increased.

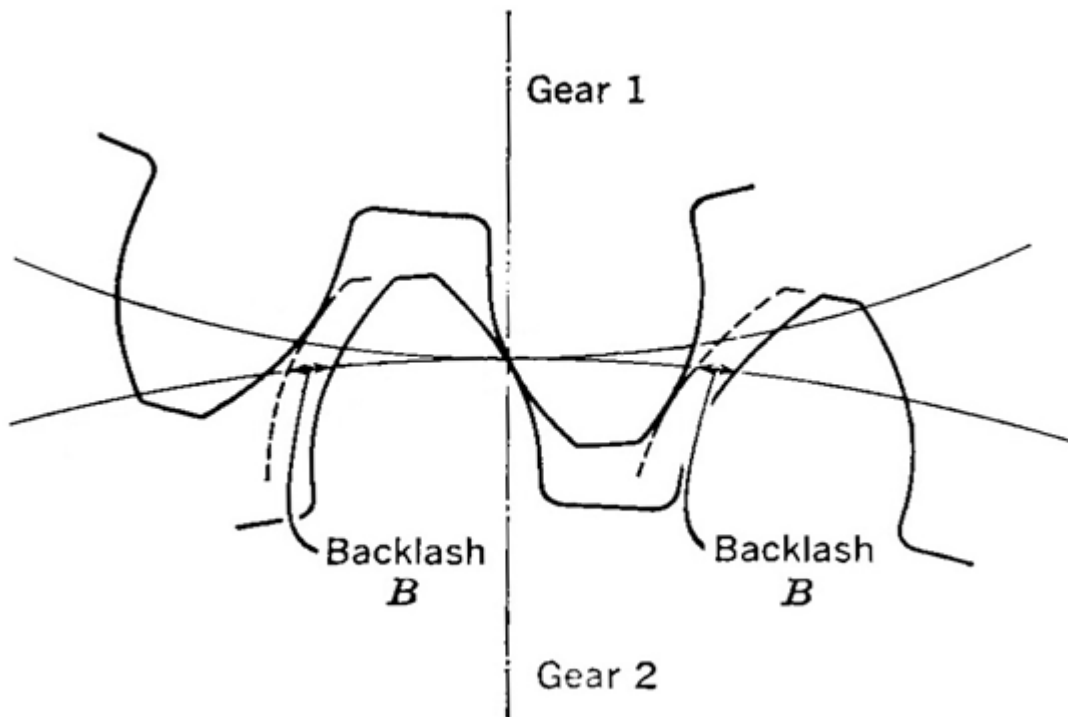


Figure 4.1 Backlash (Maitra, Handbook of Gear Design , 1989)

If cutting tool has interference while hatching gear teeth, it cut recess at the root of the gear. It has bad application to transfer power because of cutting root material. It is called undercutting. The tooth has weakest section at the root and because of undercutting it become more weakened. There are two ways to minimize the undercutting such as to use larger pressure angle and to have minimum number of teeth on spur.

4.2 Surface Stress Calculation for Spur Gear

Gears will have rapidly compressive contact surface stress between mating gears and it will cause pitting effect as a form of wear. This is called surface stress. The rapid cycle will increase dynamic load and surface stress on the gear teeth and cause to ultimate to fail. Also gear tooth moves against each other with sliding and rolling action. The sliding action with the coefficient of friction causes extra surface stress on gear teeth. The stress usually defined by Hertz formulas and called hertz stresses. The hertz equation for spur gear is determined by below equation.

$$SS = \sqrt{\left[\frac{0.7 * FD * E1 * E2 * 10^5 * \left(\frac{1}{D} + \frac{1}{D1}\right)}{\sin(RHO) * \cos(RHO) * CR * (E1 + E2)} * \frac{1}{fw} \right]} \dots\dots\dots 34$$

Where,

- SS = Surface Stress
- FD = Dynamic Force
- E1, E2 = Elastic Moduli
- D, D1 = Pinion & Gear Diameters
- RHO = Pressure Angle in Radian
- CR = Contact Ratio
- fw = Face width

4.3 Dynamic Factor For Stress Calculation

According to experiment, dynamic factor has contribution in stress calculation. It includes the dynamic effect on the gear so it is recommended to calculate it. Dynamic factor has relation with the pitch line velocity. According to manufacturing accuracy and pitch line velocity, dynamic factor has different equations which are shown below. It is denoted by Kv symbol.

$$Kv = \sqrt{\frac{78 + \sqrt{V}}{78}} \text{ For cut or milled teeth} \dots\dots\dots 35$$

$$Kv = \frac{78 + \sqrt{V}}{78} \text{ For shaped teeth} \dots\dots\dots 36$$

$$K_v = \frac{50}{50 + \sqrt{V}} \text{ For high precision cut teeth..... 37}$$

$$K_v = 20.60 \text{ Constant 38}$$

Equation 38 is used if gear is made differently than milled, shaped, and high precision cut teeth. To calculate the dynamic factor, tangential force is require to be calculated. It is denoted by FT symbol. The equation for it is given below.

$$FT = \frac{3.3 \times 10^4 \times HP}{V} \text{ 39}$$

Where, HP is horse power and V is velocity in mm/sec. Now, dynamic force is calculated by multiplying tangential force with dynamic factor. For different values of pitch line velocity, dynamic factors can be taken from the Figure 4.2 which is a graphical result from the Mathcad program.

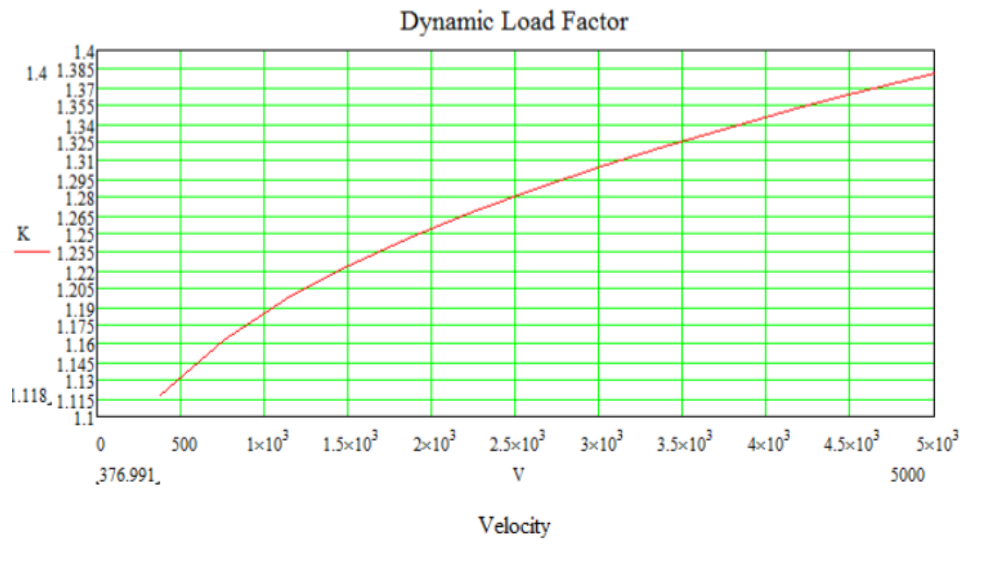


Figure 4.2 Dynamic Factor for different Pitch Line Velocity. (Mathcad Program Result)

4.4 AGMA Surface Stress Calculation

AGMA has taken the Hertz equation as a base to find out surface stress on gear and pinion. AGMA defined the surface stress as below equation.

$$SSA = C_p \times \sqrt{\frac{FT}{K \times FW \times C_a} \times C_s \times C_f \times \frac{C_m}{I}} \dots\dots\dots 40$$

Where,

C_p = Elastic coefficient

K = Dynamic load factor

D = Pinion diameter

C_a = Application factor

C_s = Size factor

C_f = Surface finish factor

C_m = Load distribution factor

I = Surface geometry factor

4.4.1 Surface Geometry Factor, I

AGMA included radii of curvature and pressure angle for gear and pinion tooth in surface stress equation. AGMA defined the equation for I is as below.

$$I = \frac{\cos(\phi)}{\left(\frac{1}{\rho_p \pm \rho_g}\right) \times dp} \dots\dots\dots 41$$

Where,

Φ = Pressure angle

ρ_p, ρ_g = Radius of curvature for gear and pinion respectively

dp = Diametral pitch

The + and – signs are used to evaluate surface stress for external and internal gears. Radius of curvature is calculated from the mesh geometry of tooth. The equation for it is given below.

4.4.2 Elastic Coefficient Factor

AGMA included material properties in the surface stress such elastic moduli and poisson ratio. AGMA defined the equation as below.

$$C_p = \sqrt{\frac{1}{\pi \left(\left(\frac{1 - \nu_p^2}{E_p} \right) + \left(\frac{1 - \nu_g^2}{E_g} \right) \right)}} \dots\dots\dots 42$$

In this thesis steel is taken in account so elastic moduli for gear and pinion is $30 \cdot 10^6$ MPA and poisson ratio as 0.3.

4.4.3 Surface Finish Factor

AGMA has not defined any specific value for surface finish factor. For the conventional gear, it is taken as 1. The value can be increased for unusually rough surface finish or presence of detrimental residual stress.

4.4.4 Application Factor

In the absence of dynamic load information, application factor is applied to increase the tensile strength on the basis of stockiness of machinery. AGMA has defined some values for the application factor which are given below.

Table 4.2 Application Factor (Nortan)

Driving Machine	Uniform	Mod.	Heavy
Uniform	1	1.25	≥ 1.75
Light	1.25	1.50	≥ 2.00
Medium	1.50	1.75	≥ 2.25

4.4.5 Load Distribution Factor

If there is any misalignment in axis, transmitted load become uneven over the teeth. This problem increases with the increment of face width. AGMA has also defined values for it such as below.

Table 4.3 Load Distribution Factor

FW	Cm
<2	1.6
6	1.7
9	1.8
9	1.8
≥20	2.0

CHAPTER 5

MATHCAD

5.1 Introduction

Engineering calculation play a crucial role in all stage of product development. In Mathcad has been the leading engineering calculation solution for more than 20 years design Mathcad is a PTC's product development system which delivers unique, highly visual interface to perform, document and share calculation and design work. Mathcad offers unparalleled capabilities to preserve valuable intellectual capabilities while simplifying compliance, reporting, verification and troubleshooting.

5.2 Values Calculated In Mathcad Program

In the Mathcad program, input values are elastic moduli, velocity ratio, pressure angle, pinion speed in rpm, horse power and pinion diameter. First calculations are done for the basic values such as gear diameter, pinion and gear teeth, circular pitch, pinion and gear addendum and dedendum, standard distance between pinion and gear and base circle radius for pinion and gear by using input values. In the first half Mathcad program pitch is taken as user dependent.

To design Mathcad program for the spur gear, it is required to focus on the Lewis form factor and AGMA geometry form factor calculations. From the equation 32, 33 and 35, required data can be identify to calculate.

Root fillet stress concentration factor (K_f), velocity (V), contact ratio (CR), and tangential load (F_T) on teeth are calculated after the basic calculations and to obtain form factor values. To calculate root fillet stress concentration factor, it is require calculating root fillet radius from the equation 37 which uses values from the results of basic calculations. Allowable error is calculated from the velocity and actual tolerance which described as error is

calculated by allowable error. In this program error is taken from the Buckingham Earle's

$$\text{graph. } Error = AE_N + \frac{(V - N \times 200) \times (AE_{N+1} - AE_N)}{200} \dots 43$$

Where,

ERR is actual error (tolerance)

AE is allowable error

V is velocity

N is constant which is calculated from following equation.

$$N = \frac{V}{200} + 1 \dots\dots\dots 44$$

Face width is calculated according to thumb rule with the increment of 0.25. The equation 12 is used to calculate length of line of action. Standard pinion and gear addendum and dedendum are required to calculate. Also, it is required to convert degree pressure angle into radian. These values are calculated from following equations respectively.

$$Ra = R + \frac{Q}{P} \dots\dots\dots 45$$

$$Ra1 = R1 + \frac{Q}{P} \dots\dots\dots 46$$

$$Rb = R \times \cos(RHO) \dots\dots\dots 47$$

$$Rb1 = R1 \times \cos(RHO) \dots\dots\dots 48$$

Q is a constant value. It is taken as 1 if the pressure angle is 14.5 degree and 0.8 if the pressure angle is other than 14.5 degree. Line of action value is used to calculate contact ratio (C) and tangential load (FT).

After getting values of contact ratio and tangential load, Lewis form factors (Y) are calculated. The dynamic load factor (FD) and velocity factor (K) are calculated to evaluate AGMA geometry factor (J).

Finally surface stress (3), pinion and gear bending stress (31), (32) and hardness number (39) are calculated after getting all values and compared with the AGMA standards.

5.3 Program

Below Mathcad program consider diameter 14 in, pressure angle 20 degree, poison ratio 0.292, elastic moduli 30×10^6 , and velocity ratio 5. The program is made to calculate surface stress, bending stress for pinion and gear, and hardness number according Lewis form factor and AGMA geometry factor.

INPUT

$$\phi = \begin{matrix} 14.5 \\ 20 \end{matrix}$$

$$\text{PressureAngle} = 14.5$$

$$\text{PressureAngle} = 20.0$$

$$\text{Pressure Angle} = 20$$

$$\mu = 0.279 \dots\dots\dots \text{Poison Ratio}$$

$$E1, E2 = 30 \times 10^6 \dots\dots\dots \text{Elastic Moduli}$$

$$VR = 5 \dots\dots\dots \text{Velocity Ratio}$$

$$T = P \times D \dots\dots\dots \text{Pinion Teeth}$$

$$D1 = VR \times D \dots\dots\dots \text{Gear Diameter}$$

$$T1 = T \times VR \dots\dots\dots \text{Gear Teeth}$$

$$Mg = \frac{T1}{T} \dots\dots\dots \text{Gear Ratio}$$

$$Pc = \frac{\pi}{P} \dots\dots\dots \text{Circular Pitch}$$

$$B = 0.10 \dots\dots\dots \text{Backlash}$$

$$T\text{Thickness(Pitch)} := \frac{Pc(Pitch) - B}{2} \dots\dots\dots \text{Tooth Thickness}$$

$$R01(Pitch) := \frac{T(Pitch) + 2}{2 \cdot (Pitch)} \dots\dots\dots \text{Pinion Addendum Radius}$$

$$R02(Pitch) := \frac{T1(Pitch) + 2}{2 \cdot (Pitch)} \dots\dots\dots \text{Gear Addendum Radius}$$

$$C(Pitch) := \frac{T(Pitch) + T1(Pitch)}{(Pitch)} \cdot \frac{1}{2} \dots\dots\dots \text{Standard Central Distance}$$

$$Rm(Pitch) := \frac{1}{2} \cdot [R01(Pitch) + (C(Pitch) - R02(Pitch))]$$

$$Rb1 := \frac{D}{2} \cdot \cos(20deg) \dots\dots\dots \text{Base Circle Radius for Pinion}$$

$$Rb2(Pitch) := Rb1 \cdot m(Pitch) \dots\dots\dots \text{Base Circle Radius for Gear}$$

$$\Phi_r(Pitch) := \arccos\left(\frac{Rb2(Pitch) + Rb1}{C(Pitch)}\right) \dots\dots\dots \text{Operating Pressure Angle}$$

$$I1(Pitch) := \sqrt{Rm(Pitch)^2 - Rb1^2}$$

$$d(Pitch) := \frac{C(Pitch)}{m(Pitch) + 1} \dots\dots\dots \text{Operating Pitch Diameter}$$

$$I(Pitch) := \frac{\cos(\Phi_r(Pitch)) \cdot C\Psi^2}{\left(\frac{1}{I1(Pitch)} + \frac{1}{I2(Pitch)}\right)} \cdot d(Pitch) \cdot mn \dots\dots\dots \text{Pitting Resistance for external gears}$$

$$Rt(Pitch) := \frac{0.35}{Pitch} \dots\dots\dots \text{Hob Teeth Radius}$$

$$M(Pitch) := \frac{D}{T(Pitch)} \dots\dots\dots \text{Module}$$

$$bp(Pitch) := 1.25 \cdot M(Pitch) \dots\dots\dots \text{Dedendum of Pinion}$$

$$Rf(Pitch) := \frac{(bp(Pitch) - Rt(Pitch))^2}{\frac{D}{2} + (bp(Pitch) - Rt(Pitch))} + Rt(Pitch)$$

$$\begin{array}{l}
 \underline{H} := \left\{ \begin{array}{l} H \leftarrow 0.22 \text{ if } PressureAngle = 14 \\ H \leftarrow 0.22 \text{ if } PressureAngle = 14.5 \\ H \leftarrow 0.18 \text{ if } PressureAngle = 20 \\ H \end{array} \right. \quad \underline{L} := \left\{ \begin{array}{l} L \leftarrow 0.20 \text{ if } PressureAngle = 14 \\ L \leftarrow 0.20 \text{ if } PressureAngle = 14.5 \\ L \leftarrow 0.15 \text{ if } PressureAngle = 20 \\ L \end{array} \right. \\
 \underline{M} := \left\{ \begin{array}{l} M \leftarrow 0.40 \text{ if } PressureAngle = 14 \\ M \leftarrow 0.40 \text{ if } PressureAngle = 14.5 \\ M \leftarrow 0.45 \text{ if } PressureAngle = 20 \\ M \end{array} \right.
 \end{array}$$

$$Factor := 3.4$$

$$Kf(Pitch) := H + \left(\frac{TThickness(Pitch)}{Rf(Pitch)} \right)^L \cdot (Factor)^M$$

$$\begin{array}{l}
 FW(Pitch) := \left\{ \begin{array}{l} n \leftarrow 19 \\ \text{for } i \in 0..n-1 \\ \left| \begin{array}{l} f_{w_i} \leftarrow \frac{8}{Pitch} + i \cdot 0.25 \\ f_{w_i} \leftarrow 0 \text{ if } f_{w_i} > \frac{12.5}{Pitch} \end{array} \right. \\ f_w \end{array} \right. \dots \text{Face-width}
 \end{array}$$

$$\underline{V} := \frac{\pi \cdot D \cdot N}{12} \dots \text{Velocity}$$

$$ERR = AE_{NI} + \frac{(V - NI \cdot 200) \cdot (AE_{NI+1} - AE_{NI})}{200} \dots \text{Error}$$

$$NI = \frac{V}{200} + 1$$

$$\begin{array}{l}
 AE := \left(\begin{array}{l}
 0.0042 \\
 0.0037 \\
 0.00338 \\
 0.00305 \\
 0.00272 \\
 0.00242 \\
 0.00219 \\
 0.00198 \\
 0.00180 \\
 0.00162 \\
 0.00150 \\
 0.00135 \\
 0.00122 \\
 0.00111 \\
 0.001 \\
 0.0009 \\
 0.0008 \\
 0.00075 \\
 0.0007 \\
 0.00065 \\
 0.0006 \\
 0.0058 \\
 0.00054 \\
 0.00053 \\
 0.00052 \\
 0.00050
 \end{array} \right)
 \end{array}
 \qquad
 \begin{array}{l}
 Error := \left(\begin{array}{l}
 2.65 \cdot 10^{-3} \\
 2.16 \cdot 10^{-3} \\
 1.74 \cdot 10^{-3} \\
 1.65 \cdot 10^{-3} \\
 1.33 \cdot 10^{-3} \\
 1.1 \cdot 10^{-3} \\
 8.5 \cdot 10^{-4} \\
 7.5 \cdot 10^{-4} \\
 6.708 \cdot 10^{-4} \\
 5.5 \cdot 10^{-4} \\
 5.4 \cdot 10^{-4} \\
 4.6 \cdot 10^{-4} \\
 0.0005 \\
 0.0005 \\
 0.0005 \\
 0.0005 \\
 0.0005 \\
 0.0005 \\
 0.0005 \\
 0.0005 \\
 0.0005 \\
 0.0005 \\
 0.0005 \\
 0.0005
 \end{array} \right)
 \end{array}$$

$$CI := \begin{cases} CI \leftarrow 0.107 \cdot Error & \text{if } PressureAngle = 14.5 \\ CI \leftarrow 0.115 \cdot Error & \text{otherwise} \end{cases}
 \qquad
 QK := \begin{cases} QK \leftarrow 1.0 & \text{if } PressureAngle = 14.5 \\ QK \leftarrow 0.8 & \text{otherwise} \end{cases}$$

$$T_{max}(Pitch) := \frac{4 \cdot QK^2 - T(Pitch)^2 \cdot \sin(RHO)^2}{2 \cdot T(Pitch) \cdot \sin(RHO)^2 - 4 \cdot QK}$$

$$Interference(Pitch) := (T_{max}(Pitch) < T(Pitch))$$

$$R := \frac{D}{2} \quad R_I := \frac{D_I}{2} \quad CRD := R + R_I$$

$$Ra(Pitch) := R + \frac{QK}{Pitch}$$

$$Rb := R \cdot \cos(RHO)$$

$$Ral(Pitch) := Rl + \frac{QK}{Pitch}$$

$$Rbl := Rl \cdot \cos(RHO)$$

$$Z(Pitch) := \sqrt{[(Ra(Pitch))^2 - Rb^2]} + \sqrt{[(Ral(Pitch))^2 - Rbl^2 - CRD \cdot \sin(RHO)]} \text{Length of Action}$$

$$CR(Pitch) := \frac{Z(Pitch)}{3.1415927 \cdot \cos(RHO)} \dots \text{Contact Ratio}$$

$$FT := \frac{3.3 \cdot 10^4 \cdot HP}{V} \dots \text{Transmitted Force}$$

LEWIS FORM FACTOR

$$Y(Pitch) := \begin{cases} Y \leftarrow 0.39 - \frac{1.99}{(D \cdot Pitch - 0.28)^{0.98}} & \text{if } PressureAngle = 14.5 \\ Y \leftarrow 0.55 - \frac{1.41}{(Pitch \cdot D - 1.89)^{0.756}} & \text{otherwise} \end{cases}$$

$$Yl(Pitch) := \begin{cases} Yl \leftarrow 0.39 - \frac{1.99}{(Dl \cdot Pitch - 0.28)^{0.98}} & \text{if } PressureAngle = 14.5 \\ Yl \leftarrow 0.550 - \frac{1.41}{(Pitch \cdot Dl - 1.89)^{0.756}} & \text{otherwise} \end{cases}$$

$$K := \begin{cases} K \leftarrow \sqrt{\left(\frac{78 + \sqrt{V}}{78}\right)} & \text{if } AIDF = 1 \\ K \leftarrow \frac{78 + \sqrt{V}}{78} & \text{if } AIDF = 2 \\ K \leftarrow \frac{50}{50 + \sqrt{V}} & \text{if } AIDF = 3 \\ K \leftarrow 20.67 & \text{otherwise} \end{cases} \dots \text{Dynamic Factor}$$

$$FD(Pitch) := \begin{cases} (FD \leftarrow FT \cdot K) & \text{if } AIDF = 1 \\ (FD \leftarrow FT \cdot K) & \text{if } AIDF = 2 \\ (FD \leftarrow FT \cdot K) & \text{if } AIDF = 3 \\ FD \leftarrow FT + \frac{[0.05 \cdot V \cdot [(FW(Pitch) \cdot C) \cdot FT]]}{0.05 \cdot V + (FW(Pitch) \cdot C \cdot FT)} & \text{otherwise} \\ \text{return } FD & \end{cases}$$

.....Dynamic Force

AGMA GEOMETRY FORM FACTOR

$$J(Pitch) := \frac{Y(Pitch)}{Kf(Pitch)} \quad JI(Pitch) := \frac{YI(Pitch)}{Kf(Pitch)}$$

Lewis stress and Hardness calculation

$$SB(fw, Pitch) := \frac{1.5 \cdot FD(Pitch) \cdot Pitch}{CR(Pitch) \cdot fw \cdot Y(Pitch)}$$

$$SBI(fw, Pitch) := \frac{1.5 \cdot FD(Pitch) \cdot Pitch}{CR(Pitch) \cdot fw \cdot YI(Pitch)}$$

$$SS(fw, Pitch) := \sqrt{\frac{0.7 \cdot FD(Pitch) \cdot E1 \cdot E2 \cdot 10^6 \cdot \left[\frac{1}{D} + \left(\frac{1}{DI} \right) \right]}{\sin(RHO) \cdot \cos(RHO) \cdot CR(Pitch) \cdot (E1 + E2)}}$$

$$BHNW(fw, Pitch) := \frac{SS(fw, Pitch) + 10^4}{400}$$

AGMA STRESS AND HARDNESS CALCULATION

Table 5.1 Load Distribution Factor

FaceWidth	Cm
<2	1.6
6	1.7
9	1.8
9	1.8
≥20	2.0

Table 5.2 Application Factor

Driving Machine	Driven Machine		
	Uniform	Mod.	Heavy
Uniform	1	1.25	≥1.75
Light	1.25	1.50	≥2.00
Medium	1.50	1.75	≥2.25

$$C_p := \left[\frac{1}{\pi \cdot \left[\frac{(1-\mu^2)}{E1 \cdot 10^6} + \frac{1-\mu^2}{E2 \cdot 10^6} \right]} \right]^{0.5} \dots\dots\dots \text{Elastic Coefficient}$$

$$K_m := 1.6 \dots\dots\dots \text{Load Distributer Factor}$$

$$K_a := 1 \dots\dots\dots \text{Application Factor}$$

$$K_s := 1 \dots\dots\dots \text{Size Factor}$$

$$K_b := 1 \dots\dots\dots \text{Rim Thickness Factor}$$

$$K_l := 1 \dots\dots\dots \text{Idler Factor}$$

$$BSA(f_w, Pitch) := \frac{FT}{f_w \cdot J(Pitch)} \cdot \frac{K_a}{K} \cdot K_s \cdot K_b \cdot K_l \cdot K_m \dots\dots\dots \text{AGMA Pinion Bending Stress}$$

$$BSA_I := BSA(FW_I, 1)$$

$$BSA1(f_w, Pitch) := \frac{FT}{f_w \cdot J1(Pitch)} \cdot \frac{K_a}{K} \cdot K_s \cdot K_b \cdot K_l \cdot K_m \dots\dots\dots \text{AGMA Gear Bending Stress}$$

$$BSA1_I := BSA1(FW_I, 1)$$

$$SSA(f_w, Pitch) := C_p \cdot \sqrt{\frac{FT}{K \cdot D \cdot f_w \cdot C_a} \cdot C_s \cdot C_f \cdot \frac{C_m}{I(Pitch)}} \dots\dots\dots \text{AGMA Surface Stress}$$

$$SSA_I := SSA(FW_I, 1)$$

$$BHNWA(fw, Pitch) := \frac{SSA(fw, Pitch) + 10^4}{400} \dots\dots\dots \text{AGMA Hardness Number}$$

$$BHNWA_I := BHNWA(FW_I, 1)$$

VARIOUS PITCHES

$$i := 0..18$$

$$Pitch := \left| \begin{array}{l} \text{for } i \in 0, 1..18 \\ \left| \begin{array}{l} A1 \leftarrow 1 \\ P_i \leftarrow i + A1 \end{array} \right. \\ P \end{array} \right.$$

$$FFacewidth^{(i)} := FW(Pitch_i)$$

$$SBall(fw, pitch) := \left| \begin{array}{l} \text{for } i \in 0..18 \\ \text{for } j \in 0..18 \\ \left| \begin{array}{l} sb_{i,j} \leftarrow NaN \text{ if } fw_{i,j} = 0 \\ sb_{i,j} \leftarrow SB(fw_{i,j}, Pitch_j) \text{ otherwise} \end{array} \right. \\ sb \end{array} \right.$$

$$SB_{all} := SBall(FFacewidth, Pitch)$$

$$SB1all(fw, pitch) := \left| \begin{array}{l} \text{for } i \in 0..18 \\ \text{for } j \in 0..18 \\ \left| \begin{array}{l} sb1_{i,j} \leftarrow NaN \text{ if } fw_{i,j} = 0 \\ sb1_{i,j} \leftarrow SB1(fw_{i,j}, Pitch_j) \text{ otherwise} \end{array} \right. \\ sb1 \end{array} \right.$$

$$SB1_{all} := SB1all(FFacewidth, Pitch)$$

$$BSAall(fw, pitch) := \left| \begin{array}{l} \text{for } i \in 0..18 \\ \text{for } j \in 0..18 \\ \left| \begin{array}{l} bsa_{i,j} \leftarrow NaN \text{ if } fw_{i,j} = 0 \\ bsa_{i,j} \leftarrow BSA(fw_{i,j}, Pitch_j) \text{ otherwise} \end{array} \right. \\ bsa \end{array} \right.$$

$BSA_{all} := BSA_{all}(FFacewidth, Pitch)$

$BSA1_{all}(fw, pitch) := \begin{cases} \text{for } i \in 0..18 \\ \quad \text{for } j \in 0..18 \\ \quad \quad \left| \begin{array}{l} bsa1_{i,j} \leftarrow \text{NaN if } fw_{i,j} = 0 \\ bsa1_{i,j} \leftarrow BSA1(fw_{i,j}, Pitch_j) \text{ otherwise} \end{array} \right. \\ \quad \quad \left| bsa1 \end{cases}$

$BSA1_{all} := BSA1_{all}(FFacewidth, Pitch)$

$SSA_{all}(fw, pitch) := \begin{cases} \text{for } i \in 0..18 \\ \quad \text{for } j \in 0..18 \\ \quad \quad \left| \begin{array}{l} ssa_{i,j} \leftarrow \text{NaN if } fw_{i,j} = 0 \\ ssa_{i,j} \leftarrow SSA(fw_{i,j}, Pitch_j) \text{ otherwise} \end{array} \right. \\ \quad \quad \left| ssa \end{cases}$

$SSA_{all} := SSA_{all}(FFacewidth, Pitch)$

$BHNW_{all}(fw, pitch) := \begin{cases} \text{for } i \in 0..18 \\ \quad \text{for } j \in 0..18 \\ \quad \quad \left| \begin{array}{l} bhnw_{i,j} \leftarrow \text{NaN if } fw_{i,j} = 0 \\ bhnw_{i,j} \leftarrow BHNW(fw_{i,j}, Pitch_j) \text{ otherwise} \end{array} \right. \\ \quad \quad \left| bhnw \end{cases}$

$BHNW_{all} := BHNW_{all}(FFacewidth, Pitch)$

$BHNWA_{all}(fw, pitch) := \begin{cases} \text{for } i \in 0..18 \\ \quad \text{for } j \in 0..18 \\ \quad \quad \left| \begin{array}{l} bhnwa_{i,j} \leftarrow \text{NaN if } fw_{i,j} = 0 \\ bhnwa_{i,j} \leftarrow BHNWA(fw_{i,j}, Pitch_j) \text{ otherwise} \end{array} \right. \\ \quad \quad \left| bhnwa \end{cases}$

$BHNWA_{all} := BHNWA_{all}(FFacewidth, Pitch)$

5.4 Results

The comparison are made in between Lewis and AGMA results for pinion and gear bending stresses, surface stresses and hardness numbers which are shown below.

Table 5.1 Pinion and Gear Bending Stress Comparison

Lewis Pinion Bending Stress	AGMA Pinion Bending Stress	Lewis Gear Bending Stress	AGMA Gear Bending Stress
501.559	1366	342.543	932.593
486.36	1324	332.163	904.333
472.056	1285	322.394	877.735
458.568	1248	313.182	852.657
445.83	1214	304.483	828.972
433.781	1181	296.254	806.567
422.366	1150	288.458	785.342
411.536	1120	281.061	765.205
401.247	1092	274.035	746.075
391.461	1066	267.351	727.878
382.14	1040	260.985	710.547
373.253	1016	254.916	694.023
364.77	993.107	249.122	678.25
356.664	971.038	243.586	663.177
348.911	949.929	238.291	648.76
341.487	929.717	233.221	634.957
334.373	910.348	228.362	621.729
327.549	891.77	223.702	609.04
320.998	873.934	219.228	596.86

Table 5.2 Surface Stress Comparison

Lewis Surface Stress	AGMA Surface Stress
17740	14240
17470	14020
17210	13810
16960	13620
16720	13420
16500	13240
16280	13070
16070	12900
15870	12740
15670	12580
15480	12430
15300	12280
15130	12140
14960	12010
14790	11880
14640	11750
14480	11630
14330	11510
14190	11390

Table 5.3 Hardness Number Comparison

Lewis Hardness Number	AGMA Hardness Number
69.345	60.598
68.668	60.054
68.021	59.535
67.402	59.038
66.809	58.562
66.24	58.105
65.694	57.667
65.169	57.245
64.663	56.84
64.177	56.449
63.707	56.072
63.255	55.709

Table 5.3 *Continued*

62.817	55.358
62.395	55.019
61.986	54.691
61.591	54.373
61.207	54.066
60.836	53.767
60.476	53.478

The following result graphs can help gear manufacturer in a real life to design spur gear.

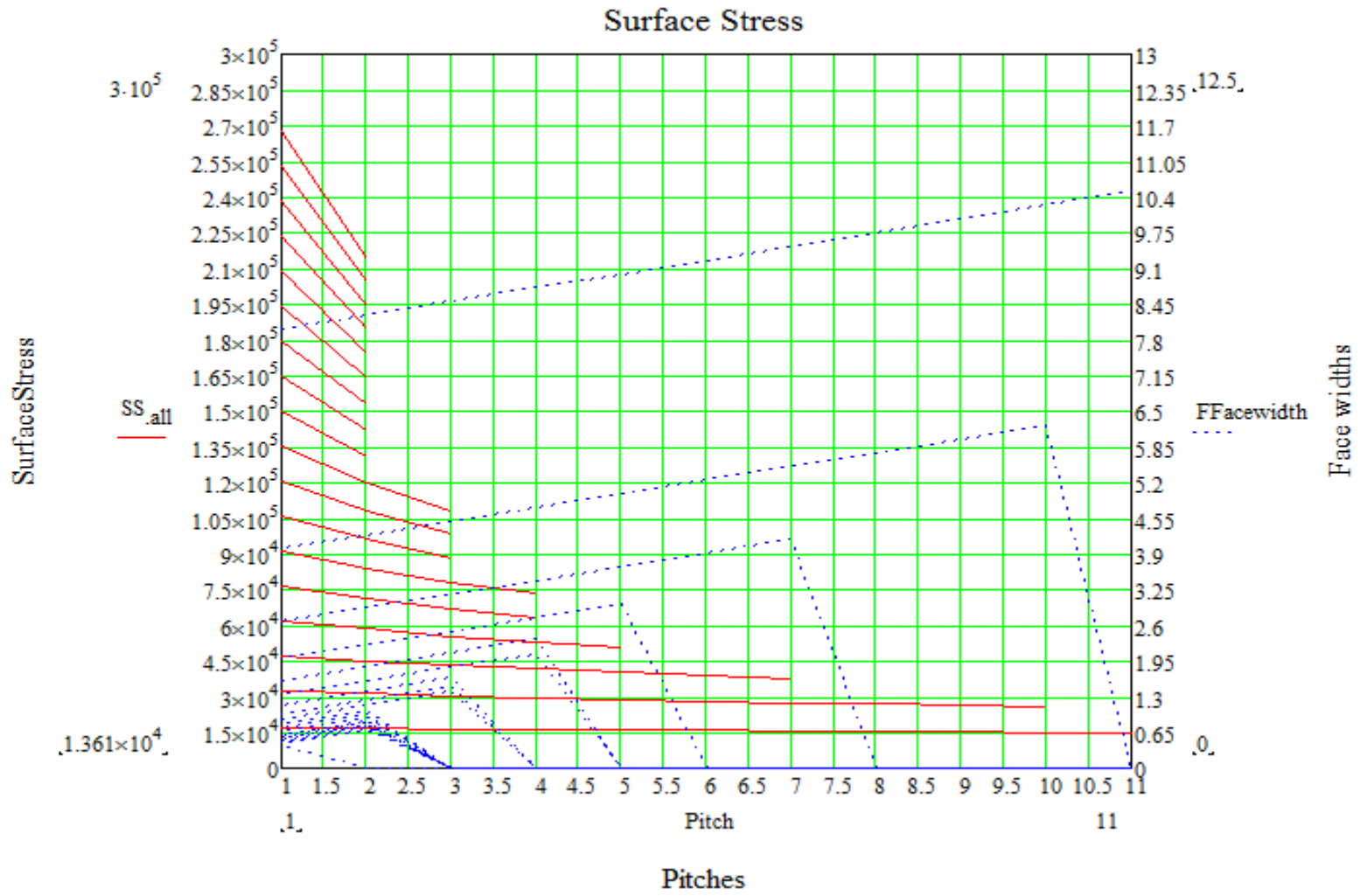


Figure 5.1 Hertz Surface Stress values for various pitches and face widths

AGMA Surface Stress

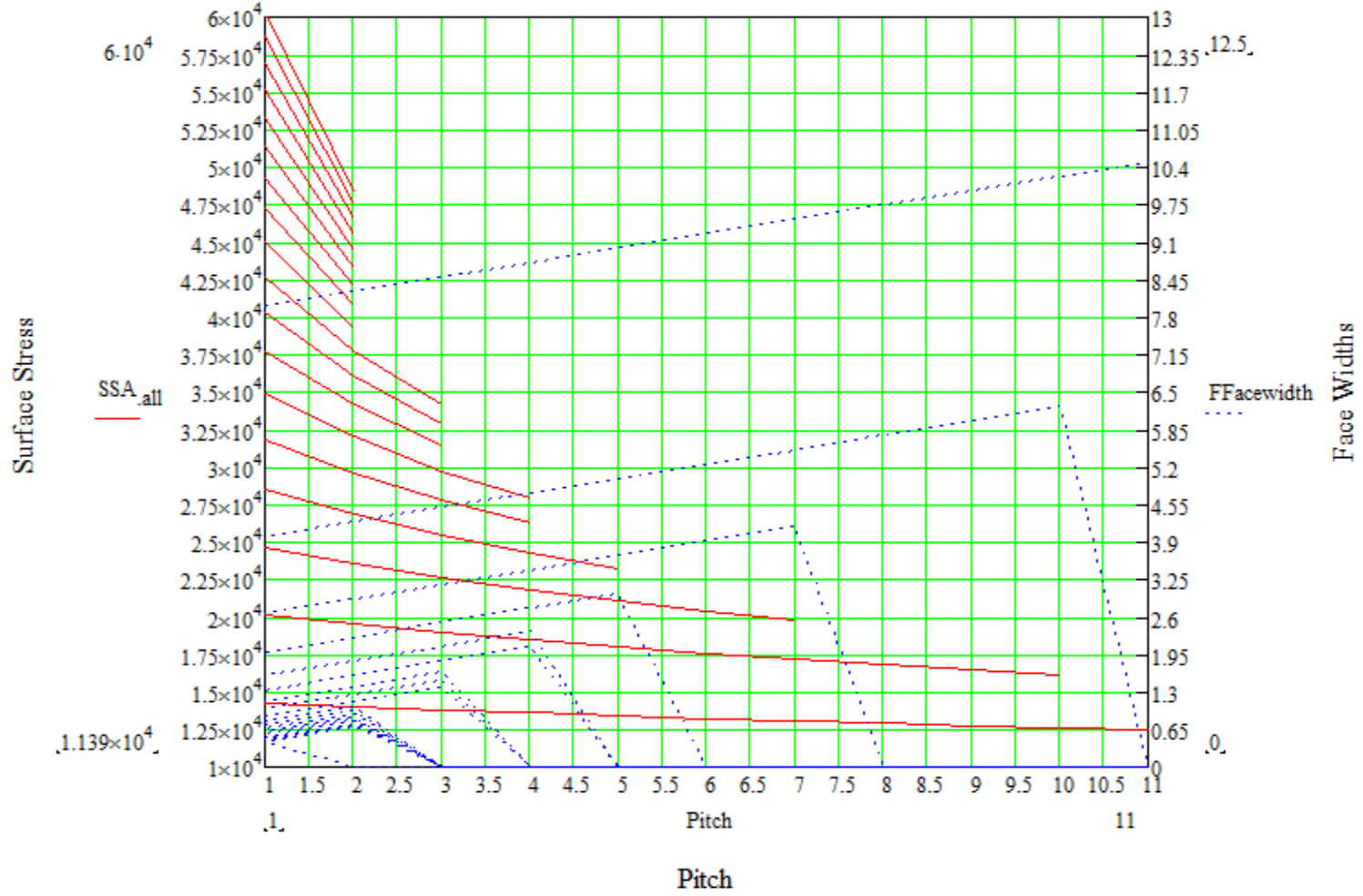


Figure 5.2 AGMA Surface Stress values for various pitches and face widths

Lewis Bending Stress

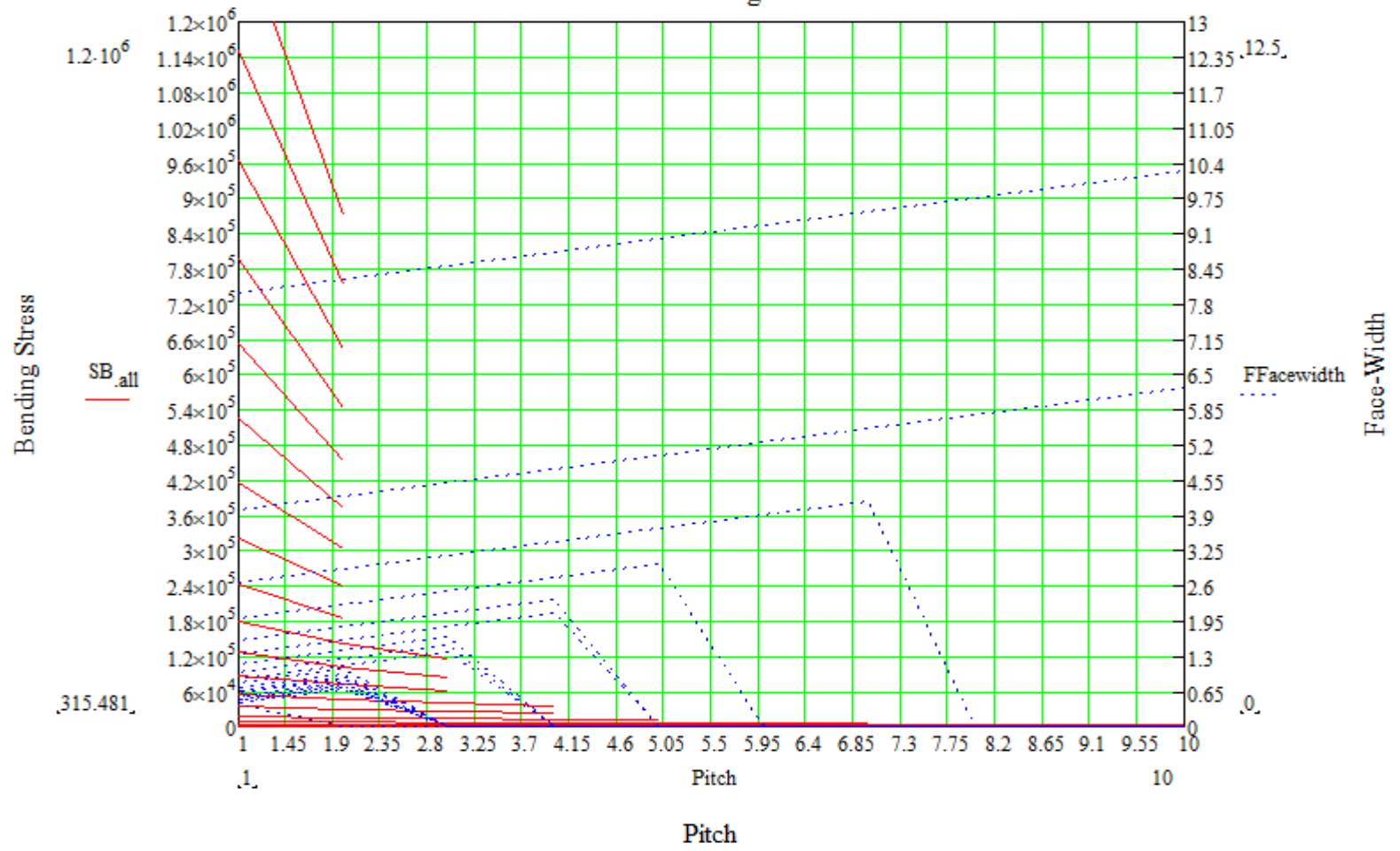


Figure 5.3 Lewis Pinion Bending Stress for various pitches and face widths

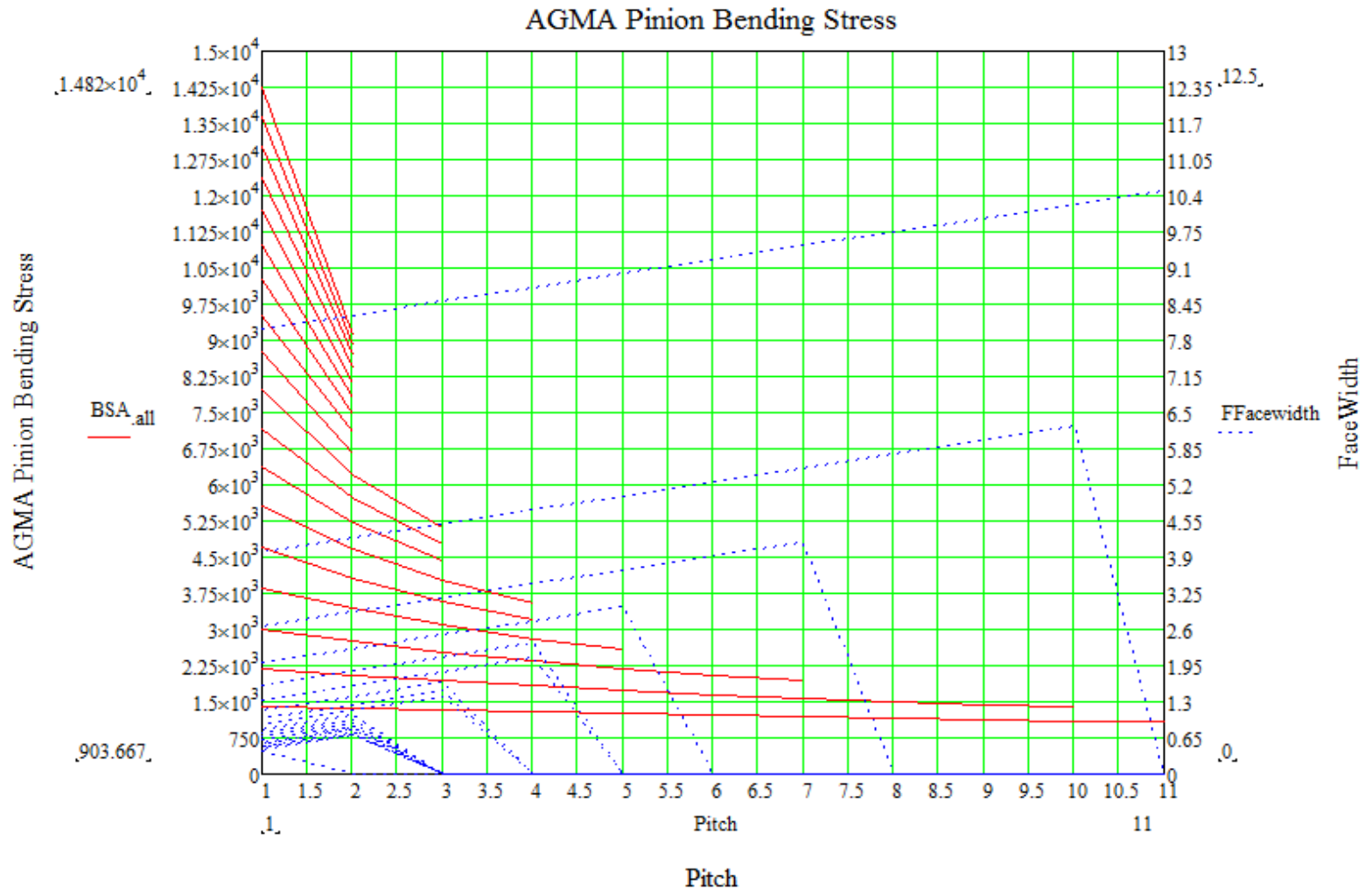


Figure 5.4 AGMA Pinion Bending Stress for Various Pitches and Face widths

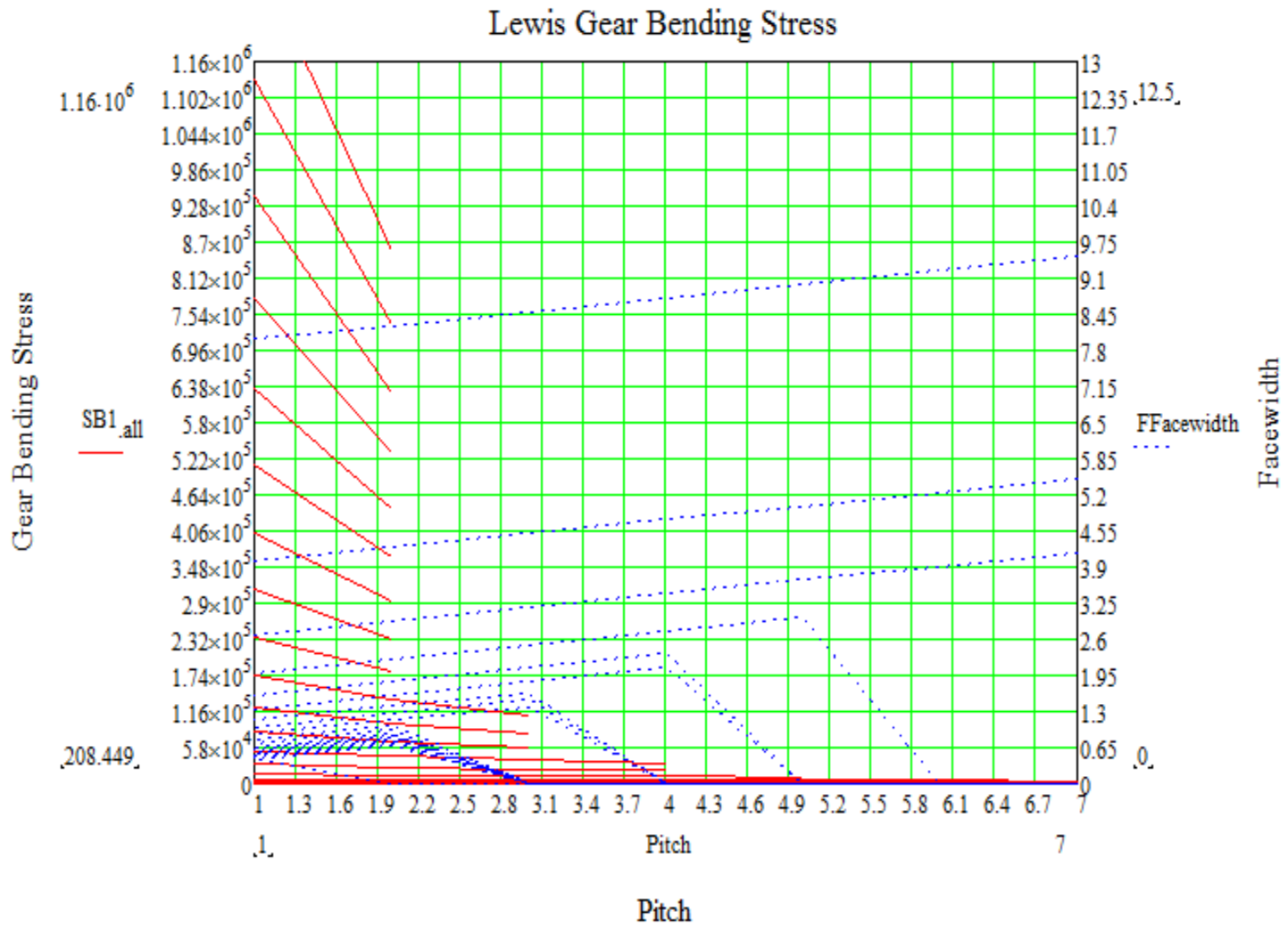


Figure 5.5 Lewis Gear Bending Stress for Various Pitches and Face Widths

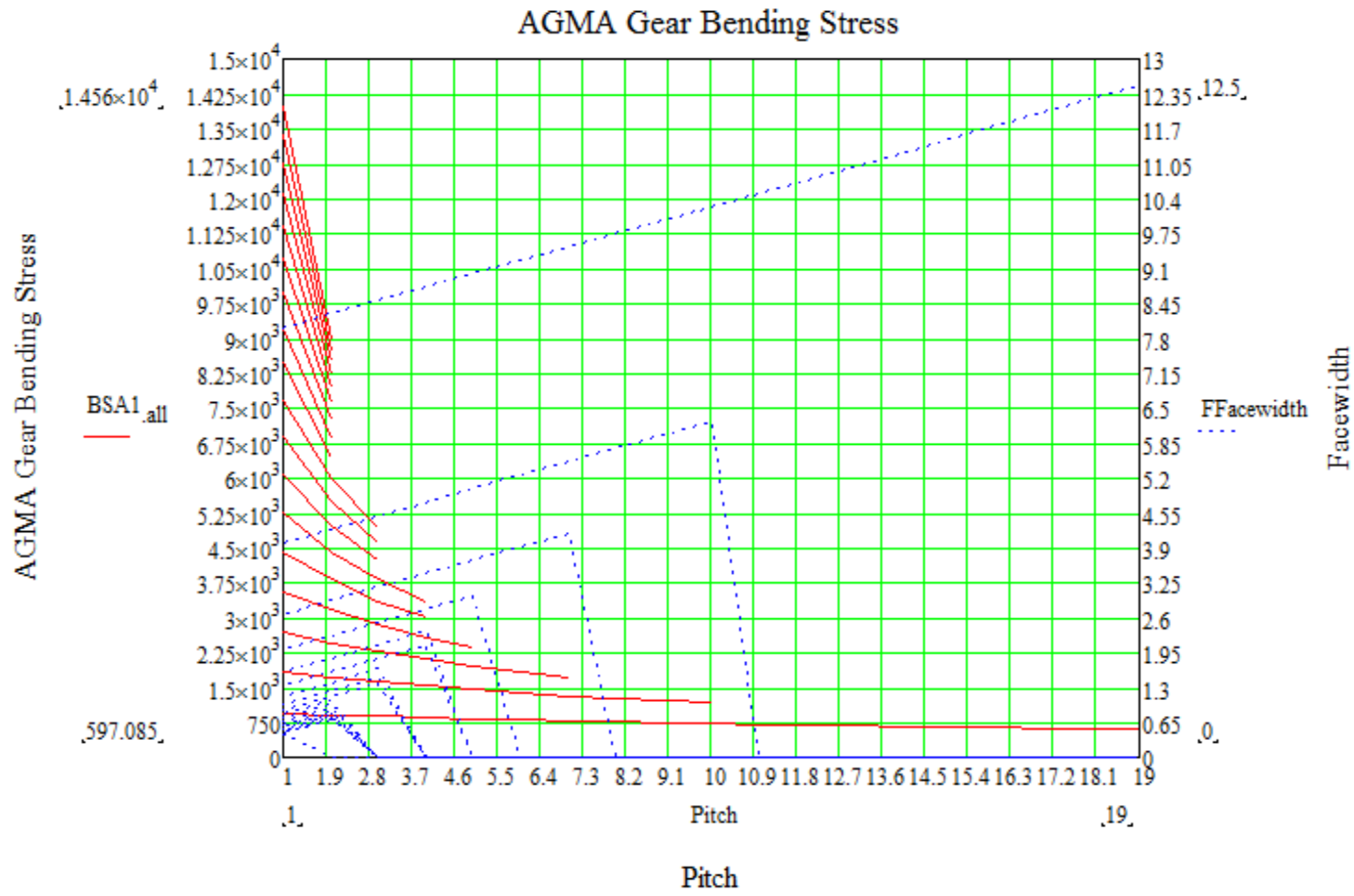


Figure 5.6 AGMA Gear Bending Stress Various Pitches and Face widths

5.5 Conclusion

From the above Mathcad results shows that Lewis method give the 19.72 percentage higher value than AGMA standards for surface stress. AGMA equation values from the Mathcad results for the pinion and gear bending stress gives 63.28 percentages than Lewis equations. For the hardness number, Lewis equation gives 12.61 percentage higher value than AGMA because it consider the maximum surface stress on the tooth while AGMA equation consider the maximum surface stress at the root of tooth. From the results, conclusion can be made that Lewis equations and form factor can give appreciable guidelines to manufacture gear although it has less accuracy than AGMA standards.

Mathcad objectives set in the beginning were met and facility for spur gear analysis is now available that is easy, fast and flexible. The single input Mathcad program can give the excellent results to evaluate gear nomenclatures one by one and also for different values of input can be easily compared for best results.

5.6 Future Work

This thesis work analyzed the spur gear's stress and hardness values. It is very ease now to compare with the AGMA standards for US gear manufacturer.

For the future work, more diameters can be included for spur gear. Helical gear can also analyze by Mathcad program and easy to compare with standards which can be useful for manufacturer.

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BIOGRAPHICAL INFORMATION

Jaydeep K Panchal received bachelor degree in Mechanical Engineering from the Charotar Institute of Technology, India. He did masters in mechanical engineering from the University of Texas, Arlington. He took the participation in F1 car racing project. He simulated 4 stroke and 6 stroke engine using Ricardo Wave software at UTA. He did the internship in Research In Motion at Irving as a Mechanical Design Engineer. He showed his active participation by designing speaker for the Blackberry tablet.