Gears

Objectives

1. Compute the forces exerted on gear teeth as they rotate and transmit power.
2. Use appropriate stress analyses to determine the relationships among the applied forces, the geometry of the gear teeth, the precision of the gear teeth and other factors specific to a given application, in order to make final decisions about those variables.
3. Describe various methods for manufacturing gears and levels of precision and quality to which they can be produced.
4. Describe suitable metallic materials from which to make the gears, in order to provide adequate performance for both strength and pitting resistance.
5. Use the Lewis Equation and the standards of the American Gear Manufacturers Association (AGMA) as the basis for completing the design of the gears.

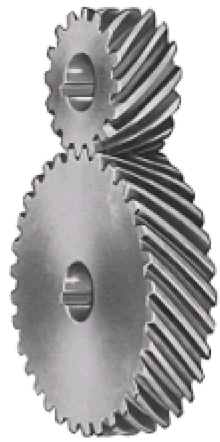
Types of Gear Pg xxx

Spur Gear

- Teeth is parallel to axis of rotation
- Can transmit power between parallel shaft
- The simplest form for gear

Helical gear

- Teeth is inclined to the axis of rotation
- Smoother than spur
- Develop thrust load (helix angle)
- Can transmit power from one shaft to a parallel and non-parallel shaft



Bevel gear

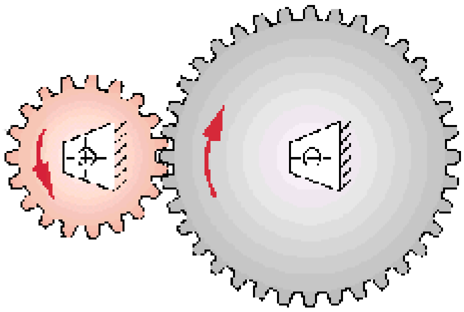
- Teeth on conical surfaces
- Transmit power between two intersecting shafts

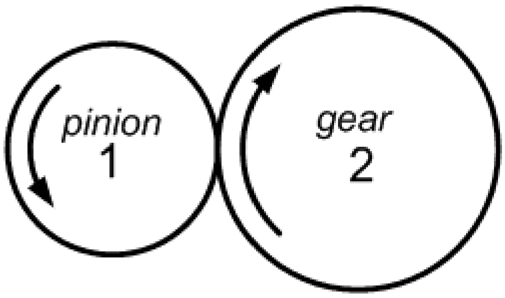
Worm gear

- Transmit power between two intersecting shafts
- Extremely low gear ratio

Pinion and Gear

According to the layman terminology, the pinion is the smaller gear. However, in the proper terminology, pinion is the driver and gear is driven. Although in most application, pinion is usually smaller than gear, it can be also be the larger of the two.





A pair of gears (pinion and gear) can be represented by 2 circles. The diameter of the two circles are based on the pitch circle diameters of the mating gears.

Pitch circle diameter, d

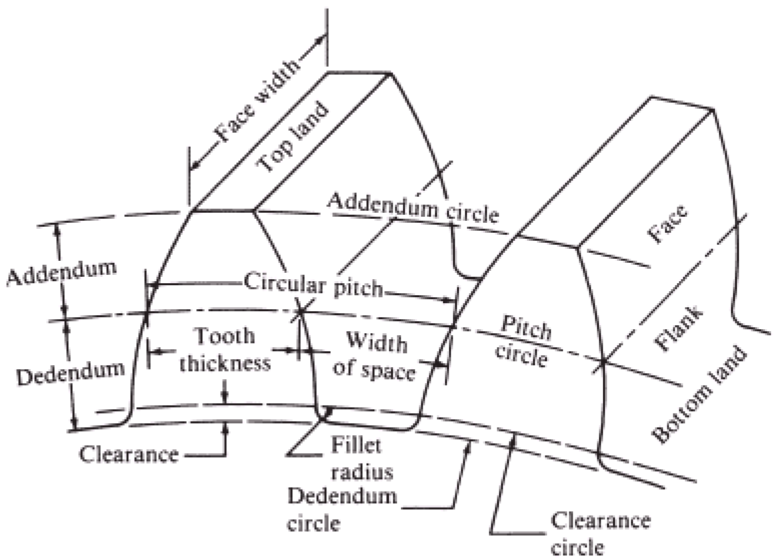
The pitch circle is the theoritical diameter upon which all gear dimensions are based. Similarly, almost all gear calculations are also based on the pitch circle diameter.

METRIC: $d = N\,m$ (mm)

IMPERIAL: $d = \dfrac{N}{P_d}$ (inch)

where N = number of gear teeth
P_d = diametral pitch (tooth per in)
m = module (mm)

"Mating gears must have same module (m) or diametral pitch P_d "



Facewidth : width of the gear tooth
addendum : distance between the top land to the pitch circle
dedundum : distance between the pitch circle to the bottom of the gear

Interference

To avoid interference, the minimum number of teeth for a one to one gear ratio is

$$N_p = \frac{4k}{6\sin^2(\phi)}\left[1 + \sqrt{1 + 3\sin^2(\phi)}\right]$$

where $k = 1.0$ for full-depth, $k = 0.8$ for stub teeth

ϕ = pressure angles (20^o, $22^1/_2{}^o$, 25^o)

If $m_G = \dfrac{N_G}{N_P} > 1$ indicating external gearset then

minimum number of teeth in the pinion,

$$N_P = \frac{2k}{(1 + 2m_G)\sin^2(\phi)}\left[m_G + \sqrt{m_G^2 + (1 + 2m_G)\sin^2(\phi)}\right]$$

to mate with the maximum number of teeth in the gear,

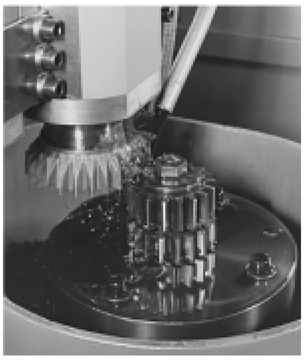
$$N_G = \frac{N_P^2\sin^2(\phi) - 4k^2}{4k - 2N_P\sin^2(\phi)}$$

If the above equation is not adhered to, then undercutting will occur.

Forming of Gear teeth

Machined by various methods including sand casting, shell molding, investment casting, die casting, etc.

Most popular methods –



- Form milling: mainly used for large gears. A milling cutter that has the shape of the tooth space is used. See diagram.
- Shaping: usually used for internal gears. Cutter used reciprocates on a vertical spindle. See diagram.
- Hobbing: similar process to milling except that both the workpiece and the cutter rotate in a coordinated manner.

Tooth Systems

The tooth system is a standard that specifies the relationships involving addendum, dedendum, working depth, tooth thickness, and pressure angle.

Refer to pg 676: Table 13-1 and Table 13-2 for spur gears specs and sizes.

Gear Train

For mating gears, $m_1 = m_2$

then

$$\frac{d_1}{d_2} = \frac{N_1}{N_2} = \frac{n_2}{n_1}$$

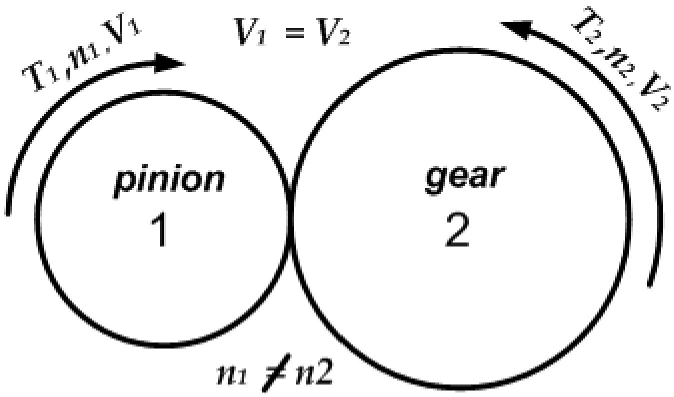
or

$$n_1 N_1 = n_2 N_2$$

$$V = \frac{\pi \cdot d\, n}{60}$$

where n is in rpm

The gear-train ratio is given by $e = \dfrac{nL}{nF} = \dfrac{rotational\_speed\_of\_last\_gear}{rotational\_speed\_of\_first\_gear}$



Gear-train ratio for a set of 2 gears

$$e = \frac{n_2}{n_1} = \frac{N_1}{N_2}$$

Idler gear

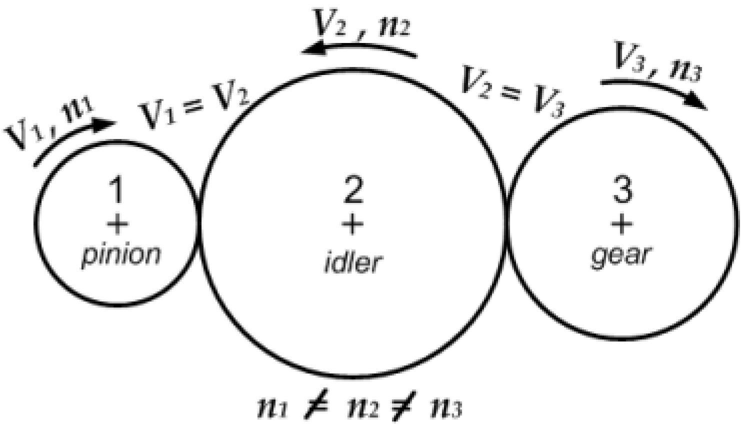
$$n_1 N_1 = n_2 N_2$$

$$n_2 = n_1 \frac{N_1}{N_2}$$

$$n_2 N_2 = n_3 N_3$$

$$n_3 = n_2 \frac{N_2}{N_3}$$

$$n_3 = n_1 \frac{N_1}{N_2}\frac{N_2}{N_3} = n_1 \frac{N_1}{N_3}$$

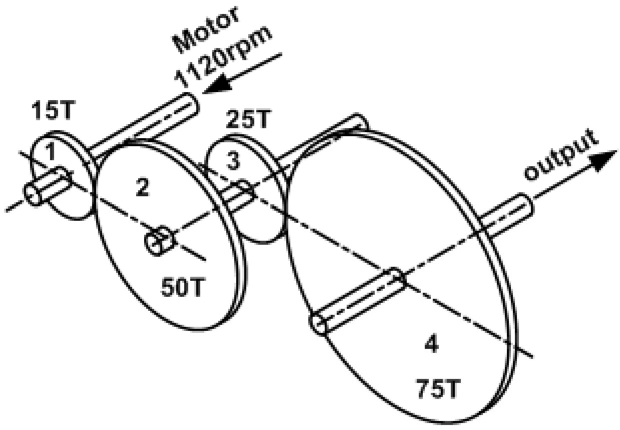


$$n_1 N_1 = n_3 N_3$$

For computing gear train with an idler gear,

$$e = \frac{n_3}{n_1}$$ (both gear rotating same direction)

Multi gearset system



The *gear-train ratio* for the first gearset

$$e_1 = \frac{N_1}{N_2} = \frac{15}{50} = \frac{n_2}{n_1} = \frac{n_2}{1120}$$

$$n_2 = 1120\,\frac{15}{50} = 336\,rpm$$

Speed $n_3 = n_2 = 336\,rpm$

The *gear-train ratio* for the 2nd gearset

$$e_2 = \frac{N_3}{N_4} = \frac{25}{75} = \frac{n_4}{n_3} = \frac{n_4}{336}$$

$$n_4 = 336\,\frac{25}{75} = 112\,rpm$$

The *gear-train ratio* for the multi-gearset system:

$$e = \frac{Pdt\_of\_driving\_gears}{Pdt\_of\_driven\_gears} = \frac{N_1 \times N_3}{N_2 \times N_4}$$

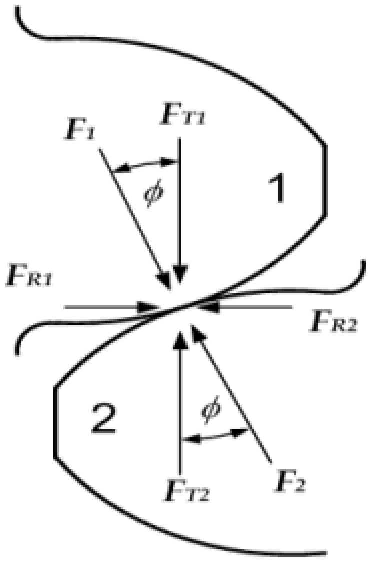
where also $e = \dfrac{nL}{nF} = \dfrac{n_4}{n_1}$

Example: See diagram above $e = \dfrac{N_1 \times N_3}{N_2 \times N_4} = \dfrac{15 \times 25}{50 \times 75} = \dfrac{1}{10}$

$$e = \frac{n_4}{n_1} = \frac{n_4}{1120}$$

Final shaft output speed $n_4 = \dfrac{1}{10} \times 1120 = 112\,rpm$

Gear teeth Force analysis



The pinion tooth 1 applied force F_1 against the gear teeth 2 at an angle equal to pressure angle. For equilibrium, gear teeth 2 reacts with equal force F_2 against pinion teeth 1 and so

$$F_1 = F_2$$

The force F_{R1} and F_{T1} is the components of F_1 . The radial component, F_{R1} acts toward the center of the gear causing bending stress on the gear shaft. F_{T1} which is the tangential component of F_1 is the force which turn the gears.

$$F_{R1} = F_{T1} \cdot \tan(\phi)$$

$$F_{T1} = W_t = \frac{H}{V}$$ where *Wt* is the tangential force

Torque at shaft 1 $$T_1 = W_t \cdot \left(\frac{d_1}{2}\right)$$ *(Nm, lbf-in)*

METRICS

$$V = \frac{\pi \cdot d \cdot n}{60(1000)}$$ *(m/s) ; d (mm), n (rpm)*

$$H = W_t \cdot V = \frac{2\pi n T}{60}$$ *(watts) ; V (m/s), T (Nm)*

$$T = \frac{30H}{\pi n}$$ *(Nm) ; H (watts), Wt (N), n (rpm)*

IMPERIAL

d (in), n (rpm) $$V = \frac{\pi d(\text{in})\, n(\text{rpm})}{12}$$ *(fpm = feet per min)*

V (ft/min), T (lbf-in) $$H = \frac{W_t(\text{lbf}) \cdot V}{33000}$$ *(hp)*

H (hp), W_t (lbf) $$T = \frac{63025H}{n}$$ *(lbf-in)*

Lewis Bending Equations

Dynamic effects are present when a pair of gear rotate at moderate and high speed.
These effects are expressed by using the Velocity factor K_V pg 727 . Unit V meter/sec

Unit : V ft/min or m/sec

IMPERIAL	METRICS	
$K_v = \frac{600+V}{600}$	$K_v = \frac{3.05+V}{3.05}$	cast iron, cast profile
$K_v = \frac{1200+V}{1200}$	$K_v = \frac{6.1+V}{6.1}$	cut or milled profile
$K_v = \frac{50+\sqrt{V}}{50}$	$K_v = \frac{3.56+\sqrt{V}}{3.56}$	hobbed or shaped profile
$K_v = \sqrt{\frac{78+\sqrt{V}}{78}}$	$K_v = \sqrt{\frac{5.56+\sqrt{V}}{5.56}}$	shaved or ground profile

Bending stress

Allowable Bending stress number $\sigma = \frac{K_v W_t P_d}{F\,Y}$ (imperial) $\sigma = \frac{K_v W_t}{F\,m\,Y}$

$$s_t < s_{at}\frac{Y_n}{SF\,K_r}$$

Contact stress number $s_c = C_p\sqrt{\frac{W_t K_o K_s K_m K_v}{F\,d_p\,I}}$

$$s_c < s_{ac}\left[\frac{Z_n C_H}{(SF)K_R}\right]$$

AGMA Stress Equations

Through Hardening

- AISI 1020, 1040, 1050, 3140, 4140, 4150, 4340, 6150, 8650
- not recommended for HB > 400

Case Hardening

- use flame hardening, induction hardening, carburizing and nitriding on surface
- available steel grade 1, grade 2 and grade 3 (see Table 9-3)

Gear quality

1. Composite variation: allowable amounts of variations of the actual tooth form from the actual tooth form
2. Specified by AGMA as quality numbers.
3. Quality numbers range from 5 to 15 with increasing precision.

Bending stress

(Imperial units)

Bending stress number

$$s_t = \frac{W_t P_d}{F\, J} K_o K_s K_m K_B K_v$$

$$s_t < s_{at} \frac{Y_n}{SF\, K_r}$$

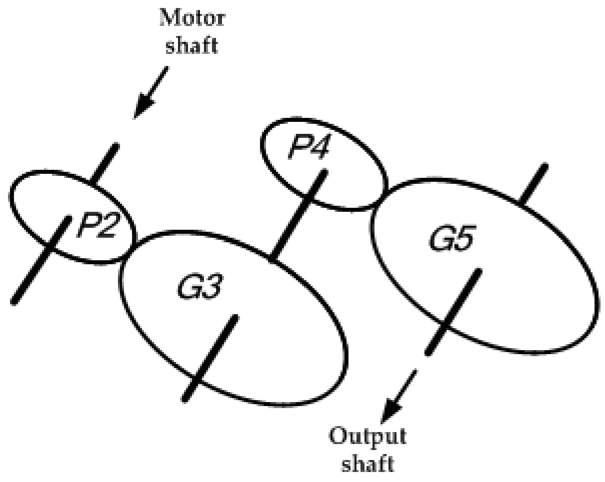
Contact stress number

$$s_c = C_p \sqrt{\frac{W_t K_o K_s K_m K_v}{F\, d_p\, I}}$$

$$s_c < s_{ac} \left[\frac{Z_n C_H}{(SF) K_R} \right]$$

Sample 1

You are required to design a gearbox for 2 level speed reduction. At both level there are 18T gear as shown in diagram below.



Propose 2 gear combinations for Gear 3 and 5 if the total reduction ratio is 24. You are constrained to a maximum speed ratio not exceeding 10 at each gearset. Your answers must be tabulated in the format given below.

	Jmlh gigi G3	Jmlh gigi G5
Gabungan 1	________	________
Gabungan 2	________	________

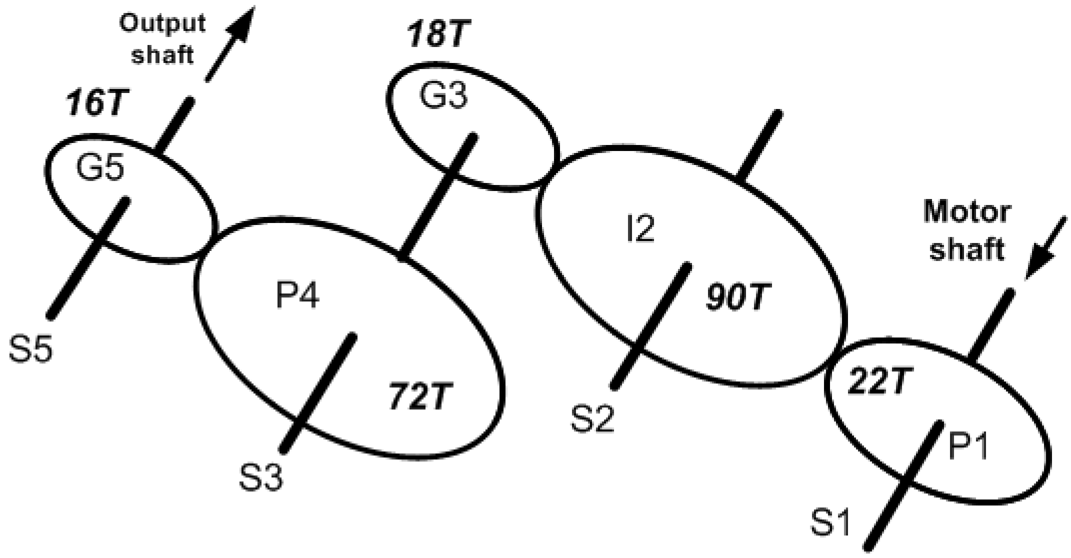
Sample 2

A motor producing 8kW at 1000 rpm clockwise is used to power a drilling machine via a gearset of 5 gears as shown in diagram below. If all gear have module size (m = 4mm) and pressure angle 20^o.

i. Calculate the drilling machine rotational speed in rpm.

ii. Calculate the torque *T* and speed (RPM) acting on gear 3 and 5.

iii. Calculate and draw the forces *F*, *Fr* and *Ft* acting on gear teeth.



Sample 1

Given $e = \frac{1}{24}$ (because 24X speed reduction in the gear set)

$$e = \frac{1}{24} = \frac{Np1 \times Np3}{Ng2 \times Ng4} = \frac{Np1}{Ng2} \times \frac{Np3}{Ng4} = \frac{1}{6} \times \frac{1}{4} = \frac{1}{8} \times \frac{1}{3} = \frac{1}{2} \times \frac{1}{12}$$

	G3	G4		
C1	108	72	ratio (1:6 & 1:4)	or vice-versa
C2	144	54	ratio (1:8 & 1:3)	or vice-versa
C3	162	48	ratio (1:12 & 1:2)	(wrong! why ???)

Sample 2

i)

$$e = \frac{Np1 \times Np4}{Ng3 \times Ng5}$$

$$e = \frac{22 \times 72}{18 \times 16} = 5.5$$

Rotation at input shaft: $n_1 = 1000rpm$

Output rotational speed: $n_5 = e\,n_1$ $n_5 = 5500\,rpm$

ii) **Analysis gear 1**

$N_1 = 22$ $\phi = 20\ deg$ $H = 8\ kW$ $m_o = 4\ mm$

$d_1 = N_1 m_o$ $d_1 = 88\,mm$

$$W_{t1} = \frac{60H}{\pi d_1 n_1} \qquad W_{t1} = 1.736 \times 10^3\,N$$

$$T_1 = W_{t1}\frac{d_1}{2} \qquad T_1 = 76.394\,N\ m$$

Analysis gear 3 (Mating gear 1&2&3)

$V_3 = V_2 = V_1$ $W_{t3} = W_{t1}$ $W_{t3} = 1.736 \times 10^3\,N$

$N_3 = 18$ $d_3 = N_3 m_o$ $d_3 = 72\,mm$

$T_3 = W_{t3} \frac{d_3}{2}$ $\quad T_3 = 62.504\,N\ m$

$n_3 = n_1 \frac{N_1}{N_3}$ $\quad n_3 = 1.222 \times 10^3\ rpm$

$F_{t3} = W_{t3}$ $\quad F_{t3} = 1.736 \times 10^3\ N$

$F_{r3} = F_{t3} \tan(\phi)$ $\quad F_{r3} = 631.938\,N$

$F_3 = \frac{F_{t3}}{\cos(\phi)}$ $\quad F_3 = 1.848 \times 10^3\ N$

speed $n_3 = 1.222 \times 10^3\ rpm$ torque $T_3 = 62.504\,N\ m$

force acting on teeth : $F_3 = 1847.7\,N$ $\quad F_{t3} = 1736.2\,N$ $\quad F_{r3} = 631.9\,N$

Analysis gear 4

$T_4 = T_3$ $\quad T_4 = 62.504\,N\ m$

$N_4 = 72$ $\quad d_4 = m_o N_4$ $\quad d_4 = 288\,mm$

$W_{t4} = \frac{2T_4}{d_4}$ $\quad W_{t4} = 434.059$

Analysis gear 5

$W_{t5} = W_{t4}$ $\quad N_5 = 16$ $\quad W_{t5} = 434.059$ $\quad F_{t5} = W_{t5}$

$T_5 = W_{t5} \frac{m_o N_5}{2}$ $\quad T_5 = 13.89\,N\ m$

$F_5 = \frac{W_{t5}}{\cos(\phi)}$ $\quad F_5 = 461.916\,N$

$F_{r5} = W_{t5} \tan(\phi)$ $\quad F_{r5} = 157.985\,N$

speed $n_5 = 5.5 \times 10^3\ rpm$ torque $T_5 = 13.89\,N\,m$

force acting on teeth :

$F_5 = 461.916\,N$ $\quad F_{t5} = 434.059\,N$ $\quad F_{r5} = 157.985\,N$