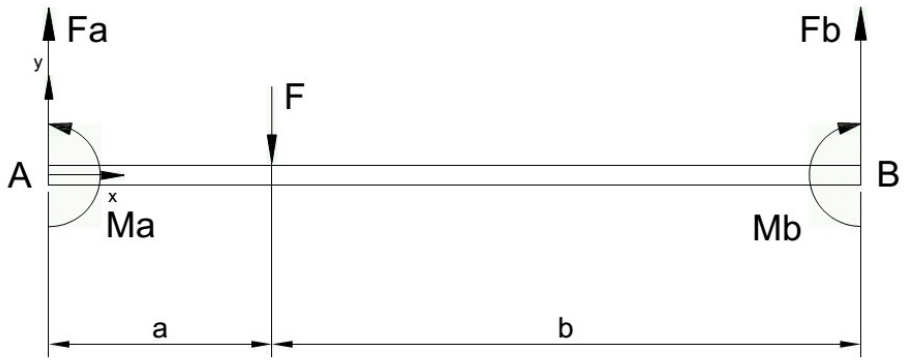
# Deflection of a Beam (both clamping points are firmly clamped):

Given:

$F := 16\ kN$ $E := 2.1 \cdot 10^5\ N \cdot mm^{-2}$ $I := 103\ cm^4$ $W_b := 25.8\ cm^3$ $a := 260\ mm$ $b := 540\ mm$

$l := a + b = 800\ mm$ $Nm := 1\ N \cdot m$ $kNm := 1\ kN \cdot m$ $F := 16\ kN$




$$F_a := F\ \frac{b^2\ (3\ a + b)}{(a+b)^3} = 12028.5\ N \qquad F_b := F \cdot \left(1 - \frac{b^2\ (3\ a + b)}{(a+b)^3}\right) = 3971.5\ N$$

$$M_a := F \cdot a\ \frac{b^2}{(a+b)^2} = 1895.4\ Nm \qquad M_b := F \cdot b\ \frac{a^2}{(a+b)^2} = 912.6\ Nm$$

$M_{bl}(x) = M_a - F_a \cdot x$ left Moment of bending

$M_{br}(x) = M_b - F_b \cdot (l - x)$ right Moment of bending

at sektion $a$ there is:

$-F_a \cdot a + M_a = -M_b + F_b \cdot b \qquad (I)$

$y''(x) = -\frac{1}{E \cdot I} \cdot M_b(x)$

$F = F_a + F_b \qquad (II)$

$-F_a \cdot a + M_a = -M_b + (F - F_a) \cdot b$ Equation $(I)$ in $(II)$

$M_b = a \cdot F_a - M_a + b \cdot (F - F_a)$

Solving the deflection:

Lösungswerte
Nebenbedingung
Gleichungslöser

$y_1(0\ mm) = 0\ mm \qquad y_1'(0\ mm) = 0$

$y_1''(x) = -\frac{1}{E \cdot I} \cdot (M_a - F_a \cdot x)$

$y_1 := \text{odesolve}\ (y_1(x), a)$

Lösungswerte
Nebenbedingung
Gleichungslöser

$y_2(a) = y_1(a) \qquad y_2(l) = 0\ mm$

$y_2''(x) = -\frac{1}{E \cdot I} \cdot (M_b - F_b \cdot (l - x))$

$y_2 := \text{odesolve}\ (y_2(x), l)$

## Diagrams of deflection, Bending Moment and shear Force :

$$y(x) := \begin{cases} \text{if } 0 \le x \le a \\ \quad y_1(x) \\ \text{if } a < x \le l \\ \quad y_2(x) \end{cases}$$

$$M_b(x) := -y''(x) \cdot E \cdot I$$

$$F_Q(x) := M_b'(x)$$

$x := 0,1\ mm \,..\, l$ $\quad x_{ymax} = 340.426\ mm$ $\quad y_{max} := y(x_{ymax}) = -0.149\ mm$

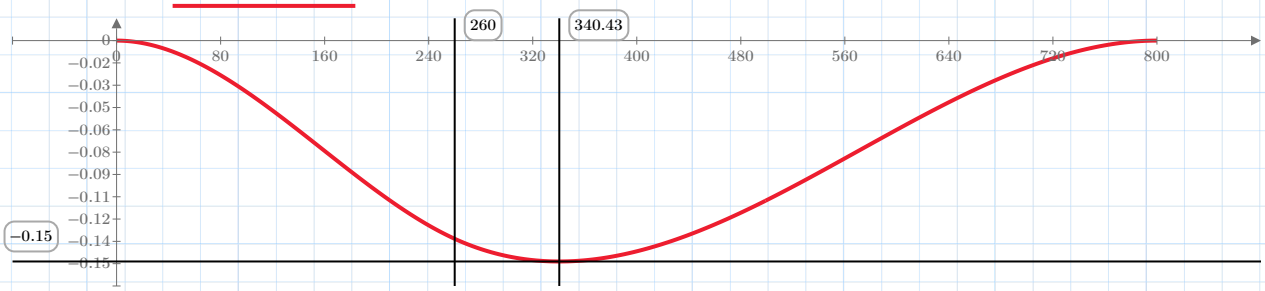
Gleichungslösen und Schätzwerte

$x := 450\ mm$

$y'(x) = 0$

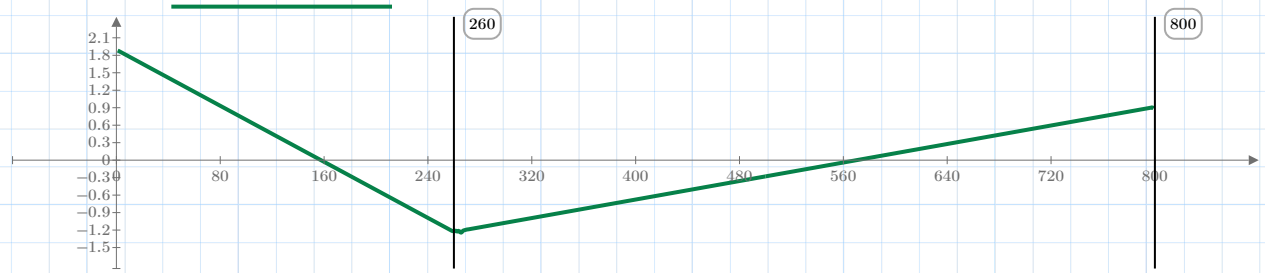
$x_{ymax} := \text{find}(x)$

$y(x)\ \ (mm)$



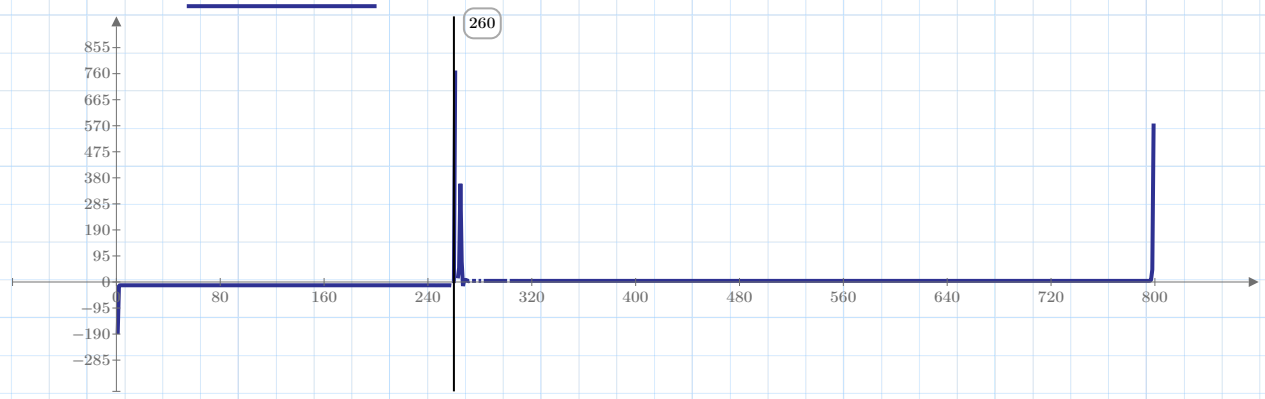

$x\ \ (mm)$

$M_b(x)\ \ (kNm)$




$x\ \ (mm)$

$F_Q(x)\ \ (kN)$




$x\ \ (mm)$