Deflection of a Beam (both clamping points are firmly clamped):
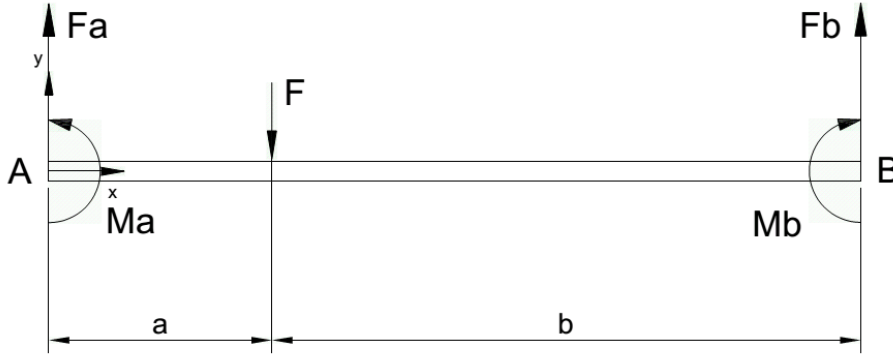
Given:

$$F := 16 \ kN \qquad E := 2.1 \cdot 10^5 \ N \cdot mm^{-2} \qquad I := 103 \ cm^4 \qquad W_b := 25.8 \ cm^3 \qquad a := 260 \ mm \qquad b := 540 \ mm$$

$$l := a + b = 800 \ mm \qquad Nm := 1 \ N \cdot m \qquad kNm := 1 \ kN \cdot m \qquad F := 16 \ kN$$

$$F_a := F \ \frac{b^2 \ (3 \ a + b)}{(a+b)^3} = 12028.5 \ N \qquad\qquad F_b := F \cdot \left(1 - \frac{b^2 \ (3 \ a + b)}{(a+b)^3}\right) = 3971.5 \ N$$

$$M_a := F \cdot a \ \frac{b^2}{(a+b)^2} = 1895.4 \ Nm \qquad\qquad M_b := F \cdot b \ \frac{a^2}{(a+b)^2} = 912.6 \ Nm$$

$$M_{bl}(x) = M_a - F_a \cdot x \qquad \text{left Moment} \qquad\qquad M_{br}(x) = M_b - F_b \cdot (l - x) \qquad\qquad \text{right Moment}$$
$$\text{of bending} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{of bending}$$

at sektion a there is: $\qquad\qquad\qquad\qquad\qquad -F_a \cdot a + M_a = -M_b + F_b \cdot b \qquad (I)$

$$y''(x) = -\frac{1}{E \cdot I} \cdot M_b(x) \qquad\qquad\qquad F = F_a + F_b \qquad (II)$$

$$-F_a \cdot a + M_a = -M_b + (F - F_a) \cdot b \qquad \text{Equation } (I) \text{ in } (II)$$

$$M_b = a \cdot F_a - M_a + b \cdot (F - F_a)$$

Solving the deflection:

$$y_1(0 \ mm) = 0 \ mm \quad y_1'(0 \ mm) = 0$$

$$y_1''(x) = -\frac{1}{E \cdot I} \cdot (M_a - F_a \cdot x)$$

$$y_1 := \text{odesolve}\,(y_1(x), a)$$

$$y_2(a) = y_1(a) \qquad y_2(l) = 0 \ mm$$

$$y_2''(x) = -\frac{1}{E \cdot I} \cdot (M_b - F_b \cdot (l - x))$$

$$y_2 := \text{odesolve}\,(y_2(x), l)$$

Diagrams of deflection, Bending Moment and shear Force :

$$y(x) := \begin{Vmatrix} \text{if } 0 \leq x \leq a \\ \quad \begin{Vmatrix} y_1(x) \end{Vmatrix} \\ \text{if } a < x \leq l \\ \quad \begin{Vmatrix} y_2(x) \end{Vmatrix} \end{Vmatrix}$$

$$M_b(x) := -y''(x) \cdot E \cdot I$$

$$F_Q(x) := M_b{}'(x)$$

$x := 450 \ mm$

$y'(x) = 0$

$x_{ymax} := \text{find}(x)$

$x := 0, 1 \ mm \, .. \, l \qquad x_{ymax} = 340.426 \ mm \qquad y_{max} := y(x_{ymax}) = -0.149 \ mm$

$y(x) \ (mm)$

$x \ (mm)$

$M_b(x) \ (kNm)$

$x \ (mm)$

$F_Q(x) \ (kN)$

$x \ (mm)$