Computer Aided Methods for Engineers (EN0718)

Bit Error Rate (BER) Analysis of an Optical Communication System

Introduction

The bit error rate (BER) of a digital communications system is defined as the ratio of the number of errors in a given time interval to the number of bits in that time interval and is an important performance parameter of any digital communications system. From this definition it is clear that the BER is a measure of the probability that any given bit will be received in error. If, for example, the maximum bit error rate specified for a system is 10^{-9} then the receiver may produce a maximum of 1 error in every 10^{9} bits of information received. The BER depends primarily on the signal to noise ratio (SNR) of the received signal which itself is a function of the transmitted signal power, the attenuation of the system and the receiver noise.

Theory

In a digital communications system the receiver samples the signal at the centre of each bit period. A binary 1 or 0 is then recorded according to whether the signal is greater or less than some threshold current or voltage. In an ideal system a 0 transmitted would result in a 0 being received and a 1 transmitted would result in a 1 being received. In a real system, however, the presence of noise can result in errors being recorded.

The bit error rate (BER) is defined by

$$BER = \frac{N_e}{N_t} \tag{1}$$

where N_e is the number of errors occurring in time t and N_t is the number of bits arriving in time t. Hence the BER is simply the probability that an error will occur in a given bit period. To determine the probability of error we need to know the probability distribution functions (PDFs) of the 0 and 1 levels of the received signal. The probability distribution functions of the 0 and 1 levels may be different since different noise mechanisms may dominate the two regimes.

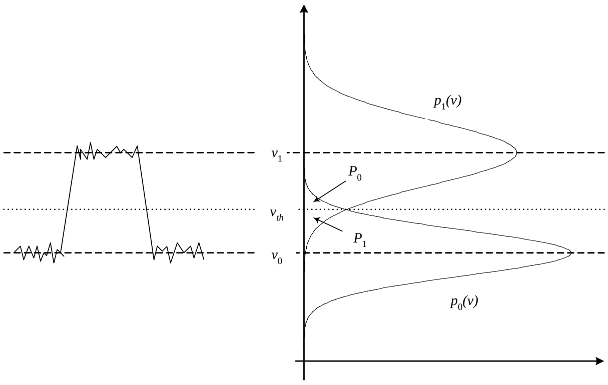

Figure 1

In many instances, however, and when thermal noise dominates, the probability distribution functions may be identical for the 0 and 1 levels. Figure 1 illustrates the PDFs $p_0(v)$ and $p_1(v)$ for the 0 and 1 levels respectively in the presence of random (Gaussian) noise.

The probability that a 0 signal is recorded in error as a 1 is given by

$$P_0 = \int_{v_{th}}^{\infty} p_0(v).dv \tag{2}$$

whilst the probability that a 1 signal is recorded in error as a 0 is given by

$$P_1 = \int_{-\infty}^{v_{th}} p_1(v).dv \tag{3}$$

The total probability of error P_e (i.e. the BER) is then given by

$$BER = P_e = a_1 P_1 + a_0 P_0 \tag{4}$$

where a_0 and a_1 are the relative probabilities of transmission of 0s and 1s respectively and

$$a_0 + a_1 = 1 \tag{5}$$

For unbiased data we have

$$a_0 = a_1 = 0.5 \tag{6}$$

In order to determine the probability of error P_e we need to know the PDFs p_0 and p_1. An exact calculation for real receivers can be difficult but if we approximate the noise voltage fluctuations as Gaussian random variables then we can illustrate the process and obtain a useful approximation for many types of noise.

Assume that the sampled, signal voltage output at the receiver may be represented by Gaussian functions with mean v_i and variance σ_i^2 where i = 0 or 1 for the 0s and 1s respectively.

Then

$$p_i(v) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left[-\frac{(v - v_i)^2}{2\sigma_i^2}\right] \tag{7}$$

so that the probabilities of errors are given by

$$P_0 = \frac{1}{\sqrt{2\pi\sigma_0^2}} \int_{v_{th}}^{\infty} \exp\left[-\frac{(v - v_0)^2}{2\sigma_0^2}\right].dv \tag{8}$$

and

$$P_1 = \frac{1}{\sqrt{2\pi\sigma_1^2}} \int_{-\infty}^{v_{th}} \exp\left[-\frac{(v - v_1)^2}{2\sigma_1^2}\right].dv \tag{9}$$

In many cases it is possible to assume that the rms noise voltages for the 0 and 1 levels are the same and hence that the distributions have the same variance. If the threshold level is set at the intersection of the signal PDFs then $P_0 = P_1$ and for an unbiased data stream with $a_0 = a_1 = 0.5$ we have

$$BER = P_e = P_1 = P_0 \qquad (10)$$

Taking $P_e = P_0$ and making the substitution $y = \frac{v - v_0}{\sqrt{2}\sigma_0}$ into (8) gives

$$BER = P_e = \frac{1}{\sqrt{\pi}} \int_{Q/\sqrt{2}}^{\infty} \exp\left(-y^2\right).dy \qquad (11)$$

where

$$Q = \frac{v_{th} - v_0}{\sigma_0} \qquad (12)$$

We can now make use of the well known error function

$$erf\left(x\right) = \frac{2}{\sqrt{\pi}} \int_0^x \exp\left(-y^2\right).dy \qquad (13)$$

and the complementary error function

$$erfc\left(x\right) = 1 - erf\left(x\right) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp\left(-y^2\right).dy \qquad (14)$$

Hence substituting into (11) we have

$$BER = \frac{1}{2}\left[1 - erf\left(Q/\sqrt{2}\right)\right] \qquad (15)$$

If the Q-factor, which is defined by the separation between the threshold level and the mean 0 and 1 signal levels in terms of the signal variance or rms noise, is known it is possible to determine the bit error rate from (15).

Assignment

You are required to produce a series of carefully documented MathCAD programmes to investigate the bit error rate of an optical communication system

MathCAD 13 is available to students using the computers in rooms E301 and E304 in Ellison Building.

(1) Assuming that the PDFs for the 0 and 1 levels are identical, produce a MathCAD programme that will graph the bit error rate as a function of the Q-factor for values of Q from 0 to 8.

[20 marks]

(2) Develop your programme to determine the bit error rate for a communication system for which the PDFs of the 0 and 1 levels are different, assuming that both are known in terms of

their mean and variance. Test your programme by determining the BER for a communication system for which the following data is known:

(i) $v_1 = 1.10$ volt

(ii) $\sigma_1 = 0.08$ volt

(iii) $v_0 = 0.20$ volt

(iv) $\sigma_0 = 0.09$ volt

(v) $a_1 = 0.55$

(vi) $a_0 = 0.45$

[Hint: You will need to establish the value of the threshold voltage v_{th} from the intersection of the PDFs p_0 and p_1.]

[20 marks]

(3) The Excel data file "EN0718 BER Simulation Data", available on Blackboard, contains a series of data sets for measurements taken from an optical communication system employing both an LED and a laser as the transmitter. Using a suitable algorithm, use MathCAD to determine the values of v_1, v_0, σ_1 and σ_0 for each data set and hence the BER to be expected in each case.

[60 marks]

Bibliography

Davis, C.C. (2000) *Lasers and electro-optics: fundamentals and engineering.* CUP. ISBN 0-521-48403-0.

Senior, J.M. (1992) *Optical fibre communications: principles and practice.* 2nd Ed. Prentice Hall. ISBN 0-13-635426-2.

Sibley, M.J.N. (1995) *Optical communications*. 2nd Ed. MacMillan. ISBN 0-333-61792-4.

Submission and Assessment

You are required to submit all of the following:

(1) A clearly documented copy of your MathCAD programme.

(2) A copy of the MathCAD file for your programme on either floppy disk or CD.

(3) A MathCAD printout from your simulation demonstrating its operation and results for the data provided.

The assignment must be submitted as specified by the Module Tutor.

You should ensure that your work meets the appropriate standard for academic practice as specified in the University regulations and procedures applying to cheating, plagiarism, and other forms of academic misconduct.

You should ensure that the MathCAD file that you submit is free from trojans, viruses etc..

Late submissions, without prior authorisation from your Programme Leader, will be awarded a mark of zero.

J. I. H. Allen
October 2007