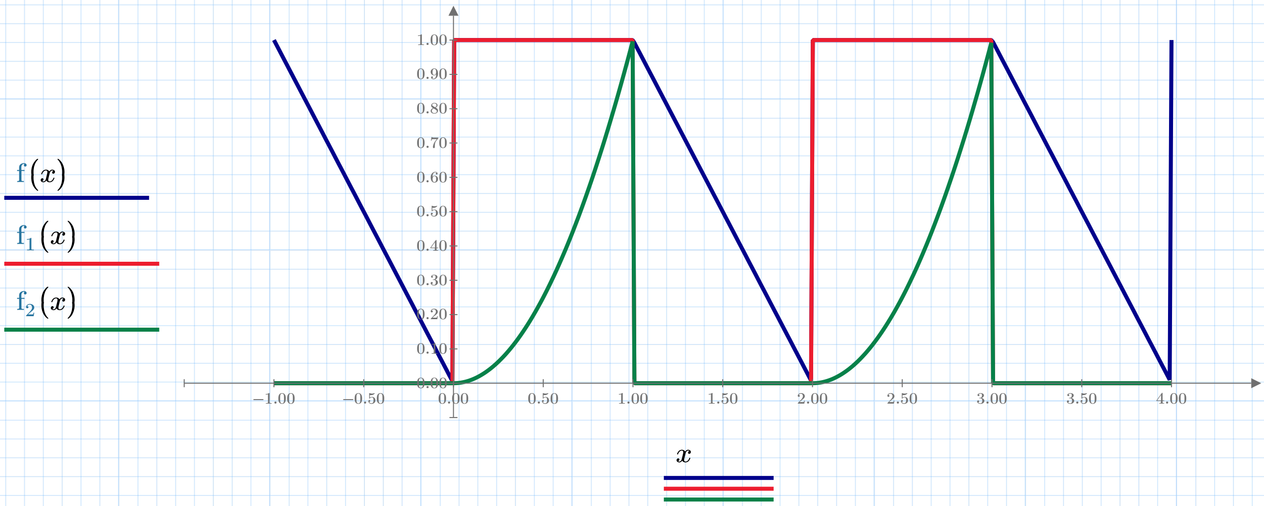
Fourieranalyse von Funktionen:

$$f(x) := \begin{cases} \text{if } (0 \le x) \wedge (x \le 1) & 1 \\ \text{else if } (-1 \le x) \wedge (x \le 0) & -x \\ \text{else if } x > 1 & f(x-2) \end{cases}$$

$$f_1(x) := \begin{cases} \text{if } (0 \le x) \wedge (x \le 1) & 1 \\ \text{else if } (-1 \le x) \wedge (x \le 0) & 0 \\ \text{else if } 1 \le x < 2 & 0 \\ \text{else if } (2 < x) \wedge (x \le 3) & 1 \\ \text{else if } (3 < x) \wedge (x \le 4) & 0 \end{cases}$$

$$f_2(x) := \begin{cases} \text{if } (-1 \le x) \wedge (x \le 0) & 0 \\ \text{else if } (0 < x) \wedge (x \le 1) & x^2 \\ \text{else if } (1 < x) \wedge (x \le 2) & 0 \\ \text{else if } (2 < x) \wedge (x \le 3) & x^2 - 4 \cdot x + 4 \\ \text{else if } (3 < x) \wedge (x \le 4) & 0 \end{cases}$$



$L := 1$ Positiver Endpunkt des Intervalls

$Nt := 20$ Reihenfolge der Fourier−Aproximation

Programme für die Fourier−Koeffizienten:

$$FC(f, N, L) := \begin{array}{|l} Z^{\langle 0 \rangle} \leftarrow \left[\begin{pmatrix} \dfrac{1}{2 \cdot L} \cdot \displaystyle\int_{-L}^{L} f(x)\, dx \\ 0 \end{pmatrix} \right] \\ \text{for } n \in 1..N \\ \quad \begin{array}{|l} Z_{n,0} \leftarrow \dfrac{1}{L} \cdot \displaystyle\int_{-L}^{L} f(x) \cdot \cos\left(\dfrac{n \cdot \pi \cdot x}{L}\right) dx \\ Z_{n,1} \leftarrow \dfrac{1}{L} \cdot \displaystyle\int_{-L}^{L} f(x) \cdot \sin\left(\dfrac{n \cdot \pi \cdot x}{L}\right) dx \end{array} \\ Z \end{array}$$

Berechnung der Fourier−Koeffizienten

$res := FC(f, Nt, L)$

$A := res^{\langle 0 \rangle}$ $\qquad B := res^{\langle 1 \rangle}$

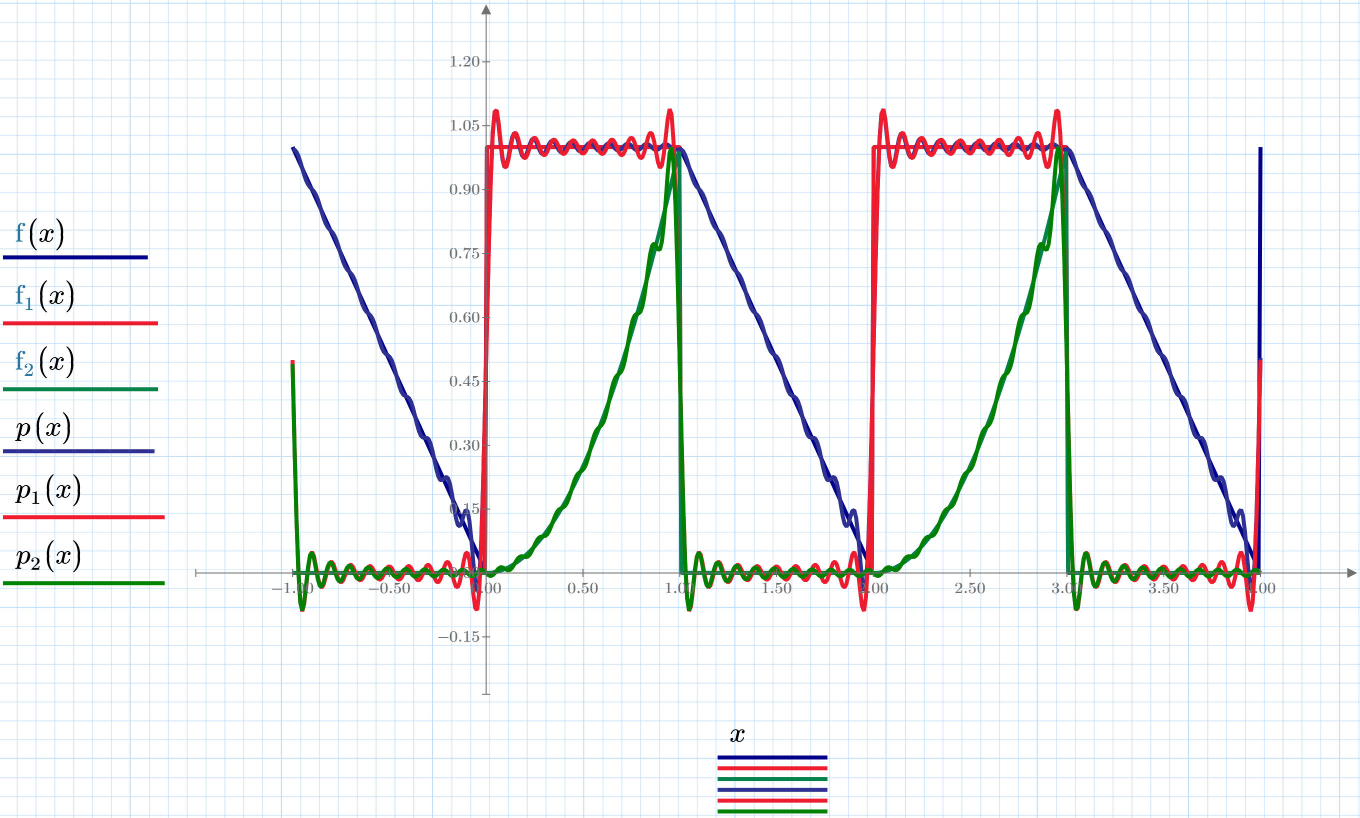
$$p(x) := A_0 + \sum_{n=1}^{Nt} \left(A_n \cdot \cos\left(\frac{n \cdot \pi \cdot x}{L}\right) + B_n \cdot \sin\left(\frac{n \cdot \pi \cdot x}{L}\right) \right)$$

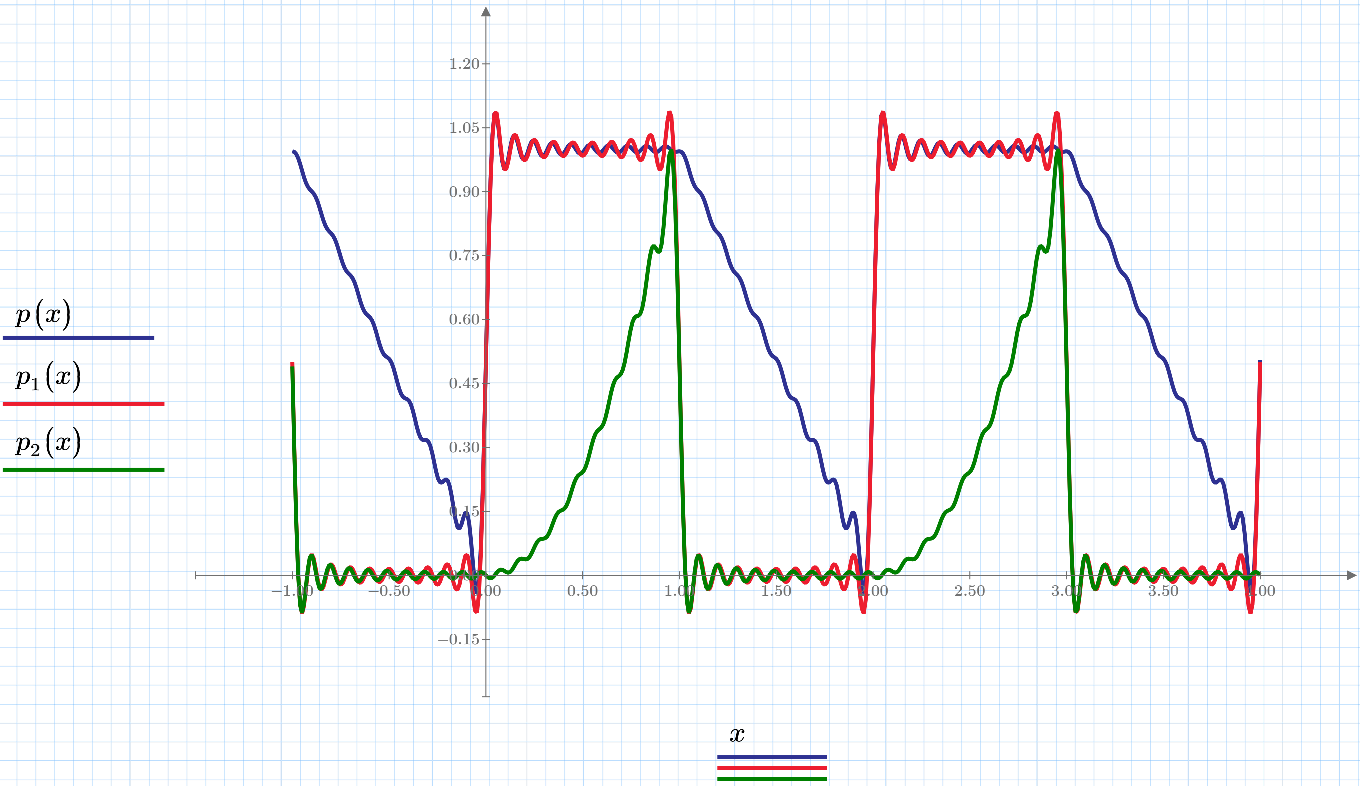
$$FC_1(f, N, L) := \begin{array}{|l} Z^{\langle 0 \rangle} \leftarrow \left[\begin{pmatrix} \dfrac{1}{2 \cdot L} \cdot \displaystyle\int_{-L}^{L} f(x)\, dx \\ 0 \end{pmatrix} \right] \\ \text{for } n \in 1..N \\ \quad \begin{array}{|l} Z_{n,0} \leftarrow \dfrac{1}{L} \cdot \displaystyle\int_{-L}^{L} f(x) \cdot \cos\left(\dfrac{n \cdot \pi \cdot x}{L}\right) dx \\ Z_{n,1} \leftarrow \dfrac{1}{L} \cdot \displaystyle\int_{-L}^{L} f(x) \cdot \sin\left(\dfrac{n \cdot \pi \cdot x}{L}\right) dx \end{array} \\ Z \end{array}$$

$$FC_2(f, N, L) := \begin{array}{|l} Z^{\langle 0 \rangle} \leftarrow \left[\begin{pmatrix} \dfrac{1}{2 \cdot L} \cdot \displaystyle\int_{-L}^{L} f(x)\, dx \\ 0 \end{pmatrix} \right] \\ \text{for } n \in 1..N \\ \quad \begin{array}{|l} Z_{n,0} \leftarrow \dfrac{1}{L} \cdot \displaystyle\int_{-L}^{L} f(x) \cdot \cos\left(\dfrac{n \cdot \pi \cdot x}{L}\right) dx \\ Z_{n,1} \leftarrow \dfrac{1}{L} \cdot \displaystyle\int_{-L}^{L} f(x) \cdot \sin\left(\dfrac{n \cdot \pi \cdot x}{L}\right) dx \end{array} \\ Z \end{array}$$

$res_1 := FC_1\left(\mathrm{f}_1, Nt, L\right)$ $\quad A := res_1^{\langle 0 \rangle}$ $\quad B := res_1^{\langle 1 \rangle}$ $\quad p_1(x) := A_0 + \sum_{n=1}^{Nt} \left(A_n \cdot \cos\left(\frac{n \cdot \pi \cdot x}{L}\right) + B_n \cdot \sin\left(\frac{n \cdot \pi \cdot x}{L}\right)\right)$

$res_2 := FC_2\left(\mathrm{f}_2, Nt, L\right)$ $\quad A := res_2^{\langle 0 \rangle}$ $\quad B := res_2^{\langle 1 \rangle}$ $\quad p_2(x) := A_0 + \sum_{n=1}^{Nt} \left(A_n \cdot \cos\left(\frac{n \cdot \pi \cdot x}{L}\right) + B_n \cdot \sin\left(\frac{n \cdot \pi \cdot x}{L}\right)\right)$



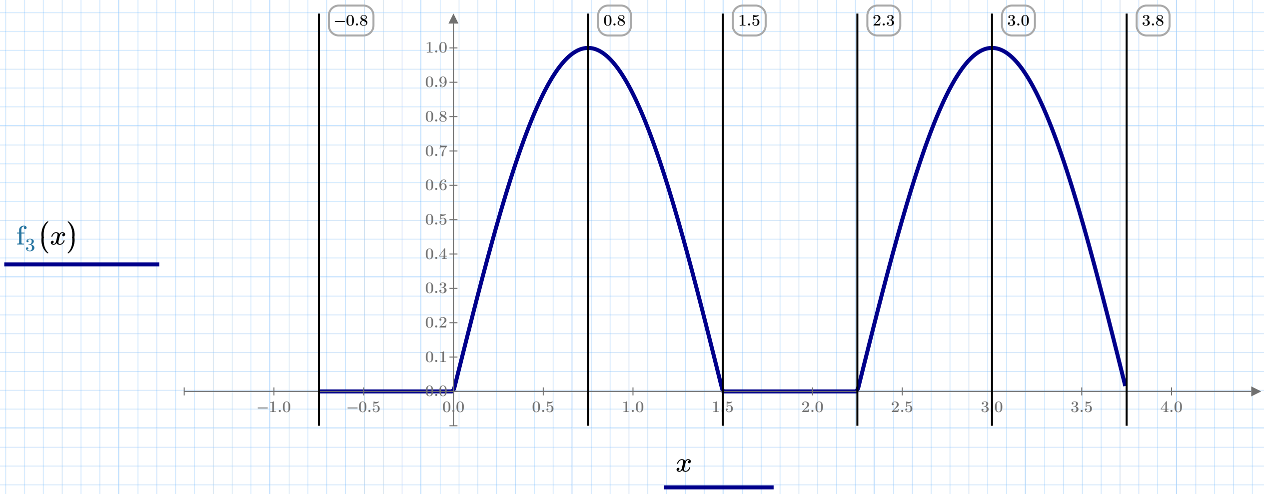


Sinusförmige Schwingung eines Gleichrichters:

$$f_3(x) := \left\| \begin{array}{l} \text{if } \left(-\frac{3}{4} \le x\right) \wedge (x \le 0) \\ \quad \| \; 0 \\ \text{else if } (0 < x) \wedge \left(x \le \frac{3}{2}\right) \\ \quad \| \; \sin\left(\frac{2 \cdot \pi}{3} x\right) \\ \text{else if } \left(\frac{3}{2} < x\right) \wedge \left(x \le \frac{9}{4}\right) \\ \quad \| \; 0 \\ \text{else if } \left(\frac{9}{4} < x\right) \wedge \left(x \le \frac{15}{4}\right) \\ \quad \| \; \sin\left(\pi \cdot \left(\frac{5}{2} - \frac{2 \cdot x}{3}\right)\right) \end{array} \right.$$

Programm für die Fourier–Koeffizienten:

$$FC_3(f, N, L) := \left\| \begin{array}{l} Z^{\langle 0 \rangle} \leftarrow \begin{bmatrix} \left(\frac{1}{2 \cdot L} \cdot \int_{-L}^{L} f(x) \, dx\right) \\ 0 \end{bmatrix} \\ \text{for } n \in 1..N \\ \quad \left\| \begin{array}{l} Z_{n,0} \leftarrow \frac{1}{L} \cdot \int_{-L}^{L} f(x) \cdot \cos\left(\frac{n \cdot \pi \cdot x}{L}\right) dx \\ Z_{n,1} \leftarrow \frac{1}{L} \cdot \int_{-L}^{L} f(x) \cdot \sin\left(\frac{n \cdot \pi \cdot x}{L}\right) dx \end{array} \right. \\ Z \end{array} \right.$$



Berechnung der Fourier–Koeffizienten

$L := 1 \qquad Nt := 5$

$res_3 := \boxed{FC_3(f_3, Nt, L)}$

$A := \boxed{res_3}^{\langle 0 \rangle} \qquad B := \boxed{res_3}^{\langle 1 \rangle}$

$$p_3(x) := \boxed{A}_0 + \sum_{n=1}^{Nt} \left(A_n \cdot \cos\left(\frac{n \cdot \pi \cdot x}{L}\right) + B_n \cdot \sin\left(\frac{n \cdot \pi \cdot x}{L}\right)\right)$$