
8 Conservation Equations and Smoke Filling

Most fire deaths are due to the inhalation of smoke and toxic gases. It is therefore important for the fire protection engineer to be well acquainted with the design methods used to control the flow of smoke in a building and to know how these are arrived at. In this chapter we state the conservation laws for mass and energy and apply these to a number of fire protection problems. The conservation equations are often presented as coupled differential equations that must be solved simultaneously by computer. We introduce a number of commonly applied assumptions that allow these equations to be considered separately and thus derive analytical solutions and iterative methods that can be applied to problems to do with the smoke filling process. We consider compartments under two types of ventilation conditions: closed compartments with only small leakage vents, which results in a dynamic pressure build-up, and compartments with openings large enough to prevent the build-up of pressures due to gas expansion. We apply the conservation equations to calculate smoke filling time and derive smoke-control methodologies for several cases.

CONTENTS

- 8.1 Terminology
 - 8.2 Introduction
 - 8.3 Conservation Equations for a Control Volume
 - 8.3.1 The Conservation of Mass
 - 8.3.2 Some Thermodynamic Properties
 - 8.3.3 The Conservation of Energy
 - 8.4 Pressure Rise in Closed Rooms
 - 8.4.1 Pressure Rise in a Closed Volume
 - 8.4.2 Pressure Rise in a Leaky Compartment
 - 8.5 Smoke Filling of an Enclosure with Leaks
 - 8.5.1 Small Leakage Areas at Floor Level
 - 8.5.2 Small Leakage Areas at Ceiling Level
 - 8.5.3 Estimating Gas Temperatures for the Floor Leak Case
 - 8.5.4 Limitations
 - 8.6 Smoke Control in Large Spaces
 - 8.6.1 Smoke Filling: The Non-Steady Problem
 - 8.6.2 Smoke Control: The Steady-State Problem
 - 8.6.3 Case 1: Natural Ventilation from Upper Layer
 - 8.6.4 Case 2: Mechanical Ventilation from Upper Layer
 - 8.6.5 Case 3: Lower Layer Pressurization by Mechanical Ventilation
 - 8.7 Summary
- References
- Problems and Suggested Answers

8.1 TERMINOLOGY

Control volume — The first law of thermodynamics and the continuity equation for mass are sometimes applied to a volume in space into which, or out from which, a substance flows. This volume is called a control volume.

Control surface — The surface that completely surrounds the control volume is called the control surface.

Total energy — The sum of the internal energy, the kinetic energy, and the potential energy of a system, or $E = U + KE + PE$, where U stands for the internal energy at constant volume. We will only be interested in the increase or decrease in total energy as the system goes from one stage to another. In our applications the changes in kinetic and potential energy are very small compared to the changes in internal energy; we shall therefore focus our attention on changes in internal energy.

Internal energy — Associated with the translation, rotation, and vibration of the molecules and the chemical energy due to bonding between atoms. Our attention will be focused on the influence that temperature and pressure have on the internal energy. Internal energy is assigned the symbol U .

Enthalpy — The sum of the internal energy and the pressure work performed by or on a system. The term arises since the combination of the two properties often appear when analyzing thermodynamic processes. Enthalpy is assigned the symbol H .

Specific properties — When a property of a system is divided by unit mass, it is called a specific property. Specific properties are assigned lowercase letters, so that specific total energy is $e = E/m$, specific internal energy is $u = U/m$, and specific enthalpy is $h = H/m$.

Specific heat — The energy required to raise the temperature of a unit mass of a substance by 1 degree; this depends on the substance. We are interested in two kinds of specific heats: specific heat at constant volume, c_v , and specific heat at constant pressure, c_p .

8.2 INTRODUCTION

General: The conservation equations of mass, momentum, and energy can be used in many ways for assessing the environmental consequences of a fire in an enclosure. In this chapter we use the conservation of mass and energy; the momentum equation will not be explicitly applied, since information needed to calculate velocities and pressures across openings will come from assumptions and specific applications discussed in earlier chapters.

Controlling the smoke extraction from a compartment can save lives, aid firefighting, and protect property. Smoke control design must be based on the conservation of mass and the conservation of energy; the first can be written to allow the mass flow rates of gases through openings to be expressed in terms of pressure differences, the second allows the gas temperatures to be calculated so that the pressure differences can be assessed.

Since the mass flows are dependent on temperature and the energy content of the compartment is dependent on mass flows, the conservation equations must be coupled and solved simultaneously by computer. However, some simple assumptions will allow the equations to be solved separately so that analytical solutions or iteration schemes can be derived. In this chapter we state the conservation equations for a specific volume in space and use these to derive equations for the prediction of a number of environmental parameters of interest.

Ventilation conditions: We direct our attention to two ventilation conditions. In a closed compartment, or a compartment with small leakages, the release of heat will cause an increase in pressure. This pressure drives the mass flow out and there will be no mass flow into the compartment. In Chapter 5, we referred to this stage of the fire as the first stage.

In a compartment with larger openings there will be little or no build-up of dynamic pressure. The opening flows are determined by the hydrostatic pressure differences across the openings, and there will be mass flow out of and into the compartment. In Chapter 5, we referred to this stage of the fire as the third stage.

Overview: We start by discussing the conservation laws for a specific volume in space and thus arrive at expressions that we will use for our applications. The applications include calculating dynamic pressures and smoke filling in a compartment with small leaks as well as transient smoke filling and steady-state smoke control for compartments with larger openings. A number of common smoke-control methods will be discussed and the employed equations will be derived.

The aim of this chapter is not to give a thorough presentation of the various smoke control design methodologies available to the fire protection engineer. Klote and Milke have provided an excellent and thorough guide to the use of design methodologies for smoke management in atria, for stairwell pressurization, for mechanical smoke removal, and for many more smoke-control applications.¹ Our aim is rather to show how the conservation equations can be used to derive such design equations from first principles and to enhance the readers' awareness of the multitude of assumptions commonly made along the way.

8.3 CONSERVATION EQUATIONS FOR A CONTROL VOLUME

The law of conservation of energy is commonly called the *first law of thermodynamics* when it is applied to problems where the effects of heat transfer and internal energy changes are included. The first law and the continuity equation for mass can be applied to a system or to a finite *control volume*.

A thermodynamical system is defined as a definite quantity of matter contained within a closed surface where the system can move in time and space. This approach is often applied when analyzing velocity distributions, diffusion processes, etc., where the state can change from one point to another. The resulting equations are in differential form and are sometimes referred to as the *point-wise equations*.

In many cases an analysis is simplified if attention is focused on a volume in space into which, or out from which, a substance flows. Such a volume is a control volume. The surface that completely surrounds the control volume is the *control surface*. This way of expressing the conservation equations is used where knowledge of the inner structure of the flow is not necessary. We shall be using the conservation equations expressed for a control volume and apply these to fire compartments.

The derivation of the equations for control volumes is given in many elementary textbooks on fluid mechanics and thermodynamics (see for example Welty et al.²), and it is assumed that the reader is acquainted with these. We use [Figure 8.1](#) to set up the conservation equations for mass and energy for a control volume.

The following definitions apply:

- CV is a control volume that is selected as a particular region in space
- CS is the control surface, the boundary of the control volume
- V is the volume of the control volume
- ρ is the density of the matter in the control volume
- \dot{m}_j is mass flow rate of fluid mixture through the control surface at stream j (out of CV is positive)
- m is the mass of the control volume
- \bar{v} is the velocity of the matter
- \bar{n} is the unit normal vector (outward) on the control surface

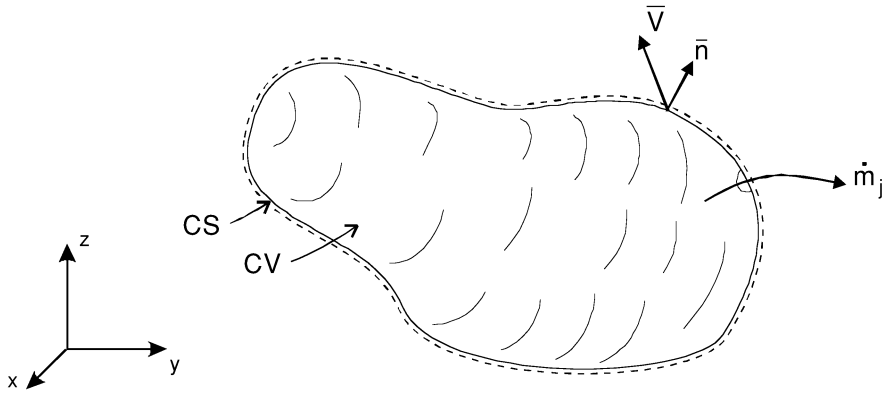


FIGURE 8.1 Control volume.

8.3.1 THE CONSERVATION OF MASS

The conservation of mass can be written as

$$\left[\text{Rate of change} \right]_{\text{mass in the CV}} + \left[\text{Net rate of mass flow} \right]_{\text{leaving the CV or } \sum_{\text{out}} - \sum_{\text{in}}} = 0$$

or

$$\frac{dm}{dt} + \sum_{j=1}^n \dot{m}_j = 0 \quad (8.1)$$

Since the mass can be written as $m = \rho V$, this can be written in integral form as

$$\frac{d}{dt} \iiint_{\text{CV}} \rho dV + \iint_{\text{CS}} \rho \bar{v} \cdot \bar{n} dS = 0 \quad (8.2)$$

Figure 8.2 shows the relationship between the velocity of the matter and the unit normal vector.

The quantity $\bar{v} \cdot \bar{n}$ is the fluid velocity normal to the CS, which gives the rate at which fluid enters (–) or leaves (+) the CS. Hence $\rho \bar{v} \cdot \bar{n} dS$ is the rate of mass flow across the surface dS .

For simplicity we denote the component of the velocity of matter normal to the CS as v_n so that $v_n = \bar{v} \cdot \bar{n}$. We can then drop the vector notation and the unit normal vector, \bar{n} .

Equation (8.2) can then be written

$$\frac{d}{dt} \iiint_{\text{CV}} \rho dV + \iint_{\text{CS}} \rho v_n dS = 0 \quad (8.3)$$

8.3.2 SOME THERMODYNAMIC PROPERTIES

Before we set up the law of conservation of energy for a control volume, it is useful to discuss some thermodynamic properties. In this section we state the law for a closed system and discuss

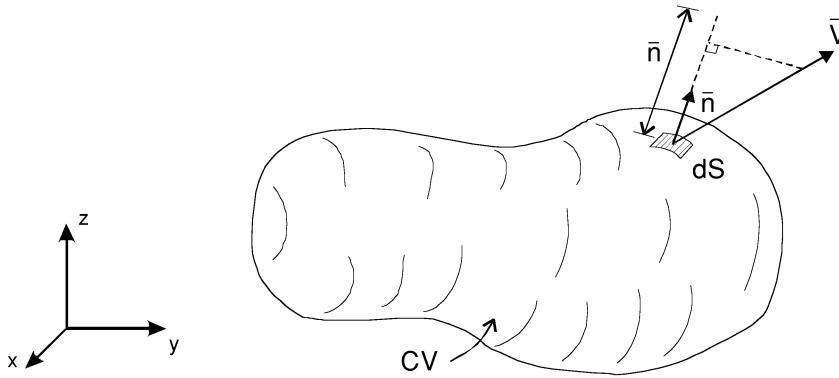


FIGURE 8.2 Mass flow rate at the control surface.

some properties of the system. This will be useful when we set up the energy conservation law for a control volume in Section 8.3.3.

For a closed system the conservation of energy can be written as

$$\left[\begin{array}{l} \text{Rate of change of} \\ \text{energy in the CV} \end{array} \right] = \left[\begin{array}{l} \text{Rate of heat} \\ \text{added to the CV} \end{array} \right] - \left[\begin{array}{l} \text{Rate of work done} \\ \text{by fluid in the CV} \end{array} \right]$$

or

$$\frac{dE}{dt} = \dot{Q} - \dot{W} \quad (8.4)$$

Before we expand on this and formulate the first law for a control volume, where mass entering and leaving the CV contains energy and also influences the energy conservation, we shall define some thermodynamic properties.

Internal energy: The total energy of the system, E , is the sum of the internal energy, the kinetic energy, and the potential energy, or $E = U + KE + PE$, where U stands for the internal energy at constant volume. The internal energy is associated with the translation, rotation, and vibration of the molecules and the chemical energy due to bonding between atoms. Our attention will be focused on the influence that temperature and pressure have on the internal energy. Since we will be interested only in the increase or decrease in internal energy, we will not need to know its absolute value, only the change.

Further, in our applications, the changes in kinetic and potential energy are very small compared to the changes in internal energy. The changes in kinetic and potential energy are traditionally neglected when solving the types of engineering problems we are interested in, and we shall follow this in our treatment.

Equation (8.4) can therefore be rewritten as

$$\frac{dU}{dt} = \dot{Q} - \dot{W} \quad (8.5)$$

Specific internal energy: When a property of the system is divided by unit mass, it is called a *specific property*. The specific internal energy is thus $u = U/m$. Similarly, the specific total energy is $e = E/m$.

Enthalpy and specific enthalpy: When analyzing certain types of thermodynamic processes we frequently encounter the combination of properties $U + PV$, the sum of the internal energy and the pressure work. This sum is termed *enthalpy* and given the symbol H . When dividing by the mass we get the specific enthalpy, h , as

$$h = u + P/\rho \quad (8.6)$$

since $m/V = \rho$.

Specific heat at constant volume and constant pressure: The energy required to raise the temperature of a unit mass of a substance by 1 degree is termed the specific heat, and depends on the substance. We are interested in two kinds of specific heats: specific heat at constant volume, c_v , and specific heat at constant pressure, c_p . These are defined symbolically through the relationships

$$du = c_v dT \quad (8.7)$$

and

$$dh = c_p dT \quad (8.8)$$

so

$$dh - du = (c_p - c_v) dT \quad (8.9)$$

Note that when integrating the above equations we get $u = \int_0^T c_v dT$ and $h = \int_0^T c_p dT$. Applying the lower integration limit as zero degrees Kelvin (0 K) will result in the total specific internal energy and enthalpy. Usually, however, we are interested only in the change in these properties, and the lower limit therefore is chosen as the initial temperature of the system.

Further, the control volumes we will be looking at in many of our applications contain smoke, which in turn consists mainly of air (or, rather, nitrogen). It is commonly assumed that the specific heat of air is constant through the temperature range we are interested in, and we can sometimes write the above as

$$u = c_v (T - T_a) \quad (8.10)$$

and

$$h = c_p (T - T_a) \quad (8.11)$$

The ideal gas law: We will use the ideal gas law in many of our applications. This law can be stated in many different ways, and we will derive some expressions that will be of use to us later. A common expression for the ideal gas law is

$$PV = nR_0 T$$

or

$$PV = \frac{m}{M} R_0 T$$

where n is the number of molecules in the system, M is the molecular mass of the substance, and R_0 is the universal gas constant, which is the same for all substances $R_0 = 8.314 \text{ J/(mol K)}$. For each substance there is a unique gas constant, R , given by

$$R = R_0/M$$

and for air $R = 287 \text{ J/(kg K)}$.

Since our applications often deal with air (or smoke), we can rewrite the ideal gas law, by noting that $V = m/\rho$, as

$$P = \rho RT \tag{8.12}$$

Relationship between specific heat and the gas constant: We can now derive a relationship between the gas constant and the specific heats of a substance, which will be of use to us in our applications. Derivating Eq. (8.6), we get

$$dh = du + d(P/\rho).$$

Rearranging and derivating Eq. (8.12) we get

$$d(P/\rho) = d(RT) = RdT$$

since R is constant. Combining these leads to $dh - du = R dT$. Substituting into Eq. (8.9) we arrive at

$$R = c_p - c_v \tag{8.13}$$

which is a result we will use later.

8.3.3 THE CONSERVATION OF ENERGY

When the law of the conservation of energy is applied to problems where the effects of heat transfer and internal energy changes are accounted for, it is commonly called the *first law of thermodynamics*. Equation (8.5) states the law for a closed system, where the common assumption is made that changes in kinetic and potential energy are negligible.

Consider the control volume given in [Figure 8.3](#). Heat is being added to the CV, work is being carried out by the CV, and mass is flowing in and out through the CS.

For a control volume where fluid flows through the control surface, account must be taken of the energy content of the fluid entering or leaving the control volume. We can restate the first law of thermodynamics, ignoring changes in kinetic and potential energy, as

$$\left[\begin{array}{l} \text{Rate of increase} \\ \text{of internal energy} \\ \text{in the CV} \end{array} \right] + \left[\begin{array}{l} \text{Net rate of energy} \\ \text{out of CV due to} \\ \text{fluid flow} \end{array} \right] = \left[\begin{array}{l} \text{Net rate of heat} \\ \text{added to} \\ \text{the CV} \end{array} \right] - \left[\begin{array}{l} \text{Rate of work} \\ \text{done by fluid} \\ \text{in the CV} \end{array} \right]$$

or in integral form, using the notation and assumptions made for Eq. (8.3),

$$\frac{d}{dt} \iiint_{CV} \rho \cdot u \cdot dV + \iint_{CS} \rho \cdot u \cdot v_n \cdot dS = \dot{Q} - \dot{W} \tag{8.14}$$

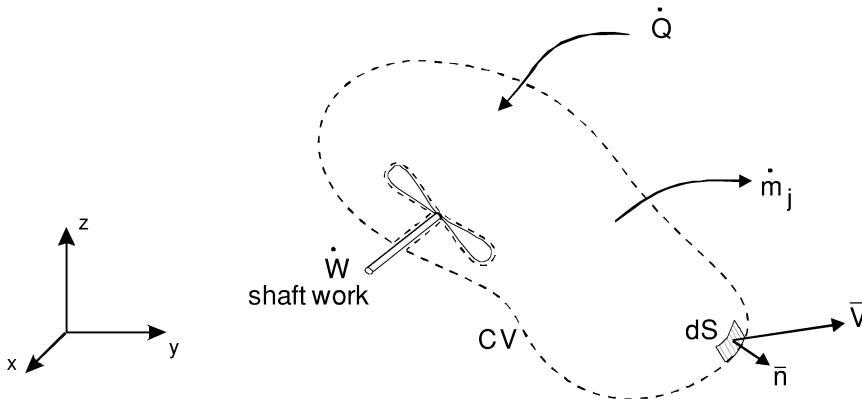


FIGURE 8.3 Control volume for the conservation of energy.

Energy can cross the control surface in three forms: as work, as heat, and as energy contained in the mass entering or leaving the control volume. It is important to distinguish between these, and we will expand on this by examining the two last terms more closely and thereby arrive at an equation we will use to solve practical problems.

Work: The work done by a constant force F on a body that is displaced a distance x is given by

$$W = Fx$$

where the work is measured in joules. Work done per unit time is denoted \dot{W} and can be written (measured in watts) as

$$\dot{W} = F \frac{dx}{dt} \tag{8.15}$$

There are several different ways of doing work, each in some way related to a force acting through a distance. In engineering problems we often distinguish three types of work. The first is the *shaft work*, W_s , the work done by the control volume on its surroundings that could cause a shaft to rotate or a weight to be raised. The second is the *pressure work*, W_p , which is done on the surroundings to overcome normal stresses on the control surface where there is fluid flow (also called flow work). The third is the *shear work*, W_τ , which is performed on the surroundings to overcome shear stresses at the control surface. We can therefore write the rate at which work is performed on the control volume surroundings as

$$\dot{W} = \dot{W}_s + \dot{W}_p + \dot{W}_\tau \tag{8.16}$$

In our applications there are no mechanical devices, such as shafts, present. The shaft work is therefore zero. The shear force at a solid surface is zero since the fluid velocity there is zero (for a fixed control volume). At openings in the control surface, the shear work can be made zero by choosing a control surface that cuts across the opening perpendicular to the flow, and we shall only be considering such control volumes. No component of the velocity vector is then in the direction of the shear force vector, and the shear work is therefore zero.

Consider [Figure 8.4](#). The pressure work can be formulated by observing that the force, F , acting on a surface dS is given as $F = P dS$.

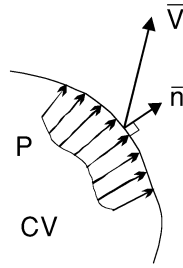


FIGURE 8.4 Force acting on the control surface.

The total force is obtained by integrating over the surface dS . Using Eq. (8.15) where dx/dt is the velocity normal to the surface dS , we can write the following equation expressing the total rate of work done on the entire control surface by normal stresses as

$$\dot{W}_p = + \iint_{CS} P v_n dS \quad (8.17)$$

The plus sign indicates that the work is being carried out by the control volume on the control surface and not vice-versa.

Heat: Heat is the form of energy transferred to or from the control volume, through the control surface, due to a temperature difference. It is important to distinguish this from the heat transferred with mass into and out of the control volume. In our applications the heat added to the control volume is due to the energy released when chemical reactions take place and heat transfer to the control volume. Here we call this term \dot{Q}_{ch} (in earlier chapters, we simply called this term \dot{Q}). Some of this heat will be transferred to the control surface; we call this \dot{q}_{loss} . We can therefore write

$$\dot{Q} = \dot{Q}_{ch} - \dot{q}_{loss} \quad (8.18)$$

We will continue to use the term \dot{Q} to describe the net rate of heat added to the control volume, but will keep in mind that this consists of the chemical energy released and the heat transfer losses to the boundaries.

Resulting equation for our applications: We can now rewrite Eq. (8.14) into a form that we will use for our applications. Replacing the work term with Eq. (8.17), we get

$$\frac{d}{dt} \iiint_{CV} \rho u dV + \iint_{CS} \rho u v_n dS = \dot{Q} - \iint_{CS} P v_n dS \quad (8.19)$$

We can now combine the second and the last term in the above equation and get

$$\frac{d}{dt} \iiint_{CV} \rho u dV + \iint_{CS} \rho \left(u + \frac{P}{\rho} \right) v_n dS = \dot{Q} \quad (8.20)$$

From the definition of enthalpy as $h = u + P/\rho$, we can use Eq. (8.6) to rewrite the second term as

$$\frac{d}{dt} \iiint_{CV} \rho u dV + \iint_{CS} \rho h v_n dS = \dot{Q} \quad (8.21)$$

We will be using this equation extensively in our applications. The first term is the time rate of change of internal energy within the control volume. The second term is the net rate of enthalpy flow out of the control volume through the control surface. The third term is the net rate of heat added to the control volume.

The main assumptions we have made so far are neglecting the changes in kinetic and potential energy and assuming that the control volume can be set up such that there will be no work carried out due to shear forces. These are commonly used engineering assumptions.

We can further simplify Eq. (8.21) by assuming that the state of the control volume is uniform: the state may change with time, but it will do so uniformly. This means that the density in Eq. (8.21) is uniform over the entire control volume and can be moved out of the integrals. Finally, by assuming that c_v and c_p are constant for the temperature ranges we are interested in, Eq. (8.10) and (8.11) will allow us to express the internal energy and the enthalpy changes of the control volume. We use these assumptions in the following sections.

8.4 PRESSURE RISE IN CLOSED ROOMS

In a closed compartment, or a compartment with small leakages, the release of heat will cause volumetric expansion of gases and an increase in compartment pressure. It is this pressure that drives the mass flow out, and there will be no mass flow into the compartment. In Chapter 5 we referred to this stage of the fire as the *first stage*.

When there is rapid accumulation of mass or energy, or when the compartment has small openings to the surroundings, this pressure rise is very rapid and any hydrostatic pressure differences with height are negligible. For example, an addition of 100 kW to a 60 m³ enclosure with a 0.01 m² opening will cause a steady-state pressure increase of ≈ 1000 Pa in a number of seconds. The hydrostatic pressure difference decreases at the rate of 10 Pa per meter as the height increases. For this case we see that the hydrostatic pressure difference is negligible and the opening flow will be determined by the pressure caused by the volumetric expansion of gases.

In this section we follow the work of Zukoski,³ who set up the conservation equations for mass and energy for the closed room case and the case where there are small leakage areas to the surroundings, and derived expressions for the pressure rise. Zukoski also considered the smoke filling process for rooms with a leakage, and we consider this in Section 8.5.

8.4.1 PRESSURE RISE IN A CLOSED VOLUME

When heat is added to an ideal gas in a fixed volume, the pressure must increase in response to the temperature according to the ideal gas law. In a building fire situation the resulting pressure and the rate of pressure rise are often kept very small by gas leaks through openings in the walls of the buildings such as cracks around windows and doors.

However, situations may arise where the enclosure can be considered to be very well sealed, such as certain compartments on ships. The purpose of this section is to derive simple equations for calculating the dynamic pressure build-up in a hermetically closed compartment. We will then use the results to show that the pressure rise is very rapid. This result can be used to justify the so-called constant pressure assumption, used when examining a leaky room fire.

We consider this problem for a very simple example; see [Figure 8.5](#). Consider a room of volume V with gases at an initial temperature, density, and pressure of T_a , ρ_a , and P_a . A small fire of constant heat output \dot{Q} is treated as a point source of heat, and any heat losses to the surrounding structure are ignored. The room is hermetically closed.

Conservation of mass: Consider Eq. (8.3). We assume that the fire is a source of heat only and the mass release rate of the fuel is neglected. The second term in Eq. (8.3) is thus zero.

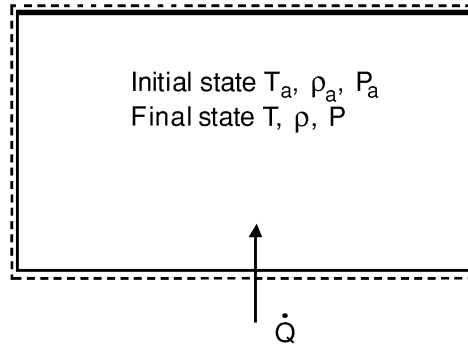


FIGURE 8.5 Hermetically closed room.

The total mass of gases in the volume will remain constant for all times and can be written as

$\iiint_{CV} \rho_a dV$, where ρ_a is the initial density.

Conservation of energy: Since the mass can be written in terms of ρ_a and since there is no fluid flow into or out of the control volume, the second term in Eq. (8.21) is zero, and the equation can be rewritten as

$$\frac{d}{dt} \iiint_{CV} \rho_a u dV = \dot{Q} \quad (8.22)$$

We can assume that the gas in the volume consists mainly of air and that the specific heat is constant over the temperature range in which we are interested. This is quite a reasonable assumption, since for the temperature range 300 to 1000 K the value of c_p for air ranges from 1.0 to 1.14 kJ/(kg K) and c_v ranges from 0.71 to 0.85 kJ/(kg K). For simplicity, these values are often taken to be those at ambient temperature, and for many of our calculations we shall use $c_p = 1.0$ kJ/(kg K) and $c_v = 0.7$ kJ/(kg K).

We can now use Eq. (8.10) and write $u = c_v (T - T_a)$. Equation (8.22) becomes

$$\frac{d}{dt} \iiint_{CV} \rho_a c_v (T - T_a) dV = \dot{Q}$$

The terms within the integral sign are constants, except dV , and the term $\iiint_{CV} dV$ is simply the volume, V . This results in

$$\frac{d}{dt} (V \rho_a c_v (T - T_a)) = \dot{Q}$$

Integrating both sides of this equation from time 0 to time t gives us

$$V \rho_a c_v (T - T_a) = \dot{Q} t \quad (8.23)$$

Resulting equation: We wish to arrive at an equation for the pressure rise. We must therefore rewrite Eq. (8.23) in terms of pressure. The ideal gas law for constant volume (Eq. (8.12)) can be applied at the initial and final states to find $P_a/T_a = P/T$, and by simple manipulation this can give

$$\frac{P - P_a}{P_a} = \frac{T - T_a}{T_a}$$

Dividing Eq. (8.23) by T_a , we find that it can be rewritten as

$$\frac{P - P_a}{P_a} = \frac{\dot{Q}t}{V\rho_a c_v T_a} \quad (8.24)$$

We have thus arrived at an expression that can be used to estimate the dynamic pressure build-up due to thermal expansion in a hermetically closed compartment. In addition to the assumptions made when deriving Eq. (8.21), some further assumptions have been made:

- The energy release rate is constant.
- The mass loss rate of the fuel is neglected in the conservation of mass.
- The specific heat does not change with temperature.
- The hydrostatic pressure difference over the height of the compartment is ignored and assumed to be negligible compared to the dynamic pressure.

EXAMPLE 8.1

A hermetically closed machine room in a ship has a volume of 60 m³. A fire starts with a constant effect of 100 kW. Estimate the pressure rise due to the expansion of the gases after 10 seconds.

SUGGESTED SOLUTION

Ignoring heat losses to the compartment boundaries, assuming $\rho_a = 1.2 \text{ kg/m}^3$, we use Eq. (8.24) to find $\frac{P - P_a}{P_a} = \frac{100 \cdot 10}{60 \cdot 1.2 \cdot 0.7 \cdot 293} = 0.068 \text{ atm}$. Multiplying by the atmospheric pressure 101 kPa gives a pressure difference of 6.8 kPa, which is a considerably high value.

Example 8.1 shows that in a very short time the pressure in a hermetically closed room rises to quite large values. A pressure difference of 6.8 kPa across a window of 0.6 m² will produce a total load of 4100 Newtons, which will probably be enough to destroy the window.

Most buildings have leaks of some sort. The above example indicates that even though a fire room may be closed, the pressure rise is very rapid and would presumably lead to sufficient leaks to prevent further pressure rise from occurring. We will use this conclusion when dealing with pressure rises in enclosures with small leaks.

8.4.2 PRESSURE RISE IN A LEAKY COMPARTMENT

Consider again a fixed volume, but this time with a small opening at floor level. The fire is considered as a source of heat only; see [Figure 8.6](#). Again we use the conservation of energy as given in Eq. (8.21), reproduced here for clarity:

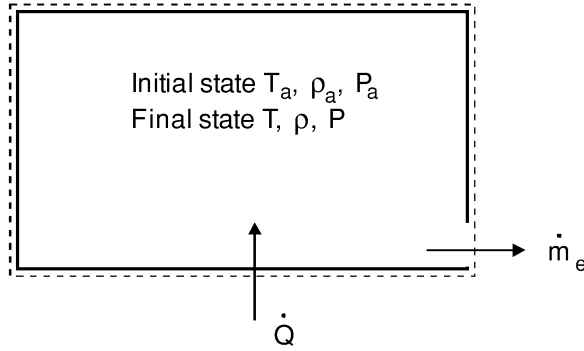


FIGURE 8.6 Control volume for a leaky compartment.

$$\frac{d}{dt} \iiint_{CV} \rho u dV + \iint_{CS} \rho h v_n dS = \dot{Q} \quad (8.21)$$

Equations (8.10) and (8.11) define u and h as $u = c_v(T - T_a)$ and $h = c_p(T - T_a)$ where T_a is some reference temperature. Here, it will be useful for us to express these in terms of total internal energy per unit mass and total enthalpy per unit mass. This will simplify our treatment. We therefore take the reference temperature T_a to be zero degrees Kelvin (0 K) and write

$$u = c_v T \quad (8.25)$$

and

$$h = c_p T \quad (8.26)$$

thus expressing the total internal energy and total enthalpy per unit mass.

First term in Equation (8.21): We are interested in arriving at an expression that allows us to evaluate the dynamic pressure in an enclosure with a leakage opening, and we must therefore write the first term in Eq. (8.21) in terms of pressure. We can do this by using the ideal gas law as given by Eq. (8.12) to express the density as

$$\rho = \frac{P}{RT} \quad (8.27)$$

Using Eq. (8.25) we can write the first term as

$$\frac{d}{dt} \iiint_{CV} \frac{P}{R \cdot T} c_v T dV \text{ which becomes } \frac{d}{dt} \iiint_{CV} P \frac{c_v}{R} dV$$

Performing the integration results in $\frac{d}{dt} \left(\frac{P c_v}{R} V \right)$, since P , c_v , and R are not dependent on volume.

This expression in turn results in $\frac{dP}{dt} \frac{V c_v}{R}$, since V , R , and c_v are not dependent on time. To summarize, we have found that

$$\frac{d}{dt} \iiint_{CV} \rho u dV = \frac{dP}{dt} \frac{V c_v}{R} \quad (8.28)$$

We shall use this result in our final expression.

Second term in Equation (8.21): Using Eq. (8.26) to express the total enthalpy per unit mass of the gases flowing out of the compartment, we can rewrite the second term in Eq. (8.21) as

$$\iint_A \rho v c_p T dA$$

since the area A is the only part of the control surface allowing mass to exit. Further, we can write v for velocity instead of v_n , since the direction of the flow is perpendicular to the surface of the opening. We know that the mass flow rate through an opening of area A can be written $\dot{m} = \rho v A$ if the density and velocity are constant over the area and the velocity normal to the surface of A .

Adopting the suffix “e” to denote “exit,” we can express the mass flow rate exiting the opening as $\dot{m}_e = \rho_e v_e A_e$. Performing the integration over the opening area we find that the second term in Eq. (8.21) can be written as $\rho_e v_e A_e c_p T_e$ or $\dot{m}_e c_p T_e$.

To summarize, we have found that

$$\iint_{CS} \rho \cdot \mathbf{h} \cdot \mathbf{v}_n \cdot dS = \rho_e \cdot v_e \cdot A_e \cdot c_p \cdot T_e = \dot{m}_e \cdot c_p \cdot T_e \quad (8.29)$$

Third term in Equation (8.21): The third term consists of the chemical heat release rate (assumed to be released as a point source) minus the heat losses to the boundary or, as expressed by Eq. (8.18), $\dot{Q} = \dot{Q}_{ch} - \dot{q}_{loss}$.

Resulting equation: Combining the three terms in Eq. (8.21), we can rewrite the equation as

$$\frac{c_v \cdot V}{R} \frac{dP}{dt} + \dot{m}_e c_p T_e = \dot{Q} \quad (8.30)$$

In Section 8.4.1 and Example 8.1 we found that the rate of pressure rise, dP/dt , was very high for the first few seconds in a closed compartment. Thus, leakage areas would be established relatively quickly, resulting in a constant level of pressure as shown in Figure 8.7. Example 8.1 therefore suggests that the constant pressure assumption is reasonable. To simplify our application of Eq. (8.30) we can now assume that $dP/dt \approx 0$ and therefore arrive at the expression

$$\dot{m}_e c_p T_e = \dot{Q} \quad (8.31)$$

If T_e and \dot{Q} are known, we can now calculate the mass flow rate out of the opening.

Equation (8.31) shows that the enthalpy flux due to mass flow out of the opening is equal to the heat addition rate. We can express this in terms of pressure by expressing the pressure difference over the opening in the well-known form

$$\Delta P = \frac{1}{2} \rho_e v_e^2 \text{ and therefore } v_e = \sqrt{2\Delta P / \rho_e}$$

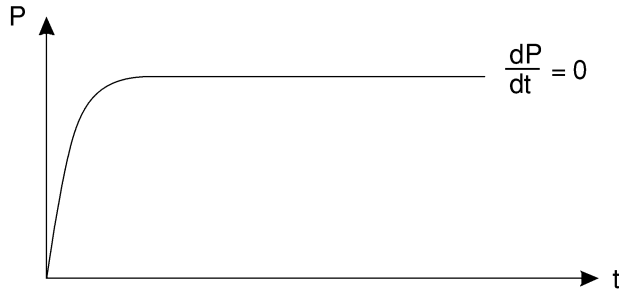


FIGURE 8.7 Rate of pressure rise in a leaky compartment.

Writing the mass flow rate as $\dot{m}_e = A_e \rho_e v_e$ and using the above expression for v_e , we can rewrite Eq. (8.31) as $A_e \sqrt{2\Delta P \rho_e} c_p T_e = \dot{Q}$. Solving for the pressure difference we get

$$\Delta P = \left(\frac{\dot{Q}}{c_p T_e A_e} \right)^2 \frac{1}{2\rho_e}$$

In order to improve our estimates of the pressure difference, we can include the flow coefficient C_d in the above expression. The value of the flow coefficient is discussed in Chapter 5 (Section 5.2.3), and for most openings the value of C_d is between 0.6 and 0.7. Including C_d in our expression leads to

$$\Delta P = \frac{1}{2\rho_e} \left(\frac{\dot{Q}}{c_p T_e A_e C_d} \right)^2 \quad (8.32)$$

We have thus arrived at two equivalent expressions to estimate mass flow rate out through an opening (Eq. (8.31)) and the pressure difference across the opening (Eq. (8.32)). The expressions are approximations, since many assumptions have been made along the way. It is advisable to keep the following assumptions in mind:

- The energy release rate is constant.
- The mass loss rate of the fuel is neglected in the conservation of mass.
- The specific heat does not change with temperature.
- The hydrostatic pressure difference over the height of the compartment is ignored and assumed to be negligible compared to the pressure due to expansion of hot gases.
- Constant pressure is assumed, neglecting the initial rate of pressure rise.
- The opening cannot be taken to be a vertically oriented slit, since the properties of the exiting gas must be uniform over the opening.
- The opening can be taken to be at floor level so the exiting gas temperature $T_e \approx T_a$. If the opening is at ceiling level, then the hot gas temperature must be known and assumed to be a constant value.
- The area of the leakage opening is assumed to be known but this is difficult to assess in real buildings.

Due to these and other assumptions made, Eq. (8.31) and (8.32) can only be expected to give answers that can be used as order of magnitude estimates.

EXAMPLE 8.2

A relatively air-tight room has a floor area of 6 m by 4 m and a height of 3 m. It has a door of width 1 m with a 1 cm high slit at the floor level. A hydraulic oil leak has ignited, causing a fire with an effect of 100 kW. Calculate the pressure difference that arises, the velocity in the opening, and the mass flow through the slit. What happens if the slit is made 10 times higher?

SUGGESTED SOLUTION

Assuming $c_p = 1.0$ kJ/(kg K), $T_e = 300$ K, and $C_d = 0.7$, we find (using Eq. (5.9)) that $\rho_e = 353/300 = 1.18$ kg/m³. Ignoring heat losses to the boundaries and using Eq. (8.32) we find

$$\Delta P = \frac{1}{2 \cdot 1.18} \left(\frac{100}{1.0 \cdot 300 \cdot 1 \cdot 0.01 \cdot 0.7} \right)^2 = 960 \text{ Pa.}$$

The velocity is $v_e = \sqrt{2\Delta P/\rho_e} = \sqrt{2 \cdot 960/1.18} = 40$ m/s (an unreasonably high value). Using Eq. (8.31) we find $\dot{m}_e = \frac{100}{1.0 \cdot 300} = 0.33$ kg/s.

Making the slit 10 times larger gives a pressure difference that is 100 times smaller (≈ 10 Pa) and a velocity 10 times smaller, but the mass flow rate is the same.

8.5 SMOKE FILLING OF AN ENCLOSURE WITH LEAKS

In this section we expand on the previous sections in Chapter 8 and use our conclusions to present simple models for calculating the time it takes to fill a compartment with smoke. We examine a volume composed of a single room where a fire of a constant heat output will cause smoke to rise and form a horizontal ceiling layer of hot gas. The room then contains two layers: the upper hot layer and the lower cold (ambient) layer, both of which are assumed to have a uniform temperature of T_g and T_a , respectively.

We follow Zukoski and assume that the opening is a leakage, located either at the floor level or at the ceiling level.³ We assume that the energy release rate results in a pressure increase due to the thermal expansion of the gases and air is pressed out through the opening. Another way of stating this limitation is to say that we assume that there is no mass flow into the room (pressure across the leak is always positive) and that the mass flow out is either cold gases (leakage at floor level) or hot gases (leakage at ceiling level) of temperature T_g or T_a .

We consider the two cases below, and in a third section we consider how the upper layer temperature can be calculated for the case where there is no mass flow into the room.

8.5.1 SMALL LEAKAGE AREAS AT FLOOR LEVEL

Figure 8.8 shows a schematic of the case we will consider. A room with a leak opening at floor level has a height H and a lower layer height z . The fire is treated as a point source of heat \dot{Q} , and no account is taken of the fuel mass flow rate. The mass flow rate out through the leakage opening is due to the expansion of the hot gases and is given by \dot{m}_e . The mass flow rate from the lower layer to the upper layer is given by \dot{m}_p , the plume mass flow rate. The control volume is chosen such that the volume of the plume is ignored; the plume is considered only as a means of transporting mass from the lower to the upper layer.

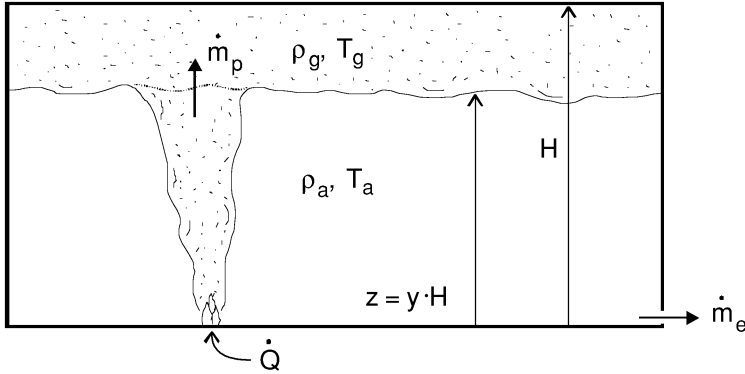


FIGURE 8.8 Simple smoke-filling model, leakage at floor level.

The conservation of mass and energy can be applied to this case in many different ways. Zukoski analyzed this case and found that applying the conservation of mass to the lower layer only, and using results from the conservation of energy arrived at earlier, resulted in a simple differential equation.³ The solution of the equation can be presented graphically and then used to solve practical smoke-filling problems for rooms with leaks. We summarize this simple smoke-filling model in the following.

We therefore choose our control volume as the lower layer and apply the conservation of mass to this volume.

The conservation of mass was earlier stated through Eq. (8.3), reproduced here for clarity:

$$\frac{d}{dt} \iiint_{CV} \rho dV + \iint_{CS} \rho v_n dS = 0 \quad (8.3)$$

In the first term, the volume of the lower layer is not constant. We must therefore express volume as $V = z S$ where z is the height of the lower layer (and is dependent on time) and S is the floor area. The second term is simply the rate of mass leaving the CV. From Figure 8.8 we see that mass leaves the lower layer through the plume as \dot{m}_p and through the leakage opening as \dot{m}_e . Performing the integration in the first term simply results in $\rho_a z S$, and Eq. (8.3) can then be rewritten as

$$\frac{d}{dt} (\rho_a z S) + \dot{m}_e + \dot{m}_p = 0 \quad (8.33)$$

where z is a function of time.

In Chapter 4 we discussed the Zukoski plume and gave the plume mass flow rate (by Eq. (4.21)) as

$$\dot{m}_p = 0.21 \left(\frac{\rho_a^2 g}{c_p T_a} \right)^{1/3} \dot{Q}^{1/3} \cdot z^{5/3} \quad (4.21)$$

We can use this result and insert it into Eq. (8.33). But first we consider the conservation of energy to arrive at an expression for \dot{m}_e .

The conservation of energy was considered in Section 8.4.2, where we found it could be written as Eq. (8.30) (reproduced here for clarity):

$$\frac{c_v \cdot V}{R} \frac{dP}{dt} + \dot{m}_e c_p T_e = \dot{Q} \quad (8.30)$$

We argued in Section 8.3.2 that the rate of pressure rise could in many cases be ignored so that $dP/dt = 0$. This gave us Eq. (8.31). Noting that the mass flow rate out contains gases with temperature T_a we can rewrite Eq. (8.31) as

$$\dot{m}_e = \frac{\dot{Q}}{c_p T_a} \quad (8.34)$$

A **differential equation for smoke-filling time** can now be achieved by combining Eq. (8.33), (4.21), and (8.34), realizing that ρ_a and S are constants:

$$\frac{dz}{dt} \rho_a S + \frac{\dot{Q}}{c_p T_a} + 0.21 \left(\frac{\rho_a g}{c_p T_a} \right)^{1/3} \dot{Q}^{1/3} z^{5/3} = 0 \quad (8.35)$$

The above differential equation cannot be solved analytically, but for a constant \dot{Q} it can very easily be solved using numerical techniques. A very convenient way to make the equation directly useful for us is to show the solution graphically. In order to make such a solution applicable to many different geometries and different heat release rates, we must first make Eq. (8.35) dimensionless, then solve it numerically and present the graphical solution in terms of nondimensional parameters. We therefore define three dimensionless parameters.

Dimensionless height is given the symbol y and is simply defined as the height of the lower layer, z , normalized by the height of the room, H , or

$$y = \frac{z}{H} \quad (8.36)$$

The dimensionless height varies from 0 to 1 and expresses the percentage of the room height below the smoke layer.

Dimensionless heat release rate is given the symbol \dot{Q}^* , and Zukoski³ defined this as

$$\dot{Q}^* = \frac{\dot{Q}}{\rho_a c_p T_a \sqrt{g} H^{5/2}} \quad (8.37)$$

This way of expressing dimensionless heat release rate has been found very useful for many other applications. When discussing flame heights in Chapter 4 we gave the dimensionless heat release rate in Eq. (4.1) where the length scale is presented by the burner diameter. For smoke-filling applications the length scale is presented by the room height.

For normal conditions, taking $\rho_a = 1.2 \text{ kg/m}^3$, $T_a = 293\text{K}$, $g = 9.81 \text{ m/s}^2$, and $c_p = 1.0 \text{ kJ/(kg K)}$, we find that Eq. (8.37) can be expressed as

$$\dot{Q}^* = \frac{\dot{Q}}{1100 H^{5/2}}$$

Note that the units of c_p in the above expression are given in terms of [kJ], and therefore the heat release rate must be given in [kW] if this simplified expression is to be used.

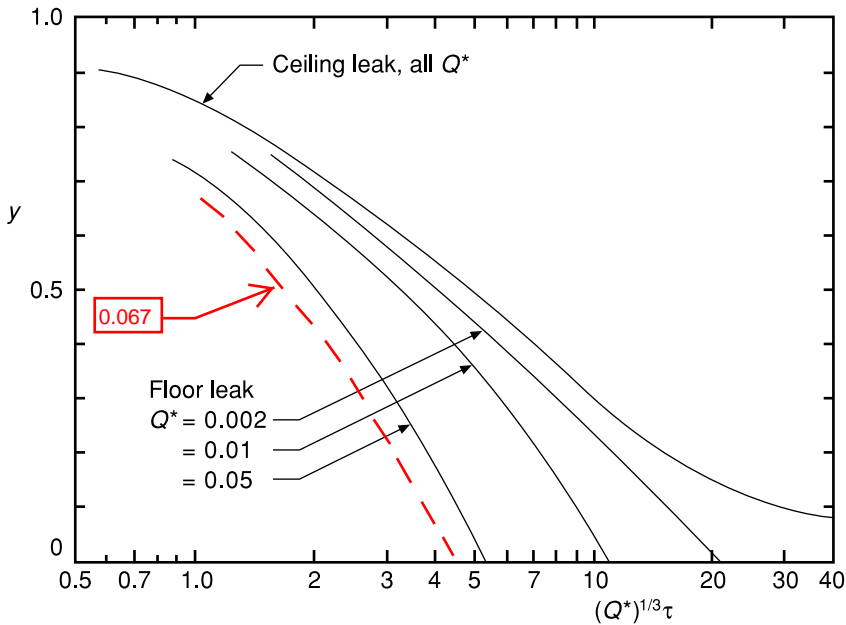


FIGURE 8.9 Dependence of ceiling layer height on time and heat release rate: solution of Eq. (8.39). (Adapted from Zukoski³.)

Dimensionless time is given the symbol τ and is defined as

$$\tau = t \sqrt{\frac{g}{H} \frac{H^2}{S}} \quad (8.38)$$

where t is time in seconds, $g = 9.81 \text{ m/s}^2$, H is the room height, and S the floor area.

Differential equation for smoke-filling time in dimensionless form: Equations (8.36), (8.37), and (8.38) can now be used to rewrite Eq. (8.35). After some manipulations we find that Eq. (8.35) can be rewritten in dimensionless form as

$$\frac{dy}{d\tau} + \dot{Q}^* + 0.21(\dot{Q}^*)^{1/3} y^{5/3} = 0 \quad (8.39)$$

Equation (8.39) cannot be solved analytically, but a numerical solution for a range of values of the dimensionless heat release rate, \dot{Q}^* , is shown in Figure 8.9, which gives the dimensionless height to the smoke layer vs. the parameter $(\dot{Q}^*)^{1/3} \tau$.

Calculational procedure: Assume that we are interested in knowing how much time it will take for the smoke layer to reach a certain height z in a room with given dimensions and a fire with a constant heat release rate. We must then

1. Calculate the dimensionless heat release rate, \dot{Q}^* , using Eq. (8.37).
2. Calculate the dimensionless height, y , using Eq. (8.36).
3. Read the value of the parameter $(\dot{Q}^*)^{1/3} \tau$ from Figure 8.9 for a given y and \dot{Q}^* and solve for τ .
4. Use this value of τ to calculate the time t using the definition of dimensionless time, τ , in Eq. (8.38).

EXAMPLE 8.3

A pool of kerosene is ignited, releasing 186 kW, in a building with floor area of 5.62 m by 5.62 m and a height of 5.95 m. Calculate the time until the smoke layer has filled half the room.

SUGGESTED SOLUTION

The dimensionless heat release rate is found using Eq. (8.37), $\dot{Q}^* = \frac{186}{1100 \cdot 5.95^{5/2}} = 0.002$.

The dimensionless height we are interested in is half the room height ($z = 2.975$ m), and Eq. (8.36) gives $y = 2.975/5.95 = 0.5$ (which of course is half the room height).

Figure 8.9 gives the factor $(\dot{Q}^*)^{1/3} \tau \approx 4$, and we can then calculate $\tau = \frac{4}{0.002^{1/3}} = 31.7$. The

time t can be solved from Eq. (8.38) as $\tau = t \cdot \sqrt{\frac{9.81}{5.95} \cdot \frac{5.95^2}{5.62^2}} = 1.44 \cdot t$. Therefore, $t = 31.7/1.44 = 22$ seconds. It therefore takes less than half a minute for a 186 kW fire to fill half the room with smoke. Figure 8.10 shows the results for other heights.

Comparison with experiments: Hägglund et al. conducted several experiments in a room of floor area of 5.62 m by 5.62 m and a height of 6.15 m.⁴ The room was closed except for a 0.25 m high by 0.35 m wide opening near the floor. The fire source was placed 0.2 m above the floor and we therefore use an effective room height of 5.95 m. The fuel was kerosene burnt in pans of various sizes with heat releases varying from 30 to 390 kW. Figure 8.10 shows the results from one such experiment where the steady-state heat release rate was given as 186 kW.

The figure shows very typical results from such comparisons. It shows that the simple Zukoski smoke-filling model³ overestimates the rate at which smoke fills the room. The main reasons for this are as follows:

- In the experiments it took up to 1 minute for the heat release rate to reach the steady-state value of 186 kW.
- Additionally, it takes time for the smoke to travel to the ceiling and spread out over it. The model treats the plume simply as a pipeline where smoke is instantaneously transported to the ceiling and across the ceiling area.
- Using the Zukoski model in this comparison we have ignored heat losses to the surrounding structure. This can be taken into account by reducing the heat release rate by some fraction of the heat release rate in the term for the enthalpy flow.

8.5.2 SMALL LEAKAGE AREAS AT CEILING LEVEL

If the leakage area is at the ceiling level, the problem is even simpler. Since we have chosen our control volume as the lower layer, the mass balance is simplified and the only mass leaving the lower layer does so through the plume.

This means that the second term in the equation for the conservation of mass is canceled, and Eq. (8.33) can be rewritten as

$$\frac{d}{dt}(\rho_a z S) + \dot{m}_p = 0$$

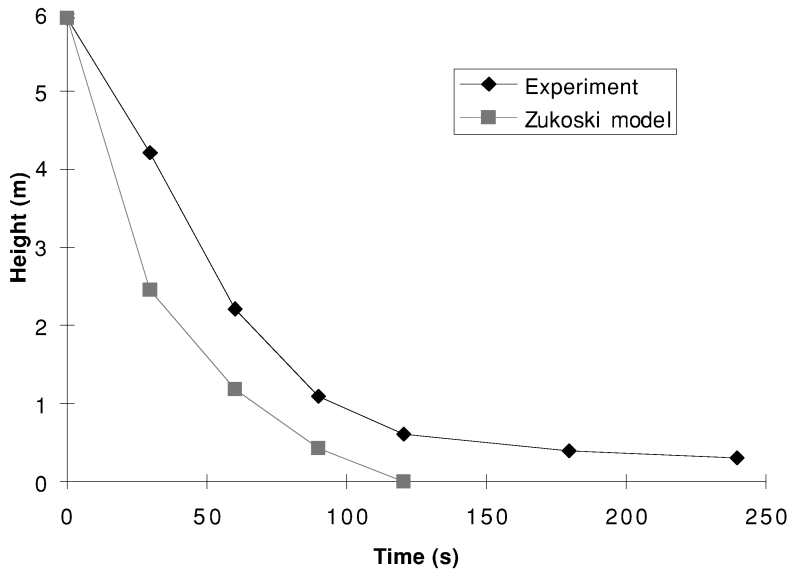


FIGURE 8.10 Comparison of experimentally observed and calculated smoke layer heights.

We can show that this also simplifies the differential equation for smoke filling (in dimensionless form) so that the second term in Eq. (8.39) is canceled and the equation can be rewritten as

$$\frac{dy}{d\tau} + 0.21(\dot{Q}^*)^{1/3} y^{5/3} = 0 \quad (8.40)$$

This differential equation can be solved immediately to give

$$y = \left[1 + \frac{2 \cdot 0.21}{3} (\dot{Q}^*)^{1/3} \tau \right]^{-3/2} \quad (8.41)$$

However, [Figure 8.9](#) also provides the solution for this case, and the calculational procedure given for the floor leak case can also be applied here.

EXAMPLE 8.4

Consider Example 8.3, but assume that the leakage opening is near the ceiling. Calculate the time taken to fill half the room volume.

SUGGESTED SOLUTION

As before, $\dot{Q}^* = 0.002$, $y = 0.5$, and $\tau = 1.44 \cdot t$. For $y = 0.5$, [Figure 8.9](#) gives a value of $(\dot{Q}^*)^{1/3} \tau \approx 4.5$. This gives $\tau = \frac{4.5}{0.002^{1/3}} = 35.6$, and therefore $t = 35.6/1.44 = 25$ seconds.

Using Eq. (8.41) instead gives $\tau = \frac{0.5^{-2/3} - 1}{0.14 \cdot 0.002^{1/3}} = 33.3$ and $t = 33.3/1.44 = 23$ seconds.

The reason for the slight difference in using Eq. (8.41) and using the graphical solution is that we have used the plume constant 0.21 in the equation, whereas Zukoski used the slightly

lower constant of 0.19 when constructing Figure 8.9.³ Considering the crudeness of this method in general and the purposes to which we shall apply it, these differences are considered to be negligible.

8.5.3 ESTIMATING GAS TEMPERATURES FOR THE FLOOR LEAK CASE

In Chapter 6 we discussed methods for calculating gas temperatures for the case where there was a flow of gases into and out of the compartment. We have been considering the case where there is no flow of upper layer gases out of the compartment and no flow of air into the compartment. This is referred to as the first stage in Section 5.3. Chapter 6 does not discuss methods for calculating gas temperatures for this case, and we present here a method of very roughly estimating the maximum temperature that can be attained in the upper layer.

Consider Figure 8.8, where at some time t , the upper layer is assumed to have a uniform temperature and density of T_g and ρ_g , respectively, and the lower layer has temperature and density of T_a and ρ_a . At this time the total mass of the upper layer, m , is

$$m = \rho_g V = \rho_g S(H - z) = \rho_g SH(1 - y) \quad (8.42)$$

We wish to obtain an expression for T_g or ρ_g . Since the temperature and mass in the upper layer change with time, we find it convenient to express the conservation of energy in terms of total energy added to the mass in the upper layer and the change in enthalpy of this mass over time 0 to t .

The first law of thermodynamics for this case can be stated as

$$Q - W = m\Delta h \quad (8.43)$$

where Q is the total heat added [in J] to the mass in the upper layer over time 0 to t and can be

expressed as $\int_0^t \dot{Q} dt$, where \dot{Q} is in [W]. This includes any heat transfer losses to the boundaries; here we make the crude assumption that $\dot{q}_{\text{loss}} = 0$. If the heat release rate is constant then this term simply becomes \dot{Q} .

W is the work carried out by the mass. The work required to push the mass into the control volume is taken care of by using enthalpies instead of internal energy in the third term of Eq. (8.43). Since there is no shaft work, W can be taken to be zero.

m is the mass of the upper layer at time t and is given by Eq. (8.42).

Δh is the change in enthalpy of this mass from time 0 to t , so $\Delta h = c_p (T_g - T_a)$.

Assuming that the heat release rate is constant, we can rewrite Eq. (8.43) as

$$\dot{Q}t = \rho_g SH(1 - y)c_p(T_g - T_a) \quad (8.44)$$

We now have the thermal state of the mass expressed in terms of both T_g and ρ_g . We can use the ideal gas law; writing $T_g \rho_g = T_a \rho_a$ we can write $T_g = T_a \rho_a / \rho_g$. The temperature term in Eq. (8.44)

can then be written as $T_g - T_a = \frac{T_a}{\rho_g}(\rho_a - \rho_g)$ and Eq. (8.44) as

$$\dot{Q}t = SH(1 - y)c_p T_a (\rho_a - \rho_g) \quad (8.45)$$

We can now calculate ρ_g and therefore T_g . However, we wish to express the first term in Eq. (8.45) in terms of $\dot{Q}^* \tau$ so we can use this in connection with the smoke-filling calculations discussed in the previous sections. Using Eq. (8.37) and (8.38) we find that $\dot{Q} \cdot t = \dot{Q}^* \tau \rho_a c_p T_a H S$, and substituting this into Eq. (8.45) gives

$$\dot{Q}^* \tau = (1 - y) \left(1 - \frac{\rho_g}{\rho_a} \right) \quad (8.46)$$

and this can be used to solve for ρ_g .

EXAMPLE 8.5

Consider the case given in Example 8.3. What is the average adiabatic temperature of the gas layer after 22 seconds, when half the room is filled with smoke?

SUGGESTED SOLUTION

From Example 8.3 we know that $\dot{Q}^* \cdot \tau = 0.002 \cdot 31.7 = 0.0634$. Since $y = 0.5$ we can solve Eq. (8.46) to give $\rho_g = 1.2 \cdot \left(1 - \frac{0.0634}{0.5} \right) = 1.048$. Using Eq. (5.9) we find

$$T_g = 353/1.048 = 337 \text{ K} = 64^\circ\text{C}.$$

When comparing the results from Eq. (8.46) with experiments, we find that the calculated temperatures are grossly overestimated. Since Eq. (8.46) expresses the adiabatic temperature, all the energy supplied by the fire is assumed to be taken up by the mass in the smoke and no heat losses are accounted for. This is in addition to other assumptions made, discussed in the previous sections. Equation (8.46) therefore cannot be used for design calculations, but only for a very rough estimate of maximum possible gas temperature of the upper layer.

Further, Eq. (8.46) is valid only for the case where there is no outflow of smoke or air from a compartment. This is referred to as the first stage in Section 5.3. For other cases the methods for calculating temperatures discussed in Chapter 6 should be used.

8.5.4 LIMITATIONS

The methods discussed in this section have several limitations. During the process of developing simple solution methodologies we have had to make a number of limiting assumptions. We must keep the following in mind:

- The fire is considered as a point source of heat only.
- The mass rate of the fuel is ignored.
- The plume is modeled as a pipeline of no volume, where mass is instantaneously transferred from the lower layer to the upper.
- The plume equation used (Zukoski plume) is valid for a weak fire source.
- No account is taken of heat losses to walls and ceiling.
- The rate of pressure rise is ignored so $dP/dt = 0$.
- No account is taken of hydrodynamic pressure differences with height; the pressure is assumed to have a single value P in the whole compartment.

With respect to the weak fire source limit, we can postulate that the method loses validity if the flame reaches the ceiling. This can be expressed numerically by considering the flame length, expressed by Eq. (4.3) as $L = 0.235 \dot{Q}^{2/5} - 1.02 D$. Taking $D = 0$ allows us to express a minimum room height in terms of heat release rate as

$$H^{5/2} > 0.23^{5/3} \dot{Q}$$

This can be expressed in terms of \dot{Q}^* since $\dot{Q}^* = \dot{Q} = \frac{\dot{Q}}{H^{5/2}} \cdot \frac{1}{\rho_a c_p T_a \sqrt{g}}$. Inserting this into the above relation and using common properties for air we find that $\dot{Q}^* < 0.036$ if the flame height does not reach the ceiling. Taking into account the fact that $D \neq 0$ we can expand this limit somewhat.

Our conclusion is that the methodologies discussed in this section should be used only if

$$\dot{Q}^* < 0.05 \tag{8.47}$$

The other limitations mentioned above should also be carefully considered.

8.6 SMOKE CONTROL IN LARGE SPACES

In a compartment with larger openings there will be little or no build-up of pressure due to the volumetric expansion of hot gases. Except for rapid accumulation of mass or energy, or for compartments with small openings, this pressure rise is small, and the pressure nominally remains at the ambient pressure. The opening flows are thus determined by the hydrostatic pressure differences across the openings, and there will be mass flow out of and into the compartment. In Chapter 5 we referred to this stage of the fire as the *third stage*.

In this section we assume that the openings and the leakage areas from the compartment to the surroundings are sufficiently large to prevent any build-up of pressure due to the volumetric expansion of hot gases.

The conservation equations for mass and energy will usually have to be solved simultaneously by computer. The equations can be solved separately if, for example, the temperature is assumed to be an average constant value throughout the process. This will allow us to express the smoke filling process as a function of time.

Alternatively, we can concentrate on smoke control as opposed to smoke filling, where the accumulation of smoke in the space is controlled by some means, such as by opening a ceiling vent to exhaust the smoke. This will allow us to examine the problem at long times, when steady state has been established. We can then solve the conservation equations by iteration.

We first consider the smoke filling problem and then study the steady-state problem for a number of smoke-control methods.

8.6.1 SMOKE FILLING: THE NON-STEADY PROBLEM

In this section we present a method for calculating the smoke-filling time for the case where no measures are taken to vent the smoke out of the compartment. This is therefore similar to the analysis presented in Section 8.5 with two main exceptions: the pressure differences causing mass flow in and out of the compartment are due to hydrostatic pressures, and we shall assume that the upper layer density, ρ_{gs} , can be taken to be some average constant value throughout the smoke-filling process. The method can therefore be used only where the space is large with respect to the heat release rate, so that the temperature rise in the upper layer can be considered to be relatively small. We discuss further the appropriateness of this assumption below.

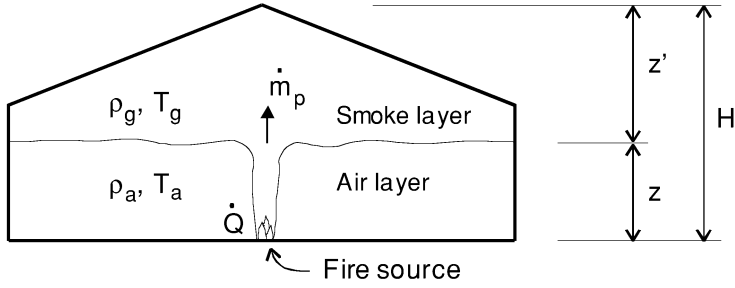


FIGURE 8.11 Schematic of the smoke filling process in a room with no venting of hot gases.

Assuming a constant average density in the upper layer for all times has the advantage that we can form an analytical solution for the smoke-filling rate, where the heat release rate does not need to be constant but can be allowed to change with time. We shall therefore stipulate a constant average value of ρ_g and use the conservation of mass to arrive at the expression for the smoke-filling rate. When this is done, the height of the smoke layer as a function of time is known and we can use the conservation of energy to check the stipulated value of ρ_g .

Consider [Figure 8.11](#) where a fire of point source \dot{Q} causes smoke filling of a compartment with height H and floor area S . At some time t , the lower layer has a thickness z and the upper layer has thickness z' , volume V_g , temperature T_g , and density ρ_g . We use the schematic and symbols in [Figure 8.11](#) to derive a simple equation for determining the upper layer interface height as a function of time.

Conservation of mass: In Section 8.5.1, where a similar case was considered, we chose our control volume as the lower layer, and therefore did not need to include ρ_g in our treatment. Here we choose our control volume as the upper layer and set up the law of the conservation of mass. The conservation of mass given by Eq. (8.3) can be written for the upper layer as

$$\frac{d}{dt}(\rho_g V_g) - \dot{m}_p = 0 \quad (8.48)$$

where the mass plume rate can be given by Eq. (4.21):

$$\dot{m}_p = 0.21 \left(\frac{\rho_a^2 g}{c_p T_a} \right)^{1/3} \dot{Q}^{1/3} \cdot z^{5/3} \quad (4.21)$$

We now make the bold assumption that ρ_g is an average constant value for all times. The first term in Eq. (8.48) (since $V_g = S z'$) can be written as $\frac{dz'}{dt} \rho_g S$. Noting that the rate at which the upper layer descends must equal the rate at which the lower layer diminishes, we can write $\frac{dz'}{dt} = -\frac{dz}{dt}$. The mass balance for the upper layer can now be written from Eq. (8.48) as

$$\frac{dz}{dt} \rho_g S + 0.21 \left(\frac{\rho_a^2 g}{c_p T_a} \right)^{1/3} \dot{Q}^{1/3} z^{5/3} = 0$$

This can be transformed to

$$\frac{dz}{z^{5/3}} = -\frac{k}{S} \dot{Q}^{1/3} dt \quad (8.49)$$

where k is a constant given by

$$k = \frac{0.21}{\rho_g} \left(\frac{\rho_a^2 g}{c_p T_a} \right)^{1/3} \quad (8.50)$$

We can now assume that the heat release rate changes as some function of time. We use the expression

$$\dot{Q} = \alpha t^n \quad (8.51)$$

where α is a growth rate factor (given in kW/s²) and n is an exponent. When $n = 0$, \dot{Q} is a constant with the numerical value of α ; when $n = 2$, we make use of the t-squared fire discussed in Section 3.4.4 and the heat release rate is given by Eq. (3.7) as $\dot{Q} = \alpha t^2$.

By substituting Eq. (8.51) into Eq. (8.49) and by integrating both sides, we get an expression for the height of the layer interface, z , in terms of time as

$$z = \left(k \frac{\alpha^{1/3}}{S} \frac{2t^{(1+n/3)}}{n+3} + \frac{1}{H^{2/3}} \right)^{-3/2} \quad (8.52)$$

For the case where $n = 0$, the heat release rate is constant with $\dot{Q} = \alpha$, and Eq. (8.52) is written as $z = \left(\frac{2k\dot{Q}^{1/3}t}{3S} + \frac{1}{H^{2/3}} \right)^{-3/2}$.

EXAMPLE 8.6a

Derive an expression for the smoke layer height using Eq. (8.49) for the case where $n = 2$ in Eq. (8.51).

SUGGESTED SOLUTION

Substituting Eq. (8.51) into Eq. (8.49) and integrating both sides we get

$$\int_H^z \frac{dz}{z^{5/3}} = -\frac{k}{S} \alpha^{1/3} \int_0^t t^{2/3} dt, \text{ since the position of the interface between the two layers goes from}$$

H to z during time 0 to t . The integral is solved to give $\frac{3}{2} \left(\frac{1}{z^{2/3}} - \frac{1}{H^{2/3}} \right) = \frac{3}{5} \frac{k}{S} \alpha^{1/3} t^{5/3}$. We can

now solve for z to give $z = \left(\frac{k}{S} \alpha^{1/3} \frac{2}{5} t^{5/3} + \frac{1}{H^{2/3}} \right)^{-3/2}$.

We now have an expression for the layer interface height as a function of time, but in the expression for the constant k the upper layer density, ρ_g , must be known. We must therefore first guess a value of ρ_g and then use the law of the conservation of energy to check our guess, and iterate forth a solution.

Conservation of energy: In Section 8.3.3 we found the conservation of energy could be written as Eq. (8.21), reproduced here:

$$\frac{d}{dt} \iiint_{CV} \rho u dV + \iint_{CS} \rho h v_n dS = \dot{Q} \quad (8.21)$$

In Section 8.4.2 we found that the first term could be written as $\frac{c_v V}{R} \frac{dP}{dt}$ and that dP/dt could be considered to be zero.

The second term in Eq. (8.21) expresses the change in enthalpy due to mass flowing into and out of the control volume. Since we have chosen the upper layer as our control volume, this is simply the enthalpy of the mass that the plume pumps into the upper layer. From the mass balance given by Eq. (8.48) we know that the plume mass flow rate is $\dot{m}_p = \frac{d}{dt}(\rho_g V_g)$. The change in enthalpy is written as $h = c_p (T_g - T_a)$ and the second term can thus be expressed as $\frac{d}{dt}(\rho_g V_g) c_p (T_g - T_a)$.

The third term gives the net energy added to the control volume. For the moment we ignore the heat losses to the boundaries, but we will take into account the heat losses in the next section.

The energy balance can now be written as

$$\frac{d}{dt}(\rho_g V_g) c_p (T_g - T_a) = \dot{Q} \quad (8.53)$$

When considering Eq. (8.48) we found that the term $\frac{d}{dt}(\rho_g V_g)$ could be written as $-\frac{dz}{dt} \cdot \rho_g \cdot S$. Expressing the heat release rate as $\dot{Q} = \alpha t^n$, substituting this into Eq. (8.53), and integrating from height H to z and from time 0 to t we find

$$(H - z) \rho_g S c_p (T_g - T_a) = \frac{\alpha t^{n+1}}{n+1} \quad (8.54)$$

In Section 8.5.3 we expressed the conservation of energy of the upper layer as Eq. (8.44), where the heat release rate was assumed to be constant. Setting $n = 0$ in Eq. (8.54) reduces the last term to \dot{Q} and we see that Eq. (8.44) and (8.54) are equivalent.

We now have an expression in terms of z , ρ_g , and T_g . We can use the ideal gas law to express T_g in terms of density, using Eq. (5.9) so that $T_g = 353/\rho_g$. Substituting this into Eq. (8.54) results in

$$\rho_g = \rho_a \left(1 - \frac{\alpha t^{n+1}}{(n+1)(H-z)S c_p 353} \right) \quad (8.55)$$

The above energy balance does not take into account any heat losses to the boundaries, and Eq. (8.55) therefore gives maximum possible temperatures (lowest possible values of density) of the upper layer. All the released energy is assumed to be contained and stored in the upper layer gases. The same assumptions were made when deriving Eq. (8.46). When comparing these equations with experiments we find that they grossly overestimate the upper layer temperature, especially for long times. Equation (8.55) therefore can be used to give a rough (minimum) estimate of the upper layer density at relatively short times.

The constant densities assumption requires that the rate of temperature rise is not excessive, and this approach therefore gives best results for compartments that are large in relation to the heat being released.

Calculational procedure:

1. Give ρ_g a guess value. This should be in the vicinity of 1.0.
2. Calculate the constant k from Eq. (8.50).
3. Calculate z at the some time t from Eq. (8.52).
4. Check ρ_g from Eq. (8.55). Note the limitation in validity of Eq. (8.55) for long times.

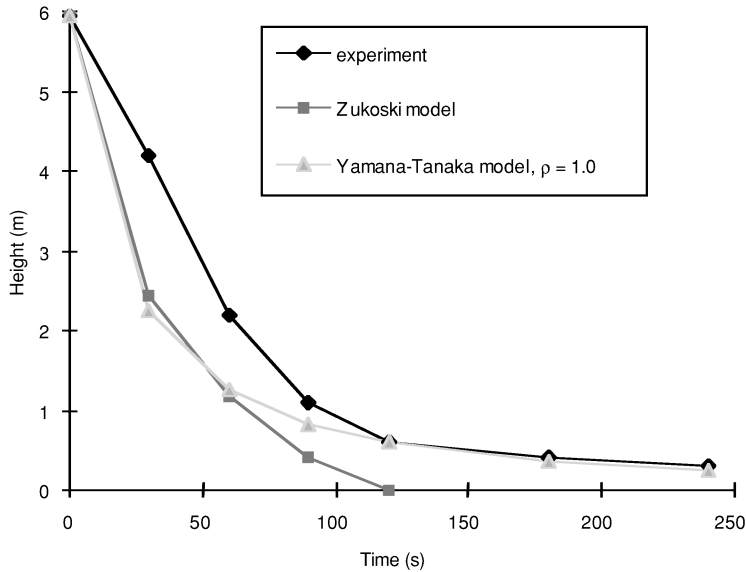


FIGURE 8.12 Comparison of experimentally measured interface height with the Zukoski³ model and the Yamana–Tanaka⁵ model.

EXAMPLE 8.6b

Use the information in Example 8.3 to calculate the smoke layer height at time $t = 60$ seconds using the Yamana–Tanaka model.

SUGGESTED SOLUTION

We give ρ_g a guess value of 1.0 kg/m^3 . Equation (8.50) gives $k = \frac{1}{1.0} \left(\frac{1.2^2 \cdot 9.81}{1.0 \cdot 293} \right)^{1/3} \cdot 0.21 = 0.0764$. For $\dot{Q} = 186 \text{ kW}$, and $n = 0$, Eq. (8.52) gives z for $t = 60 \text{ s}$ as

$$z = \left(\frac{2 \cdot 0.0764 \cdot 186^{1/3} \cdot 60}{3 \cdot 5.62 \cdot 5.62} + \frac{1}{5.95^{2/3}} \right)^{-3/2} = 1.26 \text{ m.}$$

Using Eq. (8.55) to check ρ_g we get $\rho_g = 1.2 \left(1 - \frac{186 \cdot 60}{(5.95 - 1.26) \cdot 5.62^2 \cdot 1.0 \cdot 353} \right) = 0.94 \text{ kg/m}^3$.

This is the lowest possible value of the upper layer density at $t = 60 \text{ s}$, since no account is taken of heat losses. Our assumption of $\rho_g = 1.0 \text{ kg/m}^3$ is therefore reasonable.

Comparison with experiments: In Section 8.5.1 (Figure 8.10) we compared the Zukoski model with an experiment in a room of size 5.62 by 5.62 floor area and a height of 5.95 m . Comparing the Yamana–Tanaka model with the same experiment gives the results presented in Figure 8.12.

Yamana and Tanaka also carried out a series of experiments in a large room with a floor area of 720 m^2 and a height of 26.3 m .⁶ One of the experiments involved smoke filling with no outlet for the hot gases, with a heat release rate of $\dot{Q} \approx 1300 \text{ kW}$. The smoke layer interface height was arrived at from three sources: measurements of temperature profile, optical smoke density profile, and observation by eye.

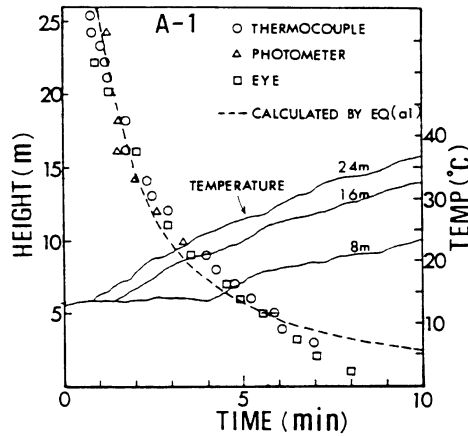


FIGURE 8.13 Height of the interface and temperature at a height of 24 m measured by Yamana and Tanaka⁶ compared to calculated interface height, assuming $\rho_g = 1.0 \text{ kg/m}^3$, using Eq. (8.52), where the calculated results have been shifted by 1 minute. (From Yamana and Tanaka⁶. With permission.)

The results are shown in Figure 8.13, where the calculated values using the above equations are shown by the dotted line. The density of the smoke layer was assumed to be 1.0 kg/m^3 . There is one deviation from the procedure outlined above: the calculated values were shifted by 1 minute. This was to account for the time taken to transport the smoke to the upper layer and across the ceiling and the time it takes for the heat release rate to attain its steady value of 1300 kW.

Figure 8.13 also shows the temperature measured at a height of 24 m. The temperature increase is relatively low, and assuming a constant average value of ρ_g is reasonable. The smoke-filling curve shown by the dotted line is referred to as the “standard filling” curve by Yamana and Tanaka,⁶ and we shall use the “standard filling” curve in later sections for comparisons.

8.6.2 SMOKE CONTROL: THE STEADY-STATE PROBLEM

In the previous section we presented the conservation of mass (Eq. (8.48)) and the conservation of energy (Eq. (8.53)) as two differential equations where the term $\frac{d}{dt}(\rho_g \cdot V_g)$ appears in both equations. For correct solutions the two equations should be coupled and solved numerically by computer. We achieved analytical solutions by assuming the density of the upper layer to be a constant average value throughout the smoke-filling process.

We now assume that some smoke-control measure has been taken where the hot gases are vented out through an opening and that the system reaches steady state at some point, i.e., that the mass rate of smoke being vented out equals the mass rate being pumped into the hot layer. We will therefore consider the mass and energy balances for the steady state, i.e., when time goes to infinity.

This will allow us to solve the mass balance and the energy balance separately, and we can then iterate forth a steady-state solution of the problem. This is what Yamana and Tanaka did and we will follow their methodology.³

Yamana and Tanaka considered a number of smoke-control measures, and we discuss three of the cases in this section:

- Case 1: Smoke control by means of a ceiling vent or an opening in the upper layer.**
- Case 2: Smoke control by means of mechanically venting the smoke from the upper layer.**
- Case 3: Pressurization of the lower layer by mechanical ventilation.**

Before we introduce these cases we will discuss the mass and energy balances to be used.

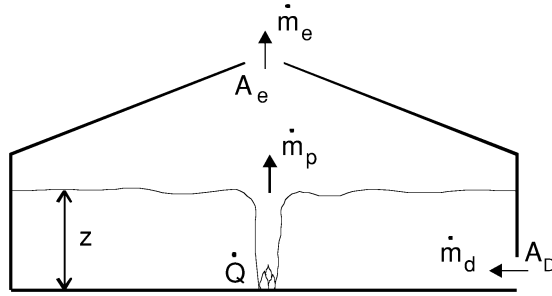


FIGURE 8.14 A room with a vent in the upper part.

Conservation of mass: Consider Figure 8.14 where a room has an inlet opening A_D at the lower part, with steady-state mass flow rate \dot{m}_d , and an outlet opening A_E at some height H_E (above the neutral layer) with a steady mass flow rate out of \dot{m}_e . The density and temperature of the upper layer are denoted ρ_g and T_g and of the lower layer ρ_a and T_a , respectively.

Rewriting Eq. (8.48), the conservation of mass for the upper layer can be written as

$$\frac{d}{dt}(\rho_g V_g) - \dot{m}_p + \dot{m}_e = 0 \quad (8.56)$$

Since we are considering the steady-state case we assume that the interface height is steady at some height z and therefore $dV_g/dt = 0$. We also assume that the temperature of the upper layer has attained a constant value and therefore that $d\rho_g/dt = 0$.

The mass balance for the upper layer can thus be written simply as

$$\dot{m}_p = \dot{m}_e$$

The same equations can be set up for the lower layer, resulting in

$$\dot{m}_d = \dot{m}_p$$

These equations are combined to give

$$\dot{m}_d = \dot{m}_p = \dot{m}_e \equiv \dot{m} \quad (8.57)$$

Note that in the above we have neglected any mass production in the room, i.e., we have neglected the burning rate of the fuel. We have shown, in Example 5.4, that even when the burning rate is very substantial, this has a very small effect on the mass flow rate through the openings.

In Chapter 5 we derived expressions for the mass flow into and out of openings and we will here use these, together with the expression for plume flow rates to calculate the steady-state layer interface height. However, we need to know the density of the upper layer as input to these equations and we must therefore set up the conservation of energy.

Conservation of energy: We have assumed that at steady state the temperature in the upper layer remains constant. Therefore, the energy produced in the upper layer, \dot{Q} , must equal the energy lost to the boundaries and the energy lost due to hot gas leaving the upper layer. The energy balance can thus be stated

$$\dot{Q} = \dot{m}_e c_p (T_g - T_a) + \dot{q}_{\text{loss}} \quad (8.58)$$

In Section 6.3.1 we set up the same energy balance, and Eq. (8.58) is identical to Eq. (6.2). As before, the term \dot{q}_{loss} is the part of the energy produced that is lost to the enclosing surfaces of the compartment. This can be expressed as the net radiative and convective heat transfer from the upper layer to the surfaces that are in contact with the upper layer. This surface area is here denoted A_w and the heat will be conducted into these surfaces.

Note that in Section 6.3 we did not know the term A_w , since we did not calculate the height of the smoke layer. In Section 6.3, we therefore used the total boundary surface area, A_T , and used experimental correlations to express the gas temperature. Here, we calculate the smoke layer height and will therefore know A_w . Further, we assume that the plume flow rate at a certain height is the same as the flow rate of hot gases out through openings. We therefore know the terms in the energy balance expressed by Eq. (8.58), and the temperature can be calculated directly.

Heat losses to boundaries: For calculation of the heat lost to the boundaries we shall use an approach similar to that taken in Section 6.3.3, summarized here for completeness: The simplest way of taking into account the heat conducted into the wall is to neglect any cooling from the boundaries and assume that the boundary surfaces have a temperature $T_s = T_g$. Solving the general heat conduction equation for this case, assuming that the boundaries are semi-infinite in thickness, results in an expression for the heat flux per unit area into the wall given by

$$\dot{q}'' = \frac{1}{\sqrt{\pi}} \sqrt{\frac{k\rho c}{t}} (T_g - T_a) \quad (8.59)$$

where k , ρ , c are the conductivity, density, and specific heat of the compartment surface material, respectively. We can then write the last term in Eq. (8.58) as

$$\dot{q}_{\text{loss}} = hA_w(T_g - T_a) \quad (8.60)$$

where

$$h = \sqrt{\frac{k\rho c}{\pi t}} \quad (8.61)$$

for a semi-infinite solid. Since we are interested only in the smoke-filling period, which lasts only a number of minutes, the semi-infinite solid assumption will be sufficient. For very thin constructions the steady-state solution of the heat conduction equation gives $h = k/\delta$, where k is the conductivity of the solid material and d is its thickness.

The expression for h , the effective heat conduction coefficient given by Eq. (8.61), is similar to the corresponding term h_k , used by McCaffrey, Quintiere, and Harkleroad (see Chapter 6, Reference 1), and given by Eq. (6.15). The only difference is that in the latter case the constant $\frac{1}{\sqrt{\pi}}$ is omitted, since the McCaffrey et al. method for calculating gas temperatures is based on experiments, and this constant is taken up in the correlation coefficients.

Material properties for the boundaries, $k\rho c$, can be taken from [Table 6.1](#) or from handbooks.

Expression for the gas temperature: We can now use the conservation of energy, Eq. (8.58), to express the upper-layer gas temperature as

$$T_g = T_a + \frac{\dot{Q}}{c_p \dot{m}_e + hA_w} \quad (8.62)$$

In the next three sections we use the above expressions when discussing the three cases mentioned earlier.

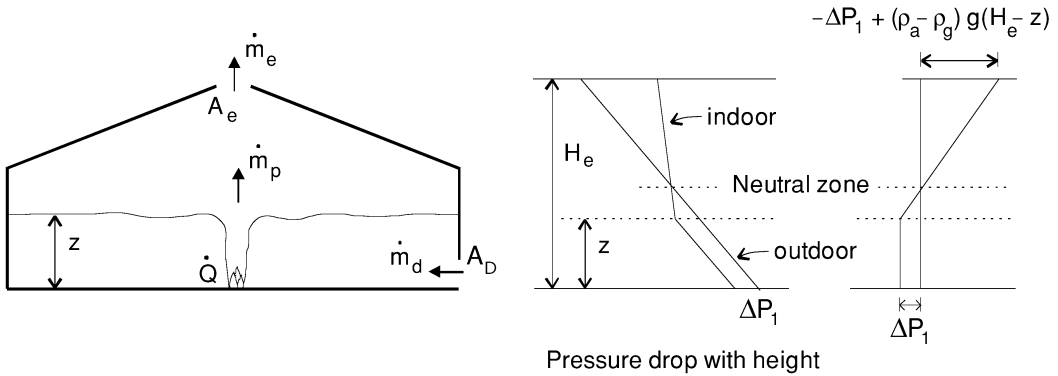


FIGURE 8.15 Schematic of a compartment with natural smoke ventilation from the upper layer and the resulting pressure differences.

8.6.3 CASE 1: NATURAL VENTILATION FROM UPPER LAYER

Here we set up a methodology where we guess the height of the layer interface, solve the mass balance and energy balance separately, and iterate forth a correct value for the steady-state interface height.

Consider [Figure 8.15](#), where \dot{m}_d and \dot{m}_e are the steady-state mass flow rates into and out of the room and \dot{m}_p is the plume mass flow rate into the upper layer. According to the mass balance (Eq. (8.57)) these are all equal and can be denoted \dot{m} .

In [Section 5.5](#) we considered mass flows through openings where there is an upper hot layer of gases and a lower cold layer. We developed expressions for the pressure difference across the opening and the rate of mass flow through these, in terms of H_N , the height of the neutral layer. Here, we express the flows in terms of the pressure difference across the openings and we use the expressions given in [Section 5.5](#) to do this.

Pressure difference across the lower opening, ΔP_1 : Using the expressions discussed in [Chapter 5](#) we know that the mass flow through the lower opening can be written

$$\dot{m}_d = C_d \rho_a v_d A_D \quad (8.63)$$

where C_d is the flow coefficient (typically ≈ 0.6) and v_d is the velocity of gases through the opening. We are interested in expressing the pressure difference across the opening, ΔP_1 , in terms of the mass flow, \dot{m}_d . From the Bernoulli equation we get $\Delta P_1 = \frac{1}{2} \rho_a v_d^2$ so that $v_d = \sqrt{\frac{2\Delta P_1}{\rho_a}}$. Inserting this into the Eq. (8.63) and solving for ΔP_1 gives

$$\Delta P_1 = \frac{\dot{m}^2}{2\rho_a (C_d A_D)^2} \quad (8.64)$$

Pressure difference across the upper opening, ΔP_u : For the upper opening we know that the mass flow rate can be expressed as

$$\dot{m}_e = C_d \rho_g v_e A_E \quad (8.65)$$

Again, we seek an expression for the pressure difference across this opening, ΔP_u . This can be expressed as the hydrostatic pressure difference $\Delta P_u = (\rho_a - \rho_g) g (H_E - H_N)$, where $(H_E - H_N)$ is the height from the neutral layer to the opening (see [Figure 8.15](#)).

Since H_N , the neutral layer height, is not known, we can express this as the total pressure difference across both openings minus the pressure difference across the lower opening. The total pressure difference across both openings is expressed as $(\rho_a - \rho_g) g (H_E - z)$, and we can therefore write

$$\Delta P_u = (\rho_a - \rho_g)g(H_E - z) - \Delta P_l \quad (8.66)$$

Mass flow rate through the upper opening: We can now use Eq. (8.65), the relationship

$v_e = \sqrt{\frac{2\Delta P_u}{\rho_g}}$ and Eq. (8.66) to express the mass flow rate through the upper opening as

$$\dot{m}_e = C_d A_E \sqrt{2\rho_g \left(-\Delta P_l + (\rho_a - \rho_g)g(H_E - z) \right)} \quad (8.67)$$

We can now proceed with our calculations.

Computational procedure:

1. Give z a guess value.

2. Calculate $\dot{m} = \dot{m}_p = 0.21 \left(\frac{\rho_a^2 g}{c_p T_a} \right)^{1/3} \dot{Q}^{1/3} z^{5/3}$. (4.21)

For $T_a = 293\text{K}$ and $\rho_a = 1.2 \text{ kg/m}^3$, $\dot{m}_p = 0.076 \dot{Q}^{1/3} z^{5/3}$.

3. Calculate $\Delta P_l = \frac{\dot{m}^2}{2\rho_a (C_d A_D)^2}$. (8.64)

4. Calculate $T_g = T_a + \frac{\dot{Q}}{c_p \dot{m}_e + h A_w}$. (8.62)

5. Calculate $\rho_g = 353/T_g$. (5.9)

6. Calculate $\dot{m}_e = C_d A_E \sqrt{2\rho_g \left(-\Delta P_l + (\rho_a - \rho_g)g(H_E - z) \right)}$. (8.67)

7. Check if $\dot{m} \approx \dot{m}_e$; if not, return to step 1.

For the special case where the lower opening A_D is very large, the pressure difference across this opening is very small. This simplifies Eq. (8.67) where ΔP_l can be taken to be ≈ 0 . If the lower opening is very small, the pressure difference across it is very large. The expression under the root sign in Eq. (8.67) can then become negative, indicating that a steady-state layer height at the guessed value of z is not possible. A new, lower value of z must then be assumed and the calculations repeated.

The value of the effective heat conduction coefficient, h , must be approximated by some means, since time is involved in Eq. (8.61). The time at which steady state is achieved can typically be taken as roughly 10 minutes. For a concrete wall, the effective heat conduction coefficient at time

10 minutes can be estimated using Eq. (8.61) and [Table 6.1](#) to be $h = \sqrt{\frac{2 \cdot 10^6}{\pi \cdot 600}} = 33 \text{ W/m}^2\text{K} = 0.033 \text{ kW/m}^2\text{K}$. For a more insulating material, fiber insulating board, the value is ten times less at $0.003 \text{ kW/m}^2\text{K}$.

The value of A_w is given as the ceiling area plus the part of the walls that are in contact with the smoke layer. Also note that in Eq. (8.62), when using \dot{Q} in [kW], h must be given in [kW/(m²K)] and c_p in [kJ/(kg K)].

EXAMPLE 8.7

Calculate the required ceiling vent area for an industrial building of floor area $30 \times 40 \text{ m}^2$ and height 10 m if the smoke layer is not to sink lower than 6 m above the floor. An appropriate design fire for this case was found to be 1000 kW, and in case of fire, a door of area 4 m^2 is opened at the floor level. The boundaries can be assumed to be concrete. The ceiling vents are assumed to open a few minutes after the fire starts.

SUGGESTED SOLUTION

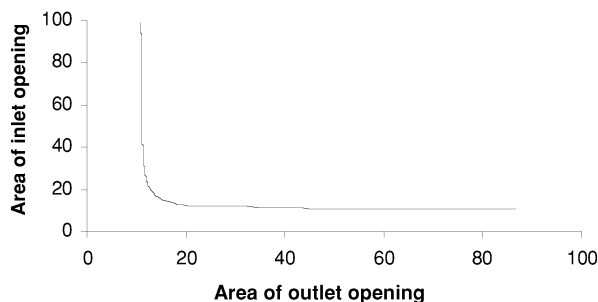
We follow the calculational procedure suggested above. The value of z is given as 6 m. Assuming $T_a = 20^\circ\text{C}$, the plume mass flow rate is calculated as

$$\dot{m} = \dot{m}_p = 0.21 \left(\frac{1.2^2 \cdot 9.81}{1.0 \cdot 293} \right)^{1/3} \cdot 1000^{1/3} \cdot 6^{5/3} = 15.14 \text{ kg/s.}$$

The pressure difference at the lower opening is $\Delta P_1 = \frac{15.14^2}{2 \cdot 1.2(0.6 \cdot 4)^2} = 16.6 \text{ Pa}$. For concrete, the effective heat conduction coefficient can be taken to be $\approx 0.033 \text{ kW}/(\text{m}^2 \text{ K})$ after 10 minutes. The boundaries in contact with the smoke $A_w = 30 \cdot 40 + 2 \cdot 30 \cdot (10 - 6) + 2 \cdot 40 \cdot (10 - 6) = 1760 \text{ m}^2$. The gas temperature is given as $T_g = 293 + \frac{1000}{1.0 \cdot 15.14 + 0.033 \cdot 1760} = 307\text{K} = 34^\circ\text{C}$. The density of the upper layer is then $\rho_g = 353/307 = 1.15 \text{ kg/m}^3$. Equation (8.67) can now be solved for A_E as

$$A_E = \frac{15.14}{0.6 \sqrt{2 \cdot 1.14 \left[\underbrace{-16.6 + (1.2 - 1.14) \cdot 9.81 \cdot (10 - 6)}_{\Delta P_u \text{ is negative}} \right]}}$$

which does not give a solution since the pressure difference at the upper opening, ΔP_u , becomes negative. This can be amended either by allowing the smoke layer to descend further than to 6 m height or by increasing the area of the lower opening. The figure below shows the relationship between A_D and A_E for this case. When the lower opening is assumed to be infinitely large, $\Delta P_1 \approx 0$ and A_E is recalculated to give $\approx 11 \text{ m}^2$. Choosing $A_D = 16 \text{ m}^2$ results in $\Delta P_1 = 1.04 \text{ Pa}$ and $A_E \approx 16 \text{ m}^2$. The design is therefore based on the assumption that at least 16 m^2 vent area will be opened at floor level in case of fire.



However, the figure above shows how important it is to provide ample area of inlet openings; a slight decrease in A_D may demand an extreme increase in A_E .

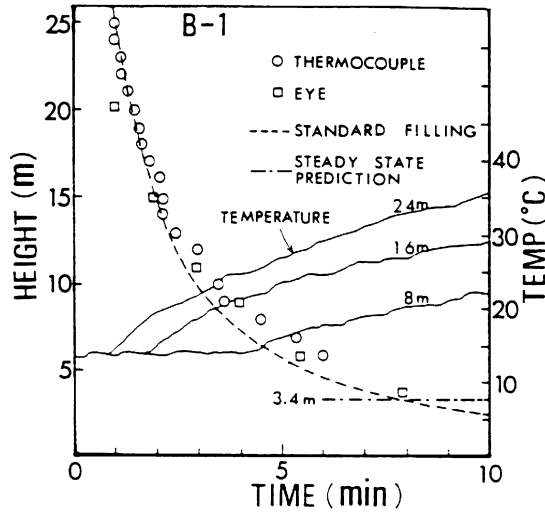


FIGURE 8.16 Smoke filling as a function of time. Steady-state layer interface height due to natural ventilation from the upper layer was calculated as 3.4 m. “Standard filling” curve shown for comparison. (From Yamana and Tanaka⁶. With permission.)

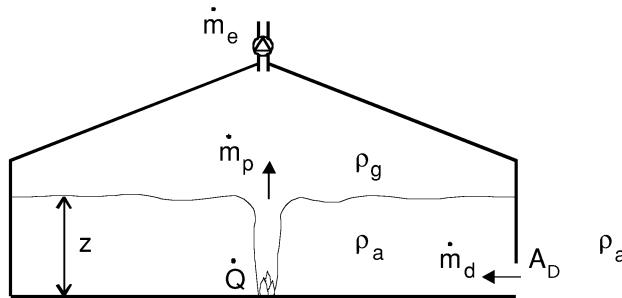


FIGURE 8.17 Schematic of a compartment with mechanical smoke ventilation.

For a large ceiling vent area there is a risk that not only smoke but also air from the lower layer will exit. A large ceiling vent area should therefore be divided into a number of vents; a rule of thumb requires each ceiling vent to be less than $2(H_E - z)^2$. In Example 8.7 this condition is fulfilled.

Comparison with experiments: The experiments by Yamana and Tanaka⁶ were summarized in Section 8.6.1. These authors also carried out experiments for Case 1: Natural ventilation from upper layer. A 1300 kW fire was allowed to smoke-fill a compartment of 26.3 m height and 720 m² floor area. The lower ventilation opening was 3.23 m² and the ceiling vent was 4.46 m². Yamana and Tanaka calculated steady-state smoke layer height for this case to be 3.4 m. The results are shown in Figure 8.16, where the layer interface height was observed by thermocouple and by eye. The dotted curve is the calculated “standard filling” curve, presented for comparison, where it is assumed that no smoke exits from the upper layer (see also Figure 8.13).

8.6.4 CASE 2: MECHANICAL VENTILATION FROM UPPER LAYER

Consider Figure 8.17, where the smoke in the upper layer is vented out by mechanical ventilation. The volumetric flow of such fans, denoted \dot{V}_e , is usually given in [m³/s] or [m³/h]. The mass flow rate out by mechanical ventilation is then given by

$$\dot{m}_e = \dot{V}_e \rho_g \quad (8.68)$$

Calculational procedure:

1. Give z a guess value.

2. Calculate $\dot{m} = \dot{m}_p = 0.21 \left(\frac{\rho_a^2 g}{c_p T_a} \right)^{1/3} \dot{Q}^{1/3} z^{5/3}$. (4.21)

3. Calculate $T_g = T_a + \frac{\dot{Q}}{c_p \dot{m}_e + hA_w}$. (8.62)

4. Calculate $\rho_g = 353/T_g$. (5.9)

5. Calculate $\dot{m}_e = \dot{V}_e \rho_g$. (8.68)

6. Check if $\dot{m} \approx \dot{m}_e$; if not, return to step 1.

EXAMPLE 8.8

Design the capacity of the mechanical ventilation system to keep the smoke layer above the height of 6 m for the building discussed in Example 8.7.

SUGGESTED SOLUTION

Example 8.7 gave the plume flow rate as 15.14 kg/s. The gas temperature and density of the upper layer were calculated to be 307K and 1.14 kg/m³, respectively. The volumetric flow rate of the fan must therefore be $\dot{V}_e = \frac{15.14}{1.14} = 13.3 \text{ m}^3/\text{s} = 48,000 \text{ m}^3/\text{h}$. This is a considerable volume flow rate. It may therefore be necessary to install a number of fans with a collective capacity of $\approx 50,000 \text{ m}^3/\text{h}$.

Comparison with experiments: Yamana and Tanaka carried out experiments for Case 2: Mechanical ventilation from upper layer.⁶ A 1300 kW fire was allowed to smoke-fill a compartment of 26.3 m height and 720 m² floor area. The smoke was removed from the upper layer by a fan of a capacity ranging from 3.2 to 6.0 m³/s. Figure 8.18 shows the results for the case where the volumetric flow of the fan is 6.0 m³/s. The steady-state smoke layer height for this case was calculated to be 3.6 m. Figure 8.18 also shows the “standard filling” curve for comparison.

8.6.5 CASE 3: LOWER LAYER PRESSURIZATION BY MECHANICAL VENTILATION

By forced ventilation into the lower layer the whole compartment becomes pressurized. This smoke control method not only results in increased pressures across the smoke vents in the upper layer, but also prevents smoke originated outside the space to enter.

Consider Figure 8.19, showing a space that is pressurized by mechanical ventilation into the lower layer. The mass flow rate from the fan is denoted \dot{m}_0 .

The mass balance for the lower layer is now written as

$$\dot{m}_0 = \dot{m}_p + \dot{m}_d \quad (8.69)$$

and for the upper layer as

$$\dot{m}_p = \dot{m}_e \equiv \dot{m} \quad (8.70)$$

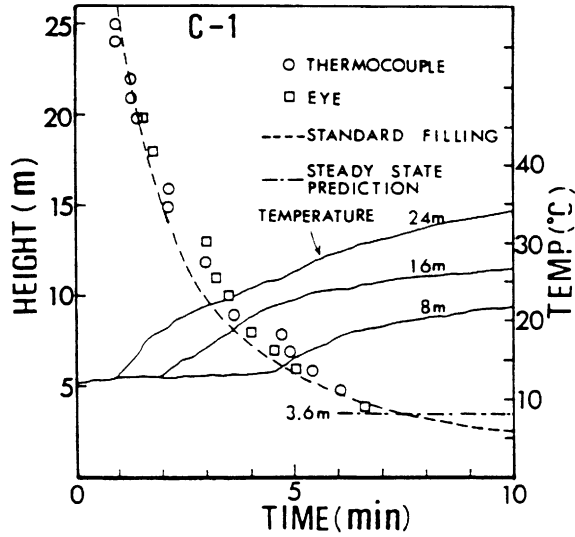


FIGURE 8.18 Smoke filling as a function of time. Steady-state layer interface height due to mechanical ventilation was calculated as 3.6 m. “Standard filling” curve shown for comparison. (From Yamana and Tanaka⁶. With permission.)

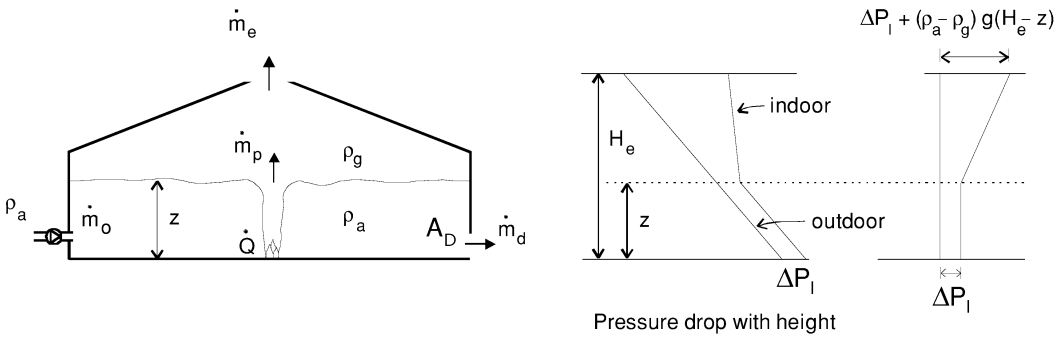


FIGURE 8.19 Schematic of a compartment pressurized by mechanical ventilation into the lower layer.

The mass flow rate through the lower opening is given by Eq. (8.63) as

$$\dot{m}_d = C_d \rho_a v_d A_D \quad (8.63)$$

Since $\dot{m}_d = \dot{m}_0 - \dot{m}_p$ and $v_d = \sqrt{\frac{2\Delta P_1}{\rho_a}}$, we can solve for ΔP_1 to give

$$\Delta P_1 = \frac{(\dot{m}_0 - \dot{m}_p)^2}{2\rho_a (C_d A_D)^2} \quad (8.71)$$

To achieve positive pressure in the lower layer requires that $\dot{m}_0 > \dot{m}_p$.

As for Case 1, we can now give an expression for the mass flow rate out through the ceiling vent as

$$\dot{m}_e = C_d A_E \sqrt{2\rho_g \left(\Delta P_1 + (\rho_a - \rho_g)g(H_E - z) \right)} \quad (8.72)$$

Note the similarity to Eq. (8.67).

Calculational procedure:

1. Give z a guess value.

$$2. \text{ Calculate } \dot{m} = \dot{m}_p = 0.21 \left(\frac{\rho_a^2 g}{c_p T_a} \right)^{1/3} \dot{Q}^{1/3} z^{5/3}. \quad (4.21)$$

$$3. \text{ Calculate } T_g = T_a + \frac{\dot{Q}}{c_p \dot{m}_e + hA_w}. \quad (8.62)$$

$$4. \text{ Calculate } \rho_g = 353/T_g. \quad (5.9)$$

$$5. \text{ Calculate } \Delta P_1 = \frac{(\dot{m}_0 - \dot{m}_p)^2}{2\rho_a (C_d A_D)^2}. \quad (8.71)$$

$$6. \text{ Calculate } \dot{m}_e = C_d A_E \sqrt{2\rho_g (\Delta P_1 + (\rho_a - \rho_g)g(H_E - z))}. \quad (8.72)$$

7. Check if $\dot{m} \approx \dot{m}_e$; if not, return to step 1.

EXAMPLE 8.9

Use the concept of lower layer pressurization to design the capacity of the mechanical ventilation system used to keep the smoke layer above the height of 6 m for the building discussed in Example 8.7. Assume a ceiling vent area of $A_E = 6 \text{ m}^2$ and an inlet opening $A_D = 4 \text{ m}^2$.

SUGGESTED SOLUTION

Example 8.7 gave the plume flow rate as 15.14 kg/s. The gas temperature and density of the upper layer were calculated to be 307K and 1.14 kg/m^3 , respectively. Since \dot{m}_0 is not given, we must use Eq. (8.72) to solve for ΔP_1 . This gives $\Delta P_1 = \frac{15.14^2}{(0.6 \cdot 6)^2 \cdot 2 \cdot 1.14} - (1.2 - 1.14) \cdot 9.81 \cdot (10 - 6) = 5.4 \text{ Pa}$. We now use Eq. (8.71) to find $\dot{m}_0 = 15.14 + \sqrt{5.4 \cdot 2 \cdot 1.2} \cdot (0.6 \cdot 4) = 23.8 \text{ kg/s}$. A standard fan may supply a flow rate of, say, $5 \text{ m}^3/\text{s} = 6 \text{ kg/s}$. Four such fans would therefore be needed in this case. Alternatively, we could decrease the area of the inlet opening or increase the area of the outlet opening, but the flow supplied by the mechanical ventilation must always exceed the plume mass flow rate of $\approx 15 \text{ kg/s}$.

Comparison with experiments: Yamana and Tanaka carried out experiments for Case 3: Lower layer pressurization by mechanical ventilation.⁶ Again, a 1300 kW fire was allowed to smoke-fill a compartment of 26.3 m height and 720 m² floor area. The lower layer was pressurized by a fan of a capacity ranging from 20 to 23.5 m³/s and the outlet opening ranged from 2.23 to 6.46 m². Figure 8.20 shows the results for the case where the volumetric flow of the fan was 20 m³/s and the outlet opening was 3.23 m². The steady-state smoke layer height for this case was calculated to be 7.5 m. Figure 8.20 also shows the “standard filling” curve, for comparison.

8.7 SUMMARY

In this chapter we discussed the enclosure fire at different stages of development. In order to summarize our findings we will use Figure 8.21 (reproduced from Figure 2.5) where we consider stages A, B, C, and D (see Section 2.3.2 and Section 5.3).

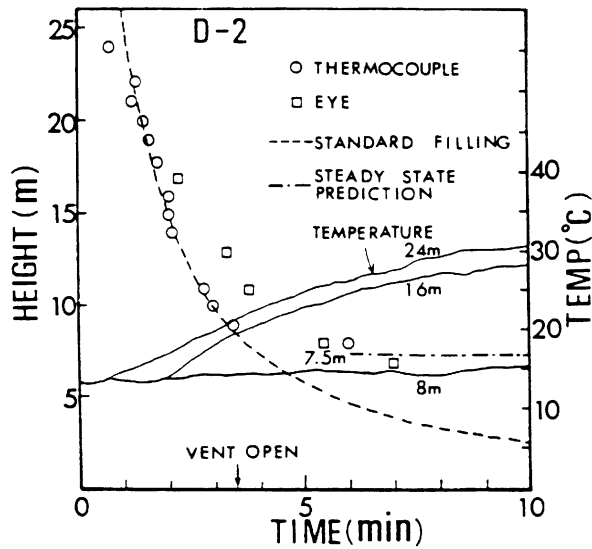


FIGURE 8.20 Smoke filling as a function of time. Steady-state layer interface height achieved by pressurization of the lower layer, calculated as 7.5 m. “Standard filling” curve shown for comparison. (From Yamana and Tanaka⁶. With permission.)

Stage A

Pressure profile: Pressure in enclosure always higher than outside. No neutral layer. Exiting flow either cold gases only (floor leak) or hot gases only (ceiling leak).

Calculation methods provided: Two cases: (a) constant \dot{Q} ; (b) $\dot{Q} = \alpha t^2$.

- (a) Hand calculations can give $H_D (= z)$ and T_g (floor leak) as a function of time and steady state ΔP and \dot{m}_g . Equations used are (8.32), (8.36)–(8.38), (8.46), and Figure 8.9. Y_{O_2} (oxygen concentration) can also be calculated, considering mass flow rate out and oxygen used for combustion.
- (b) Hand calculations can give $H_D (= z)$ and T_g (floor leak) as a function of time. T_g , ρ_g , and z are calculated by simultaneously solving the coupled Eq. (8.48) and (8.53) (the mass and energy balance). In practice, these are solved using Eq. (8.50) and (8.52) and then checking ρ_g using Eq. (8.55).

Cannot be calculated by hand: Heat losses to boundaries, \dot{q}_{loss} (adiabatic room).

Special notes: Valid for fire development until the hot layer reaches the top of the opening (when hot gases start flowing out). No heat losses are taken into account and the resulting T_g can be greatly overestimated at long times.

Stage B

Pressure profile: Pressure in enclosure always higher than outside, but exiting gas is both at temperature T_g and T_a .

Special notes: Stage B is valid if ΔP due to expansion of gases is greater than ΔP due to hydrostatic pressure difference. Stage B is usually of very short duration (few seconds if opening area is typical

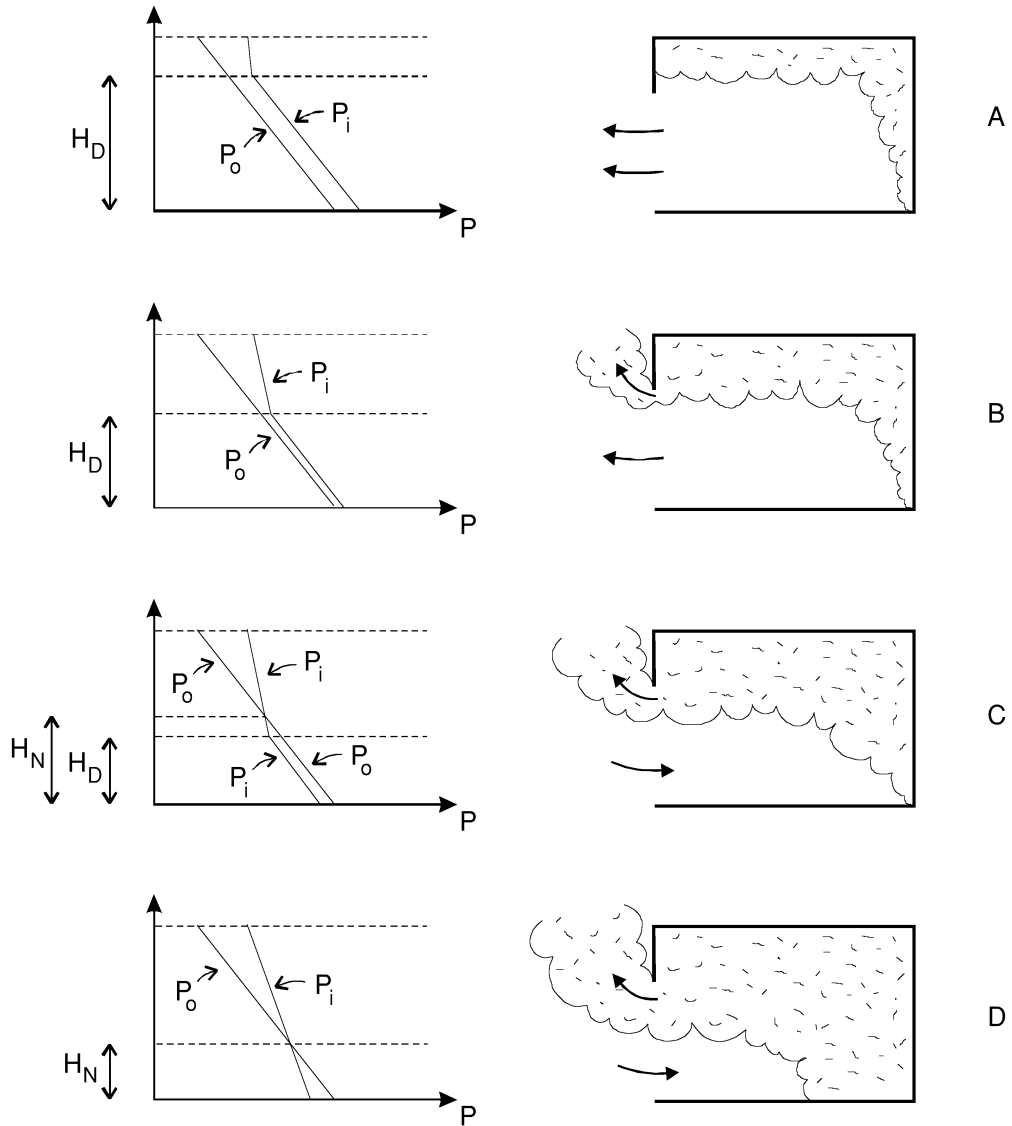


FIGURE 8.21 Pressure profile across the opening as the fire develops.

door or window), and quickly transcends to Stage C. No hand calculation methods are given here for Stage B, but many computer programs take this stage into account.

Stage C

Pressure profile: Positive pressure difference across upper part of opening, negative pressure difference across lower part (with respect to the enclosure). H_D and H_N define shape and magnitude.

Calculation methods provided: Two cases: (a) Constant \dot{Q} ; (b) Variable \dot{Q} .

- (a) Hand calculations can give *steady-state* values of $H_D (= z)$ and T_g for a number of cases (natural-, mechanical-, and positive-pressure ventilation) using Eq. (4.21), (8.62), (8.64), and (8.67)–(8.72). The steady-state ceiling vent case is also given by Eq. (5.43)–(5.46).

Alternatively, T_g as a function of time can be calculated using McCaffrey et al., Eq. (6.11). \dot{Q}_{FO} is calculated from Eq. (6.20), and \dot{q}_{loss} as a function of time can be estimated from Eq. (6.15)–(6.19).

- (b) No methods are provided for calculating H_D and H_N , one must use computer programs. T_g provided by the McCaffrey et al. correlation, Eq. (6.11), and related expressions.

Cannot be calculated by hand: H_D and H_N (but steady-state H_D can be calculated for constant \dot{Q} , as above).

Special notes: The opening is assumed so large that ΔP due to thermal expansion can be ignored. Valid for well-ventilated fire ($\dot{Q} < 1.5A_o\sqrt{H_o}$ (MW)) or until flashover occurs.

Stage D

Pressure profile: Simple pressure profile as shown in Figure 8.21. $H_D = 0$. H_N and T_g define magnitudes, but effect of T_g small for $T_g > 300^\circ\text{C}$.

Calculation methods provided: H_N , \dot{m}_g , and \dot{m}_a provided by Eq. (5.18)–(5.24). The terms in the energy balance can be calculated approximately using Eq. (6.28)–(6.30). Largest error in Eq. (6.29). Difficult to estimate how \dot{Q} varies during the growth and decay stages. When \dot{Q} is not known, Magnusson et al. provide T_g as a function of opening factor, fuel load density, and properties of bounding materials.

Cannot be calculated by hand: If \dot{Q} is known, computer programs offer solutions for T_g , where conduction through walls is more sophisticated than given by Eq. (6.29).

Special notes: \dot{Q} must be larger than \dot{Q}_{FO} . If $\dot{Q} \gg 1.5A_o\sqrt{H_o}$ (MW), this results in underventilated fires and flames through openings.

REFERENCES

1. Klote, J.H. and Milke, J.A., *Design of Smoke Management Systems*, American Society of Heating, Refrigeration and Air-Conditioning Engineers, Atlanta, GA, 1992.
2. Welty, J.R., Wilson, R.E., and Wicks, C.E., *Fundamentals of Momentum Heat and Mass Transfer*, 2nd ed., John Wiley & Sons, London, 1976.
3. Zukoski, E.E., "Development of a Stratified Ceiling Layer in the Early Stages of a Closed Room Fire," *Fire and Materials*, Vol. 2, No. 2, 1978.
4. Hägglund, B., Jansson, R., and Nireus, K., "Smoke Filling Experiments in a 6×6×6 meter Enclosure," FOA Report C 20585-D6, National Defence Research Establishment, Sweden, 1985.
5. Yamana, T. and Tanaka, T., "Smoke Control in Large Scale Spaces, Part 1: Analytical Theories for Simple Smoke Control Problems," *Fire Science and Technology*, Vol. 5, No. 1, 1985.
6. Yamana, T. and Tanaka, T., "Smoke Control in Large Scale Spaces, Part 2: Smoke Control Experiments in a Large Scale Space," *Fire Science and Technology*, Vol. 5, No. 1, 1985.
7. Drysdale, D., "Heat Transfer and Aerodynamics," *An Introduction to Fire Dynamics*, Wiley-Interscience, New York, 1992.

PROBLEMS AND SUGGESTED ANSWERS

- 8.1 In a hermetically closed machine room on a ship, a pump causes 10 l of hot transformer oil to leak out. The oil is ignited by a spark. The room has a floor area of 6 m by 4 m

and a height of 3 m. Assume that flaming combustion ceases when 40% of the oxygen content of the room has been used,

- (a) How much is then left of the transformer oil?
- (b) Calculate the pressure caused by the fire at this time.
- (c) What is the significance of the spill area?

Suggested answer: (a) 3.2 kg oil have been used, assuming combustion efficiency to be 70%, \Rightarrow 5.8 liters remain; (b) Noting that $Q \cdot t = Q$ in Eq. (8.24), $\Delta P = 5.88$ atm (not possible, no surrounding structure can withstand this pressure); (c) discuss.

- 8.2 The room described in Problem 8.1 has a 1-m-wide door with a 1-cm-high slit at the floor level. The oil spill has a rectangular area of sides 0.5 m by 0.5 m.
- (a) Calculate the pressure increase, the air velocity, and the mass flow rate through the slit.
 - (b) The slit is now made 10 times higher. How will this influence the variables calculated in (a)?

Suggested answer: (a) $\Delta P = 1060$ Pa, $v_e = 40$ m/s, $\dot{m}_e = 0.4$ kg/s; (b) ΔP is 100 times smaller, v_e is 10 times smaller, \dot{m}_e is the same.

- 8.3 A room has a floor area of 500 m² and a height of 4 m. The room is involved in a fire with a constant energy release rate of 1 MW. Calculate the time till the smoke layer is 1.5 m from the floor for
- (a) The case where the leakage areas are at the floor level.
 - (b) The case where the leakage areas are at the ceiling level.

Suggested answer: (a) 250 s; (b) 520 s.

- 8.4 A room with a floor area of 200 m² and height 6 m goes through a smoke-filling process and is filled to the height of 2 m above the floor in 3 minutes. Calculate the energy release rate that causes this, assuming leakage areas at ceiling level.

Suggested answer: 1 MW.

- 8.5 A fire occurs with an energy release rate \dot{Q} in a room of a certain geometry and a height H . The time until the smoke has reached the height $H/2$ was calculated to be 100 seconds. The same calculations are carried out in a room with the height $2H$, with all other parameters unchanged. How long time does it take for the smoke layer to reach the same absolute thickness (i.e., $H/2$)? The fire is assumed to be relatively weak and the leakage areas at the ceiling level.

Suggested answer: 21 s.

- 8.6 By making a number of assumptions, Eq. (8.30) expresses the first law of thermodynamics for the situation given in [Figure 8.8](#):

$$\frac{c_v \cdot V}{R} \frac{dP}{dt} + \dot{m}_e c_p T_e = \dot{Q} \quad (8.30)$$

Some further assumptions allow the mass flow rate out through a floor leak to be given by Equation (8.34):

$$\dot{m}_e = \frac{\dot{Q}}{c_p T_a} \quad (8.34)$$

- (a) Explain the physical meaning of the terms in Eq. (8.30) and the assumptions made for arriving at Eq. (8.34).
- (b) How is Eq. (8.34) used to derive an expression for the steady-state pressure rise?
- (c) Derive a differential equation describing the smoke-filling process depicted in [Figure 8.8](#) using Eq. (8.34), a plume equation, and the conservation of mass for the lower layer.

8.7 A room with a floor leakage has a height of 4 m and an area of 500 m². A fire with an energy release rate of 1 MW is ignited in the room. The time until the smoke layer is 1.5 m from the floor is 250 s. Give an equation for approximating the hot layer gas density at the time 250 s, when all heat losses are ignored. Calculate the corresponding gas temperature.

Suggested answer: 410°C.

8.8 In a nearly closed room a fire with a constant energy release rate of 500 kW is ignited. The room has a floor area of 100 m² and a height of 3 m. An opening near the floor is 2 m wide and 0.1 m high and one of the walls has a rectangular window with dimensions 0.9 m by 0.9 m. The flames are not in direct contact with the window. Assume a flow coefficient of 0.6.

- (a) Calculate the time until the smoke layer has reached the floor.
- (b) Will the fire go out due to oxygen deficiency in the smoke layer when the smoke layer reaches the floor? Assume the fire to go out when half the oxygen in the smoke layer has been used up.
- (c) Will the window break due to the pressure build-up? Assume that the window can withstand 1 kPa pressure rise without cracking.

Suggested answer: (a) 150 s; (b) no; (c) no.

8.9 A fire releases 100 kW in a room with a floor area of 100 m² and a height of 4 m. The room has a door which is 1 m wide and 2 m high. At the moment when the smoke layer reaches the top of the opening, the gas temperature is found to be 300°C. Ignoring any heat losses to the boundaries, calculate how long the fire has been burning.

Suggested answer: (a) 340 s.

8.10 A hotel lobby has a floor area of 6 m by 8 m and is 6 m high. A large sofa consisting mostly of polyurethane is ignited. The flame spreads across the material, causing an energy release rate that increases linearly from 0 to 2400 kW in 4 minutes, after which the sofa has burnt out.

- (a) Will the lobby be filled with smoke?
- (b) How long will it take for the smoke layer to reach the height 1 m from the floor? What will the gas layer density be at this time (ignoring heat losses to boundaries)?

Suggested answer: (a) Yes, $z < 0.3$ m for $0 < \rho_g \leq 1.2$ (because Eq. (8.55) results in a negative value, it could not be used to check ρ_g in this case); (b) through iteration, $\rho_g \approx 0.79$ kg/m³ and $t \approx 73$ s.

8.11 Do the same calculations as in Problem 8.10, using the Zukoski smoke-filling method and assuming the energy release rate to be constant at the average value of 1200 kW throughout.

Suggested answer: (a) Yes; (b) $\rho_g \approx 0.64 \text{ kg/m}^3$ and $t \approx 34 \text{ s}$.

8.12 A storage room in an industry has a floor area of 30 m by 50 m and a height of 10 m. A garage door has a width of 3 m and a height of 3 m and the room has four windows that are 0.9 m by 0.9 m, situated at a height of 1 m from the floor. The fire load consists mainly of wooden pallets ($1.2 \times 1.2 \text{ m}^2$); the maximum amount stored is two pallet loads, side by side, which are 1.4 m high. The fire is assumed to attain maximum effect after 6 minutes and firefighters are assumed to arrive at the location 15 minutes after the fire starts. Design the size of smoke vents in the ceiling with the objective of avoiding flashover and only allowing the smoke layer to be 3 m thick. Assume a flow coefficient of 0.6 for the ceiling vents and assume the walls and ceiling to be of steel sheet, isolated with mineral wool with $k_{pc} = 3.3 \cdot 10^3 \text{ W}^2\text{s/m}^4\text{K}^2$.

Suggested answer: If $\dot{Q} = 7 \text{ MW}$, then $\dot{Q}^* = 0.02 \Rightarrow$ relatively large fire. We must test both Thomas and Zukoski plumes. Thomas plume gives, for $z = 7 \text{ m}$, $C_d = 0.6$, $A_D = 9 \text{ m}^2$, $t = 6 \text{ min} \Rightarrow A_v = 9.3 \text{ m}^2$, $T_g = 310^\circ\text{C}$. Using the Zukoski plume the problem cannot be solved and z must be allowed to be a lesser value. Therefore, the choice of plume equation and inlet area is very sensitive, and the design requirement is close to a criticality. Therefore, change the design requirement or solve in alternative ways.

8.13 An atrium has a floor area of 30 m by 20 m and is 20 m high. The ceiling ventilation is designed for a fuel area of 30 m^2 with an energy release rate of 0.5 MW/m^2 . The inlet openings are very large. An effective heat conduction coefficient can be assumed to be $20 \text{ W/m}^2\text{ }^\circ\text{C}$.

- Calculate the area of ceiling vents required for ensuring a maximum thickness of the smoke layer of 5 m.
- A weak fire occurs in the atrium; a sofa burns for 3 minutes and releases energy at the constant rate of 500 kW. The ceiling vents are opened through signals from smoke detectors at the ceiling level, which activate at a temperature rise of 10°C . What will happen? Will the atrium be filled with smoke? How fast will this happen?

Suggested answer: (a) $A_E = 59 \text{ m}^2$; (b) $T_g = 28^\circ\text{C} \Rightarrow$ the smoke detectors may not activate. It will take over 10 minutes for the smoke layer to reach the height 2 m above the floor.

8.14 An industrial building is built mainly of lightweight concrete and has a floor area of 20 m by 30 m and a height of 10 m. Ceiling vents have been installed to remove smoke in case of fire. The inlet air vents consist of doors which are 6 m wide and 4 m high. The doors and the ceiling vents are assumed to open automatically in case of fire. A pool of flammable liquid on the floor has a diameter of 1.5 m and when ignited will have an energy release rate of 4600 kW. The energy release rate is assumed to increase linearly with the pool area, if the pool area increases in size. The time 5 minutes is chosen as a characteristic time.

- Calculate the area of ceiling vents required if the smoke layer is to have a thickness of maximum 3 m.
- Assume that the ceiling vents do not open but the inlet openings do. How long does it take for smoke to start flowing out through the inlet openings?

- (c) Make the same assumption as in (b). After a while, steady-state conditions will arise with regard to the flow into and out of the inlet openings. Estimate the temperature of the gases flowing out through the opening.
- (d) How large must the pool fire be (in m^2) if flashover is to be achieved? Make the calculation for two cases: the construction is made entirely of lightweight concrete and the roof is made of lightweight concrete but the floor and walls are made of concrete.

Suggested answer: (a) 15 m^2 ; (b) 106 s; (c) 180°C ; (d) 9.5 m^2 and 19.1 m^2 , respectively.

- 8.15 An industrial building has a floor area of 50 m by 30 m and is 8 m high. A 4 m^2 pool of heavy fuel oil occurs due to pipe leakage and this is ignited. An effective heat conduction coefficient can be assumed as $20 \text{ W/m}^2 \cdot ^\circ\text{C}$ and the flow coefficient is assumed to be 0.6. The smoke layer is not to sink further than 5 m from the floor.
- (a) Calculate the required area of the inlet openings if the ceiling vent is 10 m^2 .
 - (b) Calculate the required capacity of a mechanical ventilation system, designed to remove the smoke.
 - (c) Positive pressure ventilation of the lower layer is to be installed to drive out the hot smoke gases. The ceiling ventilation area is limited to 8 m^2 and the inlet openings are 13 m^2 . Calculate the required capacity of mechanical ventilation.

Suggested answer: (a) 13 m^2 ; (b) $18 \text{ m}^3/\text{s}$; (c) 23 kg/s or $19 \text{ m}^3/\text{s}$.